Multiple Access Channel with Partial and Controlled Cribbing Encoders

Himanshu Asnani
(Stanford University)

Haim Permuter
(Ben-Gurion University)

ISIT, St. Petersburg, Russia
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