Broadcast Channels with Cooperation: Capacity and Duality for the Semi-Deterministic Case

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Outline

- Channel-source duality for BCs
- Semi-deterministic BC with decoder cooperation
- Source coding dual
- Capacity results
- Summary
“There is a curious and provocative duality between the properties of a source with a distortion measure and those of a channel...”
(C. E. Shannon, 1959)
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**PTP Duality:** [Shannon, 1959], [Cover and Chiang, 2002], [Pradhan et al., 2003], [Gupta and Verdú, 2011].
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- The solutions are dual - Information measures coincide.
- A formal proof of duality is still absent.
- Solving one problem $\implies$ Valuable insight into solving dual.
Duality - Preface

Point-to-Point Case:
Duality - Preface

Point-to-Point Case:
Duality - Preface

Point-to-Point Case:

**Empirical Coordination:** \( (X, Y) \in \mathcal{T}_e^{(n)}(P_X P^*_Y | X) \)
Duality - Preface

Point-to-Point Case:

**Empirical Coordination:** \((X, Y) \in \mathcal{T}_\epsilon^{(n)}(P_X P^*_{Y|X})\)

\[\begin{array}{c}
X \\
\text{Encoder} \quad T \\
\text{Decoder} \\
Y
\end{array}\]

**Fixed-Type Code:** \((X, Y) \in \mathcal{T}_\epsilon^{(n)}(P^*_X P_{Y|X})\)

\[\begin{array}{c}
M \\
\text{Encoder} \quad X \\
\quad P_{Y|X} \\
\quad Y \\
\text{Decoder} \\
\hat{M}
\end{array}\]
Duality - Preface

Point-to-Point Case:
Point-to-Point Case:
Point-to-Point Case:

\[ R^* = I(X; Y) \]

\[ C = I(X; Y) \]
Point-to-Point Case:

\[ R^* = I(X; Y) \]

\[ C = I(X; Y) \]
Multi-User Duality - Broadcast Channels

Encoder \( (M_1, M_2) \) → Channel \( P_{Y_1,Y_2|X} \) → Decoder 1 \( \hat{M}_1(Y_1) \) → Decoder 2 \( \hat{M}_2(Y_2) \)

Encoder 1 \( X_1 \) → Decoder \( Y \) → Encoder 2 \( X_2 \)
Multi-User Duality - Broadcast Channels

Encoder

Channel

Decoder 1

Decoder 2

(Y₁, Y₂)

X

X₁

X₂

M₁, M₂

P_{Y₁, Y₂|X}

T₁(X₁)

T₂(X₂)

M₁(Y₁)

M₂(Y₂)

Y₁

Y₂

Y
Multi-User Duality - Broadcast Channels

\[
(M_1, M_2) \xrightarrow{X} (\hat{M}_1(Y_1), \hat{M}_2(Y_2))
\]

Channel:

Encoder 1: \[T_1(X_1)\]
Encoder 2: \[T_2(X_2)\]
Decoder: \[Y\]
Probabilistic relations are preserved:
Probabilistic relations are preserved:

**Broadcast Channel**

\[(X, Y_1, Y_2) \in T^{(n)}(P^*_X P_{Y_1,Y_2|X})\]

**Dual Source Coding Setting**

\[(X_1, X_2, Y) \in T^{(n)}(P_{X_1,X_2} P^*_Y|X_1,X_2)\]
Probabilistic relations are preserved:

**Broadcast Channel**

\[(X, Y_1, Y_2) \in \mathcal{T}_\epsilon^{(n)} \left( P^{*}_{X} P_{Y_1,Y_2|X} \right) \]

**Dual Source Coding Setting**

\[(X_1, X_2, Y) \in \mathcal{T}_\epsilon^{(n)} \left( P_{X_1,X_2} P^{*}_{Y|X_1,X_2} \right) \]

e.g., Markov relations, deterministic functions, etc.
Probabilistic relations are preserved:

**Broadcast Channel**

\[(X, Y_1, Y_2) \in T_{\epsilon}^{(n)} \left( P_X^* P_{Y_1,Y_2|X} \right) \]

**Dual Source Coding Setting**

\[(X_1, X_2, Y) \in T_{\epsilon}^{(n)} \left( P_{X_1,X_2}^* P_{Y|X_1,X_2} \right) \]

e.g., Markov relations, deterministic functions, etc.

