Semi-Deterministic Broadcast Channels with Cooperation

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Outline

- Motivation and past work
- AK problem with one-sided encoder cooperation
- SD-BC with one-sided decoder cooperation
- Duality
- Summary
Motivation and Past Work

- The two-encoder multiterminal source coding problem [Berger, 1978], [Tung, 1978].
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\[
(X_1^n, X_2^n, Y^n) \in \mathcal{T}_\epsilon^{(n)}(P_{X_1} P_{X_2}^* P_{Y|X_1,X_2})
\]
Motivation and Past Work

- The two-encoder multiterminal source coding problem [Berger, 1978], [Tung, 1978].

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\begin{align*}
X_1^n &\xrightarrow{} \text{Encoder 1} \xrightarrow{T_1(X_1^n)} \text{Decoder} \xrightarrow{Y^n} \\
X_2^n &\xrightarrow{} \text{Encoder 2} \xrightarrow{T_2(X_2^n)} \text{Decoder}
\end{align*}
\]

\[(X_1^n, X_2^n, Y^n) \in \mathcal{T}_{\epsilon}^{(n)}(P_{X_1, X_2, Y^n|X_1, X_2})\]

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X_2^n &\quad \text{Encoder 2} \\
T_1(X_1^n) &\quad T_2(X_2^n) \\
\text{Decoder} &\quad Y^n
\end{align*}
\]

\[
(X_1^n, X_2^n, Y^n) \in \mathcal{T}^{(n)}_\epsilon (P_{X_1} P_{X_2}^* P_{Y|X_1,X_2})
\]


- Add cooperation ability:
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\[ X_1^n, X_2^n, Y^n \in \mathcal{T}^{(n)}_\epsilon(P_{X_1,X_2}P^*_Y|X_1,X_2) \]


- Add cooperation ability:
  - Can boost performance.
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\[(X_1^n, X_2^n, Y^n) \in \mathcal{T}_{\epsilon}(n)(P_{X_1,X_2}P^*_Y|X_1,X_2)\]


- Add cooperation ability:
  - Can boost performance.
  - Milestone towards multiuser channel-source duality.
AK Problem with Cooperation - Definition

Without cooperation [Ahlswede-Körner, 1975]
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\[ X^n_1, X^n_2 \] are pairwise i.i.d. \( \sim P_{X_1, X_2} \).

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- **Encoder-Decoder Communication:** \(T_j \in [1 : 2^{nR_j}], j = 1, 2\).
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**Encoder-Decoder Communication:** \(T_j \in [1 : 2^{nR_j}], \ j = 1, 2\).

**Decoder Output:** \((X_1^n, X_2^n, Y^n) \in \mathcal{T}_\epsilon^{(n)}(P_{X_2} P_{Y|X_2} 1_{\{X_1 = f(Y)\}})\).
AK Problem with Cooperation - Solution

Theorem (Coordination-Capacity Region)

For a desired coordination distribution \( P_{X_2} P_{Y|X_2} 1\{X_1=f(Y)\} \):

\[
C_{AK} = \bigcup \left\{ \begin{array}{l}
R_{12} \geq I(V; X_1) - I(V; X_2) \\
R_1 \geq H(X_1|V, U) \\
R_2 \geq I(U; X_2|V) - I(U; X_1|V) \\
R_1 + R_2 \geq H(X_1|V, U) + I(V, U; X_1, X_2)
\end{array} \right\}
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where the union is over all \( P_{X_1, X_2} P_{V|X_1} P_{U|X_2, V} P_{Y|X_1, U, V} \) with \( P_{X_2} P_{Y|X_2} 1\{X_1=f(Y)\} \) as marginal.
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Achievability via Wyner-Ziv coding, superposition coding and Slepian-Wolf binning.
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Semi-Deterministic BC with Cooperation - Definition

