1st Semester 2012/13

Homework Set #3

Source coding (Slepien-Wolf and Wyner-Ziv settings)

1. **Slepian-Wolf for deterministically related sources.** Find and sketch the Slepian-Wolf rate region for the simultaneous data compression of (X, Y), where $y = f(x)$ is some deterministic function of x.

2. **Slepian Wolf for binary sources.** Let X_i be i.i.d. Bernoulli(p). Let Z_i be i.i.d. $\sim$ Bernoulli(r), and let $\mathbf{Z}$ be independent of $\mathbf{X}$. Finally, let $\mathbf{Y} = \mathbf{X} \oplus \mathbf{Z}$ (mod 2 addition). Let $\mathbf{X}$ be described at rate R_1 and $\mathbf{Y}$ be described at rate R_2. What region of rates allows recovery of $\mathbf{X}, \mathbf{Y}$ with probability of error tending to zero?

3. **Computing simple example of Slepian Wolf**

   Let (X, Y) have the joint pmf $p(x, y)$

p(x,y)	1	2	3
1	α	β	β
2	β	α	β
3	β	β	α

   where $\beta = \frac{1}{6} - \frac{\alpha}{2}$. (Note: This is a joint, not a conditional, probability mass function.)

   (a) Find the Slepian Wolf rate region for this source.

   (b) What is $\Pr\{X = Y\}$ in terms of α?

   (c) What is the rate region if $\alpha = \frac{1}{3}$?

   (d) What is the rate region if $\alpha = \frac{1}{9}$?

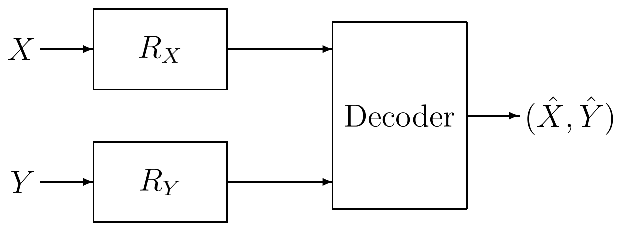
4. **An example of Slepian-Wolf**: Two senders know random variables U_1 and U_2 respectively. Let the random variables (U_1, U_2) have the following joint distribution:

$U_1 \backslash U_2$	0	1	2	$\cdots$	$m-1$
0	α	$\frac{\beta}{m-1}$	$\frac{\beta}{m-1}$	$\cdots$	$\frac{\beta}{m-1}$
1	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
2	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$m-1$	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0

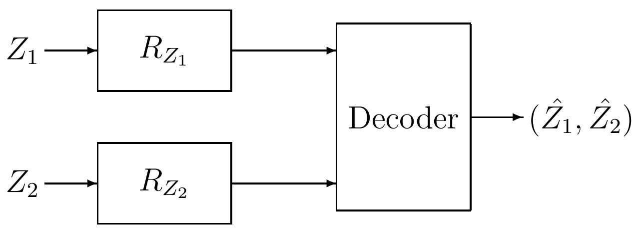
where $\alpha + \beta + \gamma = 1$. Find the region of rates (R_1, R_2) that would allow a common receiver to decode both random variables reliably.

5. **Stereo.** The sum and the difference of the right and left ear signals are to be individually compressed for a common receiver. Let Z_1 be Bernoulli (p_1) and Z_2 be Bernoulli (p_2) and suppose Z_1 and Z_2 are independent. Let $X = Z_1 + Z_2$, and $Y = Z_1 - Z_2$.

   (a) What is the Slepian Wolf rate region of achievable (R_X, R_Y)?



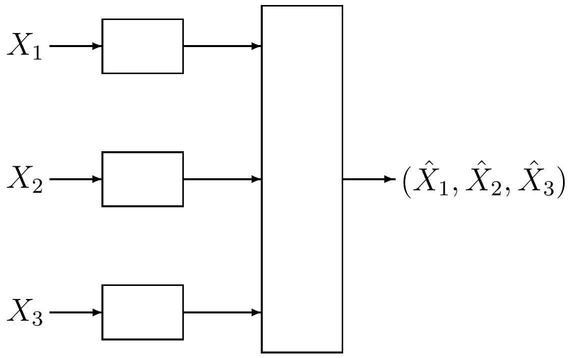
   (b) Is this larger or smaller than the rate region of (R_{Z_1}, R_{Z_2})? Why?



   There is a simple way to do this part.

6. **Distributed data compression.** Let Z_1, Z_2, Z_3 be independent Bernoulli(p). Find the Slepian-Wolf rate region for the description of (X_1, X_2, X_3) where

$$\begin{aligned} X_1 &= Z_1 \\ X_2 &= Z_1 + Z_2 \\ X_3 &= Z_1 + Z_2 + Z_3 \ . \end{aligned}$$



7. **Gaussian Wyner-Ziv** Show that for a mean square distortion where the source X and Y are jointly Gaussian, the best auxiliary random variable is Gaussian and the distortion is as Y is known to the Encoder.

8. **Wyner ziv with two decoders and one message** Consider the coordination Wyner-Ziv problem, but where the message at rate R reaches two decoders. Assume that there is a degradation of the side information at the decoders, and the first decoder has a side information Y and the second decoder does not have any side information at all. Let $\hat{X}_1$ be the action taken by Decode 1 and $\hat{X}_2$ the action taken by Decode 2. Show that in order to achieve a (weak) coordination $P_0(x,y)P(\hat{x}_2|x)P(\hat{x}_1|x,y,\hat{x}_2)$ then

$$R \geq I(X;\hat{X}_2) + I(X;U|X_2,Y) \qquad (1)$$

where the joint distribution is of the form $P_0(x,y)P(\hat{x}_2|x)P(\hat{u}|x,y,\hat{x}_2)P(\hat{x}_1|u,x,y,\hat{x}_2)$ with a marginal (summing over u) is the coordination pmf $P_0(x,y)P(\hat{x}_2|x)P(\hat{x}_1|x,y,\hat{x}_2)$. Furthermore, show that if (1) holds for some auxiliary U then there exists a code that achieves a coordination $P_0(x,y)P(\hat{x}_2|x)P(\hat{x}_1|x,y,\hat{x}_2)$.