**Additional Principles:**
Probabilistic relations are preserved:

Broadcast Channel

\[(X, Y_1, Y_2) \in T^{(n)}(P_X^* P_{Y_1,Y_2|X})\]

Dual Source Coding Setting

\[(X_1, X_2, Y) \in T^{(n)}(P_{X_1,X_2}^* P_{Y|X_1,X_2})\]

e.g., Markov relations, deterministic functions, etc.

Additional Principles:

- Causal/non-causal **encoder** CSI \[\leftrightarrow\] Causal/non-causal **decoder** SI
Probabilistic relations are preserved:

\[
(X, Y_1, Y_2) \in T_\epsilon^{(n)} \left( P_X^* P_{Y_1,Y_2 | X} \right) \quad \leftrightarrow \quad (X_1, X_2, Y) \in T_\epsilon^{(n)} \left( P_{X_1,X_2}^* P_{Y | X_1,X_2} \right)
\]

e.g., Markov relations, deterministic functions, etc.

**Additional Principles:**
- Causal/non-causal **encoder** CSI \(\leftrightarrow\) Causal/non-causal **decoder** SI
- **Decoder** cooperation \(\leftrightarrow\) **Encoder** cooperation
Multi-User Duality - Broadcast Channels

Probabilistic relations are preserved:

Broadcast Channel

\[(X, Y_1, Y_2) \in T^{(n)}(P^*_X P_{Y_1,Y_2|X})\]

Dual Source Coding Setting

\[(X_1, X_2, Y) \in T^{(n)}(P_{X_1,X_2}^* P_{Y|X_1,X_2})\]

e.g., Markov relations, deterministic functions, etc.

Additional Principles:

- Causal/non-causal encoder CSI \(\leftrightarrow\) Causal/non-causal decoder SI
- Decoder cooperation \(\leftrightarrow\) Encoder cooperation

★ Result Duality: Information measures at the corner points coincide! ★
Cooperative SD-BC vs. Cooperative WAK Problem
Cooperative SD-BC vs. Cooperative WAK Problem


BCs with Cooperation:

- Physically degraded (PD) [Dabora and Servetto, 2006].
- Relay-BC [Liang and Kramer, 2007].
- State-dependent PD [Dikstein, Permuter and Steinberg, 2014].
- Degraded message sets / PD with parallel conf. [Steinberg, 2015].
Cooperative SD-BC vs. Cooperative WAK Problem

Cooperative SD-BC vs. Cooperative WAK Problem


\[
(M_1, M_2) \rightarrow \text{Encoder} \rightarrow X \rightarrow \text{Channel} \times P_{Y_2|X} \rightarrow \text{Decoder 1} \rightarrow \hat{M}_1
\]

\[
(M_{12}(Y_1) \in [1 : 2^n R_{12}] \rightarrow \text{Decoder 2} \rightarrow \hat{M}_2
\]

\[
(M, X_1, X_2) \in T_{\epsilon}^{(n)}(P^*_X 1_{Y_1=f(X)} P_{Y_2|X}) \quad \leftrightarrow \quad (Y, X_1, X_2) \in T_{\epsilon}^{(n)}(P_{Y} 1_{X_1=f(Y)} P^*_X 2|Y)
\]

Semi-Deterministic BC

WAK Problem
Cooperative SD-BC vs. Cooperative WAK Problem


\[(M_1, M_2) \xrightarrow{X} \text{Encoder} \xrightarrow{1 \{Y_1 = f(X)\} \times P_{Y_2|X}} \text{Channel} \xrightarrow{Y_1} \text{Decoder 1} \xrightarrow{\hat{M}_1} \]
\[\xrightarrow{Y_2} \text{Decoder 2} \xrightarrow{\hat{M}_2} \]

\[(X_1 \xrightarrow{T_1} \text{Encoder 1} \xrightarrow{[1 : 2^{nR_{12}}]} T_{12}(X_1) \xrightarrow{T_2} \text{Encoder 2} \xrightarrow{Y} \text{Decoder} \xrightleftharpoons{\text{Semi-Deterministic BC}} \]

\[\text{WAK Problem} \]

\[(X, Y_1, Y_2) \in T_{\epsilon}^{(n)}(P_X \mathbb{1}_{Y_1 = f(X)} P_{Y_2|X}) \quad \text{and} \quad (Y, X_1, X_2) \in T_{\epsilon}^{(n)}(P_Y \mathbb{1}_{X_1 = f(Y)} P_{X_2|Y})\]
Cooperative SD-BC vs. Cooperative WAK Problem