Without cooperation [Gelfand and Pinsker, 1980]
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\( (M_1, M_2) \xrightarrow{Encoder} X^n \xrightarrow{\text{Channel}} Y_1^n, Y_2^n \xrightarrow{\text{Decoder 1}} \hat{M}_1 \)
\( \quad \times P_{Y_2|X} \)
\( \quad \times \{Y_1 = f(X)\} \)

\( (M_1, M_2) \xrightarrow{\text{Decoder 2}} \hat{M}_2 \)

**Messages:** \((M_1, M_2) \sim \text{Unif}[1 : 2^{nR_1}] \times [1 : 2^{nR_2}] \).
Without cooperation [Gelfand and Pinsker, 1980]

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- **Channel - Decoder Input:** \(Y_{1,i} = f(X_i)\) and \(Y_{2,i} \sim P_{Y_2|X}\).
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Decoder Cooperation: \(M_{12}(Y_1^n) \in [1 : 2^{nR_{12}}]\).

Decoders’ Output: \(\hat{M}_1(Y_1^n)\) and \(\hat{M}_2(M_{12}, Y_2^n)\).
Semi-Deterministic BC with Cooperation - Solution

Theorem (Capacity Region)

The capacity region is:

\[
C_{BC} = \bigcup \left\{ \begin{array}{l}
R_{12} \geq I(V; Y_1) - I(V; Y_2) \\
R_1 \leq H(Y_1) \\
R_2 \leq I(V, U; Y_2) + R_{12} \\
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Achievability via rate splitting, Marton coding and Wyner-Ziv-like coding for cooperation protocol.
Semi-Deterministic BC with Cooperation - Converse

Outline

Difficulty: Unique structure
Semi-Deterministic BC with Cooperation - Converse

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1. Outer bound the achievable region using 3 auxiliaries \((A, B, C)\).
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1. Outer bound the achievable region using 3 auxiliaries $(A, B, C)$.

2. Choose auxiliaries probabilistically as a function of the codebook:

\[
V = \begin{cases} (A, C) , & \text{w.p. } \lambda \\ \emptyset , & \text{w.p. } 1 - \lambda \end{cases} ; \quad U = (A, B, C)
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Generalization of [Lapidoth and Wang, 2013].
“There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel...”
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- The solutions of the problems are dual.
  - Information measures admit dual forms.
  - Optimization domain may vary.
- A formal proof of duality is still absent.
- Solving one problem \(\implies\) Valuable insight into solving dual.
Duality - Preface

Point-to-Point Case:
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Point-to-Point Case:

\[ X^n \rightarrow \text{Encoder} \rightarrow T(X^n) \rightarrow \text{Decoder} \rightarrow Y^n \]

\[ M \rightarrow \text{Encoder} \rightarrow X^n \rightarrow P_{Y|X} \rightarrow Y^n \rightarrow \text{Decoder} \rightarrow \hat{M} \]
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Duality - Preface

Point-to-Point Case:

\[ R^* = \min_{P_{Y|X}} I(X;Y) \]

\[ C = \max_{P_X} I(X;Y) \]
Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Encoder 1

$X_1^n$ → $T_1(X_1^n)$ → Decoder

$T_{12}(X_1^n)$

Encoder 2

$X_2^n$ → $T_2(T_{12}, X_2^n)$ → Decoder

(Y_1^n, Y_2^n) → Decoder 1 → $\hat{M}_1$

Channel

$1_{\{Y_1 = f(X)\}} \times P_{Y_2|X}$

Encoder

$(M_1, M_2)$ → $X^n$ → $Y_1^n$ → $\hat{M}_1$

Decoder 2

$Y_2^n$ → $\hat{M}_2$
AK Problem vs. Semi-Deterministic BC:

- **Encoder 1:** $X_1^n \rightarrow T_1(X_1^n) \rightarrow T_{12}(X_1^n) \rightarrow \text{Decoder} \rightarrow Y^n$
- **Encoder 2:** $X_2^n \rightarrow T_2(T_{12}, X_2^n)$

