Some Practice question for the Final

1. **Huffman code**
   Give a Huffman encoding into an alphabet of size $D = 4$ of the following probability mass function:

$$\mathbf{p} = (\frac{8}{36}, \frac{7}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36})$$

2. **Ternary Huffman word lengths**
   Which of the following sequences of word lengths *cannot* be the word lengths of a 3-ary Huffman code and which *can*?

   (a) $\mathbf{L} = (1, 1, 2, 2, 3, 3, 3)$

   (b) $\mathbf{L} = (1, 1, 2, 2, 3, 3)$

   (c) $\mathbf{L} = (1, 1, 2, 2, 3)$

   (d) $\mathbf{L} = (1, 2, 2, 2, 2, 2, 2)$

   (e) $\mathbf{L} = (1, 2, 2, 2, 2)$

3. **Cascade**
   Consider the two discrete memoryless channels $(\mathcal{X}, p_1(y|x), \mathcal{Y})$ and $(\mathcal{Y}, p_2(z|y), \mathcal{Z})$. Let the channel transition matrices for the cascade channels in the previous problem be

$X\backslash Y$	1	e	0
1	0	1	0
e	0	1	0
0	0	0	1

$p_1(y|x)$

$Y\backslash Z$	1	e	0
1	1	0	0
e	0	1	0
0	0	1	0

$p_2(z|y)$

   (a) What is the capacity C_1 of $p_1(y|x)$?

   (b) What is the capacity C_2 of $p_2(z|y)$?

(c) We now cascade these channels. Thus $p_3(z|x) = \sum_y p_1(y|x)p_2(z|y)$. What is the capacity C_3 of $p_3(z|x)$?

(d) Now let us actively intervene between channels 1 and 2, rather than passively transmit y^n. What is the capacity of channel 1 followed by channel 2 if you are allowed to decode the output y^n of channel 1 and then reencode it as $\tilde{y}^n$ for transmission over channel 2? (Think $W \longrightarrow x^n(W) \longrightarrow y^n \longrightarrow \tilde{y}^n(y^n) \longrightarrow z^n \longrightarrow \hat{W}$.)

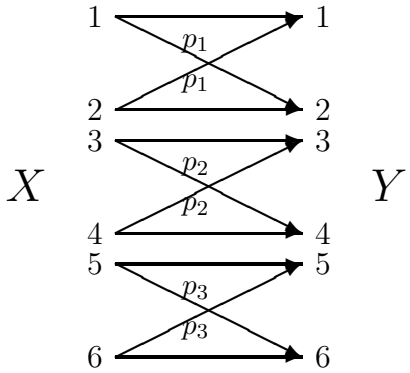
(e) What is the capacity of the cascade in part c) if the receiver can view *both* Y and Z?

4. **Noisy typewriter**
Find the capacity of th m-input channel in which $Y = X + Z \pmod m$, where $X \in \{0, 1, 2, \ldots, m-1\}$ and

$$Z = \begin{cases} 1, & \text{w.p. } \frac{3}{4} \\ 0, & \text{w.p. } \frac{1}{4} \end{cases}$$

5. **Several BSC's**

(a) What is the capacity of the 6-input, 6-output channel:



(b) What is the capacity if $p_1 = p_2 = p_3 = 0$?

(c) What is the capacity if $p_1 = p_2 = p_3 = \frac{1}{2}$?