Semi-Deterministic BC

\[(X, Y_1, Y_2) \in T_\epsilon^{(n)}(P_X^* 1_{Y_1=f(X)} P_{Y_2|X})\]

WAK Problem

\[(Y, X_1, X_2) \in T_\epsilon^{(n)}(P_Y 1_{X_1=f(Y)} P_{X_2|Y})\]
Theorem (Coordination-Capacity Region)

For a desired coordination PMF \( P_{X_2}P_{Y|X_2}1\{X_1 = f(Y)\} \):

\[
C_{WAK} = \bigcup \left\{ \begin{array}{l}
R_{12} \geq I(V; X_1) - I(V; X_2) \\
R_1 \geq H(X_1|V, U) \\
R_2 \geq I(U; X_2|V) - I(U; X_1|V) \\
R_1 + R_2 \geq H(X_1|V, U) + I(V, U; X_1, X_2)
\end{array} \right\}
\]

where the union is over all \( P_{X_1, X_2}P_{V|X_1}P_{U|X_2,V}P_{Y|X_1,U,V} \) with \( P_{X_2}P_{Y|X_2}1\{X_1 = f(Y)\} \) as marginal.
Theorem (Coordination-Capacity Region)

For a desired coordination PMF \( P_{X_2} P_{Y|X_2} \mathbb{1}_{\{X_1 = f(Y)\}} \):

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Achievability via Wyner-Ziv coding, superposition coding and Slepian-Wolf binning.
Cooperative WAK Problem - Solution

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Achievability via Wyner-Ziv coding, superposition coding and Slepian-Wolf binning.
For fixed joint PMFs and $R_{12}$:

\[ I(U; X_2 | V) + I(V; X_1) \]

\[ I(U; X_2 | V) - I(U; X_1 | V) \]

\[ H(X_1 | V, U) \]

\[ H(X_1) \]

\[ R_1 \]

\[ R_2 \]
Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

$$R_{12} = I(V; X_1) - I(V; X_2)$$

$(R_1, R_2)$ at Lower Corner Point:
$$
\left( H(X_1) , I(U; X_2|V) - I(U; X_1|V) \right)
$$

$(R_1, R_2)$ at Upper Corner Point:
$$
\left( H(X_1|V, U) , I(U; X_2|V) + I(V; X_1) \right)
$$
Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

\[ R_{12} = I(U; X_2|V) + I(V; X_1) \]
\[ R_{12} = I(U; X_2|V) - I(V; X_1|V) \]

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Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

$$R_1 \rightarrow R_2$$

$I(U; X_2|V) + I(V; X_1)$

$I(U; X_2|V)$

$I(U; X_2|V) - I(U; X_1|V)$

$H(X_1|V, U)$

$H(X_1)$

$R_{12} = I(V; X_1) - I(V; X_2)$

$(R_1, R_2)$ at Lower Corner Point:

$(H(X_1), I(U; X_2|V) - I(U; X_1|V))$

$(R_1, R_2)$ at Upper Corner Point:

$(H(X_1|V, U), I(U; X_2|V) + I(V; X_1))$

$R_{12} = I(V; Y_1) - I(V; Y_2)$

$(R_1, R_2)$ at Lower Corner Point:

$(R_1, R_2)$ at Upper Corner Point:
For fixed joint PMFs and $R_{12}$:

$$I(U; X_2|V) + I(V; X_1)$$

$$I(U; X_2|V) - I(U; X_1|V)$$

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Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

\[ R_{12} = I(V; X_1) - I(V; X_2) \]

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Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

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R_{12} = I(V; X_1) - I(V; X_2)
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$(R_1, R_2)$ at Lower Corner Point:

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Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

\[
\begin{align*}
I(U; X_2|V) & + I(V; X_1) \\
I(U; X_2|V) & - I(U; X_1|V)
\end{align*}
\]

\[
\begin{align*}
H(X_1|V, U) & \quad H(X_1) \\
H(Y_1) & \quad H(Y_1)
\end{align*}
\]

Cooperative WAK Problem

\[
R_{12} = I(V; X_1) - I(V; X_2)
\]