**Decoder:**
- Decoder 1: $Y_1^n \rightarrow \hat{M}_1$
- Decoder 2: $Y_2^n \rightarrow \hat{M}_2$
Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Encoder 1

\[ X_1^n \]

Encoder 2

\[ X_2^n \]

Decoder

\[ Y^n \]

Encoder

\[ (M_1, M_2) \]

Channel

\[ \left\{ Y_1 = f(X) \right\} \times P_{Y_2|X} \]

Decoder 1

\[ \hat{M}_1 \]

Decoder 2

\[ \hat{M}_2 \]
Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:
Duality - Multi-User Case

AK Problem vs. Semi-Deterministic BC:

Probabilistic relations are preserved:

\[
(X^n, Y^n_1, Y^n_2) \in \mathcal{T}_\epsilon^{(n)} \left( P_X^* \mathbb{1}_{Y_1 = f(X)} P_{Y_2 | X} \right) \iff (Y^n, X^n_1, X^n_2) \in \mathcal{T}_\epsilon^{(n)} \left( P_Y \mathbb{1}_{X_1 = f(Y)} P^*_{X_2 | Y} \right)
\]
Duality - Corner Point Correspondence

For fixed joint distributions and $R_{12}$:

\[
\begin{align*}
I(U; Y_2 | V) + I(V; Y_1) \\
I(U; Y_2 | V) - I(U; Y_1 | V)
\end{align*}
\]

\[
\begin{align*}
H(Y_1 | V, U) \quad & H(Y_1) \\
H(X_1 | V, U) \quad & H(X_1)
\end{align*}
\]
Duality - Corner Point Correspondence

For fixed joint distributions and $R_{12}$:

\[
\begin{align*}
R_2 & \quad R_1 \\
I(U; Y_2|V) + I(V; Y_1) & \quad H(Y_1|V, U) \quad H(Y_1) \\
I(U; Y_2|V) - I(U; Y_1|V) & \quad H(Y_1|V, U) \\
0 & \quad R_1 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Semi-Deterministic BC with Cooperation</th>
<th>Ahlswede-Körner Problem with Cooperation</th>
</tr>
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<tbody>
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I(U; Y_2|V) &- I(U; Y_1|V) \\
H(Y_1|V, U) & \\
H(Y_1) & \\
R_2
\end{align*}
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I(U; X_2|V) &+ I(V; X_1) \\
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H(X_1|V, U) & \\
H(X_1) & \\
R_1
\end{align*}

**Semi-Deterministic BC with Cooperation**

- $R_{12} = I(V; Y_1) - I(V; Y_2)$
- $(R_1, R_2)$ at Lower Corner Point: $\left( H(Y_1), I(U; Y_2|V) - I(U; Y_1|V) \right)$
- $(R_1, R_2)$ at Upper Corner Point: $\left( H(Y_1|V, U), I(U; Y_2|V) + I(V; Y_1) \right)$

**Ahlswede-Körner Problem with Cooperation**

- $R_{12} = I(V; X_1) - I(V; X_2)$
- $(R_1, R_2)$ at Lower Corner Point: $\left( H(X_1), I(U; X_2|V) - I(U; X_1|V) \right)$
- $(R_1, R_2)$ at Upper Corner Point: $\left( H(X_1|V, U), I(U; X_2|V) + I(V; X_1) \right)$
Duality - Corner Point Correspondence

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\]

\[
(R_1, R_2) \text{ at Upper Corner Point: } \left( H(X_1|V, U), I(U; X_2|V) + I(V; X_1) \right)
\]
Semi-Deterministic BC with Cooperation

Ahlswede-Körner Problem with Cooperation

<table>
<thead>
<tr>
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</tr>
<tr>
<td>((R_1, R_2)) at Lower Corner Point: (\left( H(Y_1), I(U; Y_2</td>
<td>V) - I(U; Y_1</td>
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</tr>
</tbody>
</table>
Summary

- AK problem with cooperation.
Summary

- AK problem with cooperation.
- SD-BC with cooperation.
Summary

- AK problem with cooperation.
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- Duality:
Summary

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- Duality:
  - Transformation principles.
Summary

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Summary

- AK problem with cooperation.
- SD-BC with cooperation.
- Duality:
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- Probabilistic converse.
AK problem with cooperation.
SD-BC with cooperation.
Duality:
  ▶ Transformation principles.
  ▶ Corner point correspondence.
Probabilistic converse.
AK problem with cooperation.
SD-BC with cooperation.