(R$_1$, R$_2$) at Lower Corner Point:

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(H(X_1), I(U; X_2|V) - I(U; X_1|V))
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Cooperative Semi-Deterministic BC

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R_{12} = I(V; Y_1) - I(V; Y_2)
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Corner Point Correspondence

For fixed joint PMFs and $R_{12}$:

For Cooperative WAK Problem:

- $R_{12} = I(V; X_1) - I(V; X_2)$
- $(R_1, R_2)$ at Lower Corner Point: $(H(X_1), I(U; X_2|V) - I(U; X_1|V))$
- $(R_1, R_2)$ at Upper Corner Point: $(H(X_1|V, U), I(U; X_2|V) + I(V; X_1))$

For Cooperative Semi-Deterministic BC:

- $R_{12} = I(V; Y_1) - I(V; Y_2)$
- $(R_1, R_2)$ at Lower Corner Point: $(H(Y_1), I(U; Y_2|V) - I(U; Y_1|V))$
- $(R_1, R_2)$ at Upper Corner Point: $(H(Y_1|V, U), I(U; Y_2|V) + I(V; Y_1))$
Theorem (Capacity Region)

The capacity region is:

\[ C_{BC} = \bigcup \left\{ \begin{array}{l}
R_{12} \geq I(V; Y_1) - I(V; Y_2) \\
R_1 \leq H(Y_1) \\
R_2 \leq I(V, U; Y_2) + R_{12} \\
R_1 + R_2 \leq H(Y_1|V, U) + I(U; Y_2|V) + I(V; Y_1)
\end{array} \right\} \]

where the union is over all \( P_{V,U,Y_1,X} P_{Y_2|X} 1\{Y_1 = f(X)\} \).
The capacity region is:

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where the union is over all \( P_{V,U,Y_1,X} P_{Y_2|X} 1\{Y_1 = f(X)\} \).

Later: Achievability and converse proofs for an alternative region.
Theorem (Capacity Region)

The capacity region is:

\[ C_{BC} = \bigcup \left\{ \begin{align*}
R_{12} & \geq I(V;Y_1) - I(V;Y_2) \\
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- Later: Achievability and converse proofs for an alternative region.
- \( C_{BC} \) emphasizes duality.
Cooperative Semi-Deterministic BC - Achievability Outline

![Diagram of Cooperative Semi-Deterministic BC]

- Encoder (Enc)
- Channel
- Decoder 1 (Dec 1)
- Decoder 2 (Dec 2)

Symbols:
- Enc
- Channel
- Dec 1
- Dec 2
- X
- Y₁
- Y₂
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}) \), \( j = 1, 2 \):

![Diagram](image-url)
Rate Splitting: $M_j = (M_{j0}, M_{jj}), \ j = 1, 2$:
- $(M_{10}, M_{20})$ - Common message;
**Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:

- $(M_{10}, M_{20})$ - Common message;
- $(M_{11}, M_{22})$ - Private messages.
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Common message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton (with common message).
**Rate Splitting:** \( M_j = (M_{j0}, M_{jj}) \), \( j = 1, 2 \):
- \((M_{10}, M_{20})\) - Common message;
- \((M_{11}, M_{22})\) - Private messages.

**Codebook Structure:** Marton (with common message).

\[ y_1\text{-codebook} \sim P^n_{Y_1|V} \]

\[ u\text{-codebook} \sim P^n_{U|V} \]
Cooperative Semi-Deterministic BC - Achievability Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Common message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton (with common message).

- **Cooperation:**

```
\begin{center}
\begin{tikzpicture}
\node[draw, rounded corners] (channel) at (0,0) {Channel};
\node[draw, fill=white] (enc) at (-3,0) {Enc};
\node[draw, fill=white] (dec1) at (3,0) {Dec 1};
\node[draw, fill=white] (dec2) at (3,-1.5) {Dec 2};
\draw[->] (enc) -- node[above] {$X$} (channel);
\draw[->] (channel) -- node[above] {$Y_1$} (dec1);
\draw[->] (channel) -- node[above] {$Y_2$} (dec2);
\end{tikzpicture}
\end{center}
```

- $y_1$-codebook $\sim P^n_{Y_1|V}$
- $u$-codebook $\sim P^n_{U|V}$
- $v(m_{10}, m_{20})$
Cooperative Semi-Deterministic BC - Achievability Outline

**Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), j = 1, 2: \)
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**Codebook Structure:** Marton (with common message).

**Cooperation:**
1. Partition common message c.b. into \(2^{nR_{12}}\) bins.

\[\begin{array}{c}
\text{Enc} \quad X \\
\text{Channel} \quad \begin{cases} 
Y_1 \\
Y_2 
\end{cases} \\
\text{Dec 1} \quad \text{Dec 2}
\end{array}\]

\(y_1\)-codebook \(\sim P^n_{Y_1|V}\)

\(u\)-codebook \(\sim P^n_{U|V}\)
Rate Splitting: $M_j = (M_{j0}, M_{jj}), j = 1, 2$:
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Codebook Structure: Marton (with common message).

Cooperation:
1. Partition common message c.b. into $2^{nR_{12}}$ bins.
2. Convey bin number via link.
Cooperative Semi-Deterministic BC - Achievability Outline

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- ** Cooperation:**
  1. Partition common message $c.b.$ into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.

- **User 2 Gain:**

\[ y_1 \text{-codebook } \sim P_{Y_1|V}^n \]
\[ u \text{-codebook } \sim P_{U|V}^n \]
Rate Splitting: $M_j = (M_{j0}, M_{jj}), j = 1, 2$:
- $(M_{10}, M_{20})$ - Common message;
- $(M_{11}, M_{22})$ - Private messages.

Codebook Structure: Marton (with common message).

Cooperation:
1. Partition common message c.b. into $2^{nR_{12}}$ bins.
2. Convey bin number via link.

User 2 Gain: Reduced search space of common message c.w. by $R_{12}$. 
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  1. Partition common message c.b. into $2^{nR_{12}}$ bins.
  2. Convey bin number via link.

- **User 2 Gain**: Reduced search space of common message c.w. by $R_{12}$.
  $\implies$ More channel resources for private message.
Rate Splitting: $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
- $(M_{10}, M_{20})$ - Common message;
- $(M_{11}, M_{22})$ - Private messages.

Codebook Structure: Marton (with common message).

Cooperation:
1. Partition common message c.b. into $2^{nR_{12}}$ bins.
2. Convey bin number via link.

User 2 Gain: Reduced search space of common message c.w. by $R_{12}$.

\[ \Rightarrow \] More channel resources for private message.
Summary

- Channel-source duality for BCs.
Summary

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- Cooperative semi-deterministic BCs:
Summary

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  - Source coding dual - Cooperative WAK problem.
Summary

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Summary

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Summary

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Thank you!
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

\[(M_1, M_2)\]

\[\begin{align*}
&\text{Enc} \\
&\text{Dec 1} \\
&\text{Dec 2} \\
&\text{Dec}
\end{align*}\]

Channel

\[\mathbb{1}_{Y_1 = f(X, S)} \times P_{Y_2 | X, S} \]

Enc 1

Dec 1

\[\hat{M}_1\]

Dec 2

\[\hat{M}_2\]

Enc 2

Dec

\[\hat{M}_1\]

\[\hat{M}_2\]

\[\begin{align*}
&X_1 \\
&T_1 \\
&X_2 \\
&T_2
\end{align*}\]

\[\begin{align*}
&Y \\
&Z
\end{align*}\]
Multi-User Duality - Additional Examples

State-Dependant Semi-Deterministic BC vs. Dual:

\[ (M_1, M_2) \]

\[ \text{Enc} \]

\[ \mathbf{X} \]

\[ \mathbf{S} \]

\[ \text{Channel} \]

\[ \mathbb{I}_{\{Y_1 = f(X, S)\}} \]

\[ \times P_{Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Dec} \]

\[ \mathbf{X}_1 \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \mathbf{Y} \]

\[ \mathbf{X}_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ \mathbf{Z} \]

\[ R_1 \]

\[ R_2 \]

\[ I(U; Y_2) \]

\[ -I(U; S) \]

\[ I(U; Y_2) - I(U; S) \]

\[ -I(U; Y_1 | S) \]

\[ H(Y_1 | S, U) \]

\[ H(Y_1 | S) \]

\[ I(U; X_2) \]

\[ -I(U; Z) \]

\[ I(U; X_2) - I(U; Z) \]

\[ -I(U; X_1 | Z) \]

\[ H(X_1 | Z, U) \]

\[ H(X_1 | Z) \]

\[ R_2 \]
State-Dependant Output-Degraded BC vs. Dual:
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

\[ \text{Channel: } P_{Y_1,Y_2|X,S} \]

\[ \text{Dec 1: } \hat{M}_1 \]

\[ \text{Dec 2: } \hat{M}_2 \]

\[ \text{Enc: } X \]

\[ \text{Dec: } Y \]

\[ (M_1, M_2) \]

\[ S \]
Multi-User Duality - Additional Examples

State-Dependant Output-Degraded BC vs. Dual:

\[ (M_1, M_2) \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \text{Dec} \]

\[ X_1 \]

\[ Y \]

\[ X_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ Z \]

\[ \text{Dec} \]

\[ X \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \text{Dec} \]

\[ X_1 \]

\[ Y \]

\[ X_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ Z \]

\[ \text{Dec} \]

\[ X \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \text{Dec} \]

\[ X_1 \]

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\[ X_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ Z \]

\[ \text{Dec} \]

\[ X \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

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\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \text{Dec} \]

\[ X_1 \]

\[ Y \]

\[ X_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ Z \]

\[ \text{Dec} \]

\[ X \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]

\[ T_1 \]

\[ \text{Dec} \]

\[ X_1 \]

\[ Y \]

\[ X_2 \]

\[ \text{Enc 2} \]

\[ T_2 \]

\[ Z \]

\[ \text{Dec} \]

\[ X \]

\[ \text{Enc} \]

\[ P_{Y_1, Y_2 | X, S} \]

\[ \text{Dec 1} \]

\[ \hat{M}_1 \]

\[ \text{Dec 2} \]

\[ \hat{M}_2 \]

\[ \text{Channel} \]

\[ S \]

\[ Z \]

\[ \text{Enc 1} \]
Multi-User Duality - Additional Examples

Action-Dependant Output-Degraded BC vs. Dual:
Action-Dependant Output-Degraded BC vs. Dual:

\[ (M_1, M_2) \rightarrow \text{Enc} \rightarrow X^n \rightarrow \text{Channel} \rightarrow Y_1^n, Y_2^n \rightarrow \text{Dec 1, Dec 2} \rightarrow \hat{M}_1, \hat{M}_2 \]

\[ A^n(M_1, M_2) \rightarrow S^i \rightarrow \text{Enc} \rightarrow S^i \rightarrow \text{Dec} \rightarrow Y^n \]

\[ P_{Y_1, Y_2|X, S} \]

\[ P_{S|A} \]

\[ Z^i \rightarrow P_{Z|X_1, X_2, A} \]
Multi-User Duality - Additional Examples

Action-Dependent Output-Degraded BC vs. Dual:

\[
(M_1, M_2) \xrightarrow{X^n} P_{Y_1, Y_2|X, S} \xrightarrow{Y_1^n} \hat{M}_1 \leftarrow \text{Dec 1}
\]

\[
(M_1, M_2) \xrightarrow{A^n(M_1, M_2)} S^i \xrightarrow{P_{S|A}} \xrightarrow{S^i} \text{Enc}
\]

\[
A^n(M_1, M_2) \xrightarrow{S^i} \xrightarrow{P_{S|A}} \xrightarrow{S^i} \text{Enc}
\]

\[
X^n \xrightarrow{Y_1^n} \hat{M}_2 \xrightarrow{Y_2^n} \text{Dec 2}
\]

\[
X_1^n \xrightarrow{T_1} \text{Enc 1} \xrightarrow{T_2} \text{Enc 2}
\]

\[
X_2^n \xrightarrow{T_1} \text{Enc 1} \xrightarrow{T_2} \text{Enc 2}
\]

\[
Y^n \xrightarrow{Z^i} \text{Dec}
\]

\[
A^n(T_1, T_2) \xrightarrow{Z^i} \text{Dec}
\]

\[
P_{Z|X_1, X_2, A}
\]

\[
R_2 \leftarrow I(U; Y_2) \leftarrow I(V, A; Y_1, Y_2|U) \leftarrow R_1
\]

\[
R_2 \leftarrow I(U; X_2) \leftarrow I(V, A; X_1, X_2|U) \leftarrow R_1
\]
AK Problem with Cooperation - Achievability Outline

Encoder 1

$X_1$ → $T_1(X_1)$ → Decoder

$T_{12}(X_1)$

Encoder 2

$X_2$ → $T_2(T_{12}, X_2)$ → $Y$

Goldfeld/Permuter/Kramer

BCs with Cooperation: Capacity and Duality
AK Problem with Cooperation - Achievability Outline

Encoder 1

$X_1 \rightarrow T_1(X_1) \rightarrow \text{Decoder} \rightarrow Y$

Encoder 2

$X_2 \rightarrow T_{12}(X_1) \rightarrow T_2(T_{12}, X_2)$

<table>
<thead>
<tr>
<th>Rate</th>
<th>Corner Point 1</th>
<th>Corner Point 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{12}$</td>
<td>$I(V; X_1) - I(V; X_2)$</td>
<td>$I(V; X_1) - I(V; X_2)$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>$H(X_1)$</td>
<td>$H(X_1</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$I(U; X_2</td>
<td>V) - I(U; X_1</td>
</tr>
</tbody>
</table>
AK Problem with Cooperation - Achievability Outline

Encoder 1
\[ T_1(X_1) \]
\[ T_{12}(X_1) \]
Encoder 2
\[ T_2(T_{12}, X_2) \]
Decoder
\[ Y \]

Rate | Corner Point 1 | Corner Point 2
--- | --- | ---
\( R_{12} \) | \( I(V; X_1) - I(V; X_2) \) | \( I(V; X_1) - I(V; X_2) \)
\( R_1 \) | \( H(X_1) \) | \( H(X_1|V,U) \)
\( R_2 \) | \( I(U; X_2|V) - I(U; X_1|V) \) | \( I(U; X_2|V) + I(V; X_1) \)

**Cooperation:** Wyner-Ziv scheme to convey \( V \) via cooperation link.
**Cooperation**: Wyner-Ziv scheme to convey $V$ via cooperation link.

**Corner Point 1**: $V$ is transmitted to dec. by Enc. 1 within $X_1$. 

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**AK Problem with Cooperation - Achievability Outline**

![Diagram of encoder and decoder with cooperation](image)

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- **Cooperation:** Wyner-Ziv scheme to convey $V$ via cooperation link.
- **Corner Point 1:** $V$ is transmitted to dec. by Enc. 1 within $X_1$.
- **Corner Point 2:** $V$ is explicitly transmitted to dec. by Enc. 2.
Converse:
Converse:

- Standard techniques while defining

\[ V_i = (T_{12}, X_1^{n\setminus i}, X_2^{n, i+1}), \]
\[ U_i = T_2, \]

for every \( 1 \leq i \leq n \).
Converse:

- Standard techniques while defining

\[ V_i = (T_{12}, X_{1}^{n \setminus i}, X_{2,i+1}^{n}) , \]
\[ U_i = T_2 , \]

for every \( 1 \leq i \leq n \).

- Time mixing properties.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), \ j = 1, 2\):

\[
\begin{align*}
\text{Enc} & \quad X^n \\
\text{Channel} & \quad Y^n_1, Y^n_2 \\
\text{Dec 1} & \\
\text{Dec 2} &
\end{align*}
\]
Semi-Deterministic BC with Cooperation - Achievability

Outline

- Rate Splitting: $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting**: \( M_j = (M_{j0}, M_{jj}), j = 1, 2:\)
  - \((M_{10}, M_{20})\) - Public message;
  - \((M_{11}, M_{22})\) - Private messages.

- **Codebook Structure**: Marton:

\[ Y^n \]
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), j = 1, 2: \)
  - \((M_{10}, M_{20})\) - Public message;
  - \((M_{11}, M_{22})\) - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: \((M_{10}, M_{20}) \rightarrow V.\)
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V$.
  - Private Messages - Superposed on $V$.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V$.
  - Private Messages - Superposed on $V$:
    1. $M_{11} \rightarrow Y_1$.

\[
\begin{array}{c}
\text{Enc} \\
X^n
\end{array}
\xrightarrow{\text{Channel}}
\begin{array}{c}
Y_1^n \\
\text{Dec 1}
\end{array}
\begin{array}{c}
Y_2^n \\
\text{Dec 2}
\end{array}
\]
Semi-Deterministic BC with Cooperation - Achievability Outline

- **Rate Splitting:** \( M_j = (M_{j0}, M_{jj}), \ j = 1, 2: \)
  - \((M_{10}, M_{20})\) - Public message;
  - \((M_{11}, M_{22})\) - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: \((M_{10}, M_{20}) \rightarrow V.\)
  - Private Messages - Superposed on \(V:\)
    1. \(M_{11} \rightarrow Y_1;\)
    2. \(M_{22} \rightarrow U.\)
Semi-Deterministic BC with Cooperation - Achievability

**Outline**

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj}), j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V$.
  - Private Messages - Superposed on $V$:
    1. $M_{11} \rightarrow Y_1$;
    2. $M_{22} \rightarrow U$. 
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
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  - $(M_{11}, M_{22})$ - Private messages.

- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V$.
  - Private Messages - Superposed on $V$:
    1. $M_{11} \rightarrow Y_1$;
    2. $M_{22} \rightarrow U$.

- **Decoding:** Joint typicality decoding.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
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- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of $V^n - 2^{nR_{12}}$ bins.
Semi-Deterministic BC with Cooperation - Achievability

Outline

- **Rate Splitting:** $M_j = (M_{j0}, M_{jj})$, $j = 1, 2$:
  - $(M_{10}, M_{20})$ - Public message;
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- **Codebook Structure:** Marton:
  - Public Message: $(M_{10}, M_{20}) \rightarrow V$.
  - Private Messages - Superposed on $V$:
    1. $M_{11} \rightarrow Y_1$;
    2. $M_{22} \rightarrow U$.

- **Decoding:** Joint typicality decoding.

- **Cooperation:** Bin number of $V^n$ - $2^{nR_{12}}$ bins.

- **Gain:** Dec. 2 reduces search space of $V$ by $R_{12}$. 
Semi-Deterministic BC with Cooperation - Converse

Outline

Via telescoping identities:
Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$. 
Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,
Via telescoping identities:

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$$H(M_2) - n\epsilon_n$$
Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: \( V_i = (M_{12}, Y_{1i}^{i-1}, Y_{2,i+1}^n) \) and \( U_i = M_2. \)

2. Telescoping identities [Kramer, 2011], e.g.,

\[
H(M_2) - n\epsilon_n \leq I(M_2; Y_2^n | M_{12}) + I(M_2; M_{12})
\]
Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: $V_i = (M_{12}, Y_{1,i-1}^i, Y_{2,i+1}^n)$ and $U_i = M_2$.

2. Telescoping identities [Kramer, 2011], e.g.,

$$H(M_2) - n\epsilon_n \leq I(M_2; Y_2^n | M_{12}) + I(M_2; M_{12})$$
Via telescoping identities:

1. **Auxiliaries:** \( V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \) and \( U_i = M_2 \).

2. **Telescoping identities** [Kramer, 2011], e.g.,

\[
H(M_2) - n\epsilon_n \leq I(M_2; Y_{2,i}^n|M_{12}) + I(M_2; M_{12})
\]

\[
= \sum_{i=1}^{n} \left[ I(M_2; Y_{2,i}^n|M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i+1}^n|M_{12}, Y_1^i) \right] + I(M_2; M_{12})
\]
Via telescoping identities:

1. Auxiliaries: \( V_i = (M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^{n}) \) and \( U_i = M_2 \).

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\]

\[
= \sum_{i=1}^{n} \left[ I(M_2; Y_{2,i} | M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^{n}) - I(M_2; Y_{1,i} | M_{12}, Y_{1}^{i-1}, Y_{2,i+1}^{n}) \right]
\]

\[+ I(M_2; M_{12})\]
Semi-Deterministic BC with Cooperation - Converse Outline

Via telescoping identities:

1. Auxiliaries: \( V_i = (M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \) and \( U_i = M_2 \).

2. Telescoping identities [Kramer, 2011], e.g.,

\[
H(M_2) - n\epsilon_n \leq I(M_2; Y_{2,i}^n | M_{12}) + I(M_2; M_{12})
= \sum_{i=1}^{n} \left[ I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) - I(M_2; Y_{2,i}^n | M_{12}, Y_1^{i-1}) \right] + I(M_2; M_{12})
= \sum_{i=1}^{n} \left[ I(M_2; Y_{2,i} | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) - I(M_2; Y_1,i | M_{12}, Y_1^{i-1}, Y_{2,i+1}^n) \right] + I(M_2; M_{12})
\]

\( \star \) Replaces 2 uses of Csiszár Sum Identity.