Duality:
- Transformation principles.
- Corner point correspondence.

Probabilistic converse.


Thank you!
AK Problem with Cooperation - Achievability Outline

Encoder 1

Encoder 2

Decoder

$X_1^n$ to $Y^n$

$X_2^n$

$T_1(X_1^n)$

$T_2(T_{12}, X_2^n)$

$T_{12}(X_1^n)$
AK Problem with Cooperation - Achievability Outline

Encoder 1

Encoder 2

Decoder

$X_1^n$ → $T_1(X_1^n)$

$X_2^n$ → $T_2(T_{12}, X_2^n)$

$T_{12}(X_1^n)$

$Y^n$

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**AK Problem with Cooperation - Achievability Outline**

- **Encoder 1**
  - $X_1^n$ to $T_1(X_1^n)$
  - $T_{12}(X_1^n)$

- **Encoder 2**
  - $X_2^n$ to $T_2(T_{12}, X_2^n)$

- **Decoder**
  - $Y^n$

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- **Cooperation:** Wyner-Ziv scheme to convey $V^n$ via cooperation link.
AK Problem with Cooperation - Achievability Outline

Rate | Corner Point 1 | Corner Point 2
--- | --- | ---
$R_{12}$ | $I(V; X_1) - I(V; X_2)$ | $I(V; X_1) - I(V; X_2)$
$R_1$ | $H(X_1)$ | $H(X_1|V, U)$
$R_2$ | $I(U; X_2|V) - I(U; X_1|V)$ | $I(U; X_2|V) + I(V; X_1)$

**Cooperation:** Wyner-Ziv scheme to convey $V^n$ via cooperation link.
**Corner Point 1:** $V^n$ is transmitted to dec. by Enc. 1 within $X_1^n$. 
**AK Problem with Cooperation - Achievability Outline**

![Diagram of cooperative transmission system]

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- **Cooperation**: Wyner-Ziv scheme to convey $V^n$ via cooperation link.
- **Corner Point 1**: $V^n$ is transmitted to dec. by Enc. 1 within $X_1^n$.
- **Corner Point 2**: $V^n$ is explicitly transmitted to dec. by Enc. 2.
Semi-Deterministic BC with Cooperation - Achievability

Outline
Rate Splitting: $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$: 

![Diagram of the rate splitting process with two decoders and a channel between encoder and decoders]
Rate Splitting: $M_j = (M_{j0}, M_{jj}), j = 1, 2$:  
- $(M_{10}, M_{20})$ - Public message;
Semi-Deterministic BC with Cooperation - Achievability

Outline

- Rate Splitting: $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
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Semi-Deterministic BC with Cooperation - Achievability

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Semi-Deterministic BC with Cooperation - Achievability

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Semi-Deterministic BC with Cooperation - Achievability

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  1. Partition common message c.b into \( 2^{nR_{12}} \) bins.
Semi-Deterministic BC with Cooperation - Achievability

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- **Gain at Dec. 2:**
Semi-Deterministic BC with Cooperation - Achievability

**Outline**

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}) \), \( j = 1, 2 \):
  - \((M_{10}, M_{20})\) - Public message;
  - \((M_{11}, M_{22})\) - Private messages.

- **Codebook Structure:** Marton (with common message).

- **Cooperation:**
  1. Partition common message c.b into \( 2^{nR_{12}} \) bins.
  2. Convey bin number via link.

- **Gain at Dec. 2:** Reduced search space of common message c.w by \( R_{12} \).
Semi-Deterministic BC with Cooperation - Converse

Outline

Via telescoping identities:
Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$. 
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\[
H(M_2) - n\epsilon_n \leq I(M_2; Y_2^n | M_{12}) + I(M_2; M_{12})
\]
Semi-Deterministic BC with Cooperation - Converse

Outline

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2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq I(M_2; Y_{2,n}^n | M_{12}) + I(M_2; M_{12})$$

$$= \sum_{i=1}^{n} \left[ I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n | M_{12}, Y_1^i) \right] + I(M_2; M_{12})$$
Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: \( V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \) and \( U_i = M_2 \).

2. Telescoping identities [Kramer, 2011], e.g.,

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H(M_2) - n \epsilon_n \leq I(M_2; Y_2^n | M_{12}) + I(M_2; M_{12}) \\
= \sum_{i=1}^{n} \left[ I(M_2; Y_2^n_i | M_{12}, Y_1^{i-1}) - I(M_2; Y_2^n_{i+1} | M_{12}, Y_1^i) \right] + I(M_2; M_{12}) \\
= \sum_{i=1}^{n} \left[ I(M_2; Y_2,i | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_1,i | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \right] + I(M_2; M_{12})
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Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

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2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq I(M_2; Y_{2,i}^n|M_{12}) + I(M_2; M_{12})$$

$$= \sum_{i=1}^{n} I(M_2; Y_{2,i}^n|M_{12}, Y_{1}^{i-1}) - I(M_2; Y_{2,i+1}^n|M_{12}, Y_{1}^i) + I(M_2; M_{12})$$

$$= \sum_{i=1}^{n} I(M_2; Y_{2,i}^n|M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_{1,i}^n|M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^n) + I(M_2; M_{12})$$

Replaces 4 uses of Csiszár Sum Identity!
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

\[ Y_1^n = f(X^n, S^n) \times P_{Y_2 | X^n, S^n} \]

\[ (M_1, M_2) \]

\[ X^n \]

\[ S^n \]
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

\[(M_1, M_2) \xrightarrow{Enc} X^n \xrightarrow{\text{Channel}} \begin{cases} Y_1^n \xrightarrow{\text{Dec 1}} \hat{M}_1 \\ Y_2^n \xrightarrow{\text{Dec 2}} \hat{M}_2 \end{cases} \xrightarrow{\text{Dec}} Y^n\]

\[I(U;Y_2) - I(U;S)\]

\[I(U;Y_2) - I(U;S)\]

\[0 \quad H(Y_1|S,U) \quad H(Y_1|S) \quad R_1\]

\[0 \quad H(X_1|Z,U) \quad H(X_1|Z) \quad R_1\]

\[I(U;X_2) - I(U;Z)\]

\[I(U;X_2) - I(U;Z)\]

\[0 \quad H(X_1|Z,U) \quad H(X_1|Z) \quad R_1\]
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

\[ S^n \]

\[ M_1, M_2 \]

\[ X^n \]

\[ Y_1^n, Y_2^n | X, S \]

\[ \text{Channel} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ Z^n \]

\[ X_1^n \]

\[ T_1 \]

\[ Y^n \]

\[ X_2^n \]

\[ T_2 \]
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

\[ (M_1, M_2) \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ Y_1^n \rightarrow \hat{M}_1 \]

\[ Y_2^n \rightarrow \hat{M}_2 \]

\[ I(U; X_2) - I(U; Z) \]

\[ I(U; Y_2) - I(U; S) \]

\[ I(X; Y_1, Y_2 | U, S) \]

\[ I(Y; X_1, X_2 | U, Z) \]

Goldfeld/Permuter/Kramer  Semi-Deterministic Broadcast Channels with Cooperation 14 / 15
Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:
Multi-User Duality - Additional Examples

**Action-Dependant Output-Degraded BC vs. Dual:**

\[
(M_1, M_2) \xrightarrow{Enc} X^n \xrightarrow{Y_1^n} \text{Dec 1} \xrightarrow{\hat{M}_1} \\
A^n(M_1, M_2) \xrightarrow{S^i} P_{S|A} \xrightarrow{S^i} \\
\]

\[
\text{Channel} \xrightarrow{P_{Y_1,Y_2|X,S}} \text{Dec 2} \xrightarrow{\hat{M}_2} \\
X^n \xrightarrow{Y_2^n} \\
\]

\[
\text{Enc 1} \xrightarrow{T_1} \text{Enc 2} \xrightarrow{T_2} \text{Dec} \xrightarrow{Z^i} \\
X^n_1 \xrightarrow{X^n_2} \\
\]

\[
P_{Z|X_1,X_2,A} \xrightarrow{Z^i} \\
\]

Goldfeld/Permuter/Kramer
Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:

\[
(M_1, M_2) \xrightarrow{X^n} \text{Enc} \xrightarrow{P_{Y_1,Y_2|X,S}} \text{Channel} \xrightarrow{Y_1^n,Y_2^n} \text{Dec 1, Dec 2} \xrightarrow{\hat{M}_1, \hat{M}_2} \text{Output}
\]

\[
A^n(M_1, M_2) \xrightarrow{S^i} \text{Enc} \xrightarrow{P_{S|A}} \xrightarrow{S^i} \text{Dec 1, Dec 2} \xrightarrow{\hat{M}_1, \hat{M}_2} \text{Output}
\]

\[
\text{Enc 1} \xrightarrow{T_1} \text{Dec} \xrightarrow{Y^n} \text{Output}
\]

\[
\text{Enc 2} \xrightarrow{T_2} \text{Dec} \xrightarrow{Z^i} \text{Output}
\]

\[
\begin{align*}
R_2 & \quad \begin{align*}
I(U; Y_2) & \quad \begin{align*}
0 & \quad I(V, A; Y_1, Y_2|U) \quad \begin{align*}
R_1
\end{align*}
\end{align*}
\end{align*}
\end{align*}
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R_1
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\end{align*}
\end{align*}
\end{align*}
\]
Achieving Corner Point 1:

\[\left( I(V; X_1|X_2), H(X_1), I(U; X_2|X_1, V) \right).\]
Achieving Corner Point 1:

\[
\left( I(V; X_1|X_2) , \ H(X_1) , \ I(U; X_2|X_1, V) \right).
\]

- **Cooperation:** Wyner-Ziv coding to convey \( V^n \) from Encoder 1 to Encoder 2.
Achieving Corner Point 1:

\[
(I(V; X_1|X_2), H(X_1), I(U; X_2|X_1, V)).
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- **Cooperation**: Wyner-Ziv coding to convey \(V^n\) from Encoder 1 to Encoder 2.
- **Encoder 1 to Decoder**: Conveys \(X_1^n\) to the decoder in a lossless manner.
Achieving Corner Point 1:

\[
  ( I(V; X_1|X_2) , H(X_1) , I(U; X_2|X_1, V) ).
\]

- **Cooperation**: Wyner-Ziv coding to convey $V^n$ from Encoder 1 to Encoder 2.
- **Encoder 1 to Decoder**: Conveys $X_1^n$ to the decoder in a lossless manner.
- **Encoder 2 to Decoder**: The decoder knows $X_1^n$ and therefore $V^n$. Wyner-Ziv coding to convey $U^n$. 

Achieving Corner Point 2:

\[ \left( I(V; X_1|X_2) , H(X_1|V,U) , I(U; X_2|V) + I(V; X_1) \right). \]
Achieving Corner Point 2:

\[
( I(V; X_1|X_2) , \ H(X_1|V,U) , \ I(U; X_2|V) + I(V; X_1) ) .
\]

**Cooperation:** Same.
Achieving Corner Point 2:

\[
(I(V; X_1|X_2), H(X_1|V,U), I(U; X_2|V) + I(V; X_1)).
\]

- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows $V^n$. Conveys the index of $V^n$ and uses superposition coding to convey $U^n$. 
Achieving Corner Point 2:

\[
( I(V; X_1|X_2), H(X_1|V, U), I(U; X_2|V) + I(V; X_1) ).
\]

- **Cooperation:** Same.
- **Encoder 2 to Decoder:** Knows \( V^n \). Conveys the index of \( V^n \) and uses superposition coding to convey \( U^n \).
- **Encoder 1 to Decoder:** The decoder knows \((V^n, U^n)\). Binning scheme to convey \( X_1^n \) in a lossless manner.
Converse:
Converse:

- Standard techniques while defining

\[ V_i = (T_{12}, X_{1}^{n\backslash i}, X_{2,i+1}^{n}), \]
\[ U_i = T_2, \]

for every \( 1 \leq i \leq n \).
Converse:

- Standard techniques while defining

\[ V_i = (T_{12}, X_{1}^{n \setminus i}, X_{2,i+1}^{n}), \]
\[ U_i = T_2, \]

for every \( 1 \leq i \leq n. \)

- Time mixing properties.
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$: 

![Rate Splitting Diagram]

$$X^n \xrightarrow{Enc} Y^n_1 \xrightarrow{Dec 1}$$

$$Y^n_2 \xrightarrow{Dec 2}$$
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting**: $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
Semi-Deterministic BC with Cooperation - Achievability Outline

**Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), j = 1, 2: \)
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Semi-Deterministic BC with Cooperation - Achievability Outline

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  - \( (M_{11}, M_{22}) \) - Private messages.

- **Codebook Structure:** Marton:
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj}), \ j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V^n$. 

![Diagram of channel with encoders and decoders]
Semi-Deterministic BC with Cooperation - Achievability

**Outline**

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V^n$.
  - Private Messages - Superposed on $V^n$:
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting**: \( M_j = (M_{j0}, M_{jj}), \ j = 1, 2 \):
  - \((M_{10}, M_{20})\) - Public message;
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- **Codebook Structure**: Marton:
  - Public Message: \((M_{10}, M_{20}) \rightarrow V^n\).
  - Private Messages - Superposed on \( V^n \):
    1. \( M_{11} \rightarrow Y_1^n \).

![Diagram of encoder and channels](image)
Semi-Deterministic BC with Cooperation - Achievability

**Outline**

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
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  - Public Message: $(M_{10}, M_{20}) \rightarrow V^n$.
  - Private Messages - Superposed on $V^n$:
    1. $M_{11} \rightarrow Y_1^n$;
    2. $M_{22} \rightarrow U^n$. 

\[ X^n \xrightarrow{Enc} Channel \rightarrow Y_1^n, Y_2^n \xrightarrow{Dec 1, 2} \]
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
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  - Private Messages - Superposed on $V^n$:
    1. $M_{11} \rightarrow Y_1^n$;
    2. $M_{22} \rightarrow U^n$. 

![Diagram of the semi-deterministic broadcast channel with cooperation.](attachment:channel_diagram.png)
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), \ j = 1, 2: \)
  - \((M_{10}, M_{20})\) - Public message;
  - \((M_{11}, M_{22})\) - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: \((M_{10}, M_{20}) \rightarrow V^n.\)
  - Private Messages - Superposed on \(V^n:\)
    1. \(M_{11} \rightarrow Y_1^n;\)
    2. \(M_{22} \rightarrow U^n.\)

- **Decoding:** Joint typicality decoding.
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- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of \( V^n \) - \( 2^{nR_{12}} \) bins.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
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- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V^n$.
  - Private Messages - Superposed on $V^n$:
    1. $M_{11} \rightarrow Y_1^n$;
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- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of $V^n$ - $2^nR_{12}$ bins.

- **Gain:** Dec. 2 reduces search space of $V^n$ by $R_{12}$.
Achievability: Split $M_i = (M_{i0}, M_{ii})$, $i = 1, 2$. Code construction:
Achievability: Split $M_i = (M_{i0}, M_{ii})$, $i = 1, 2$. Code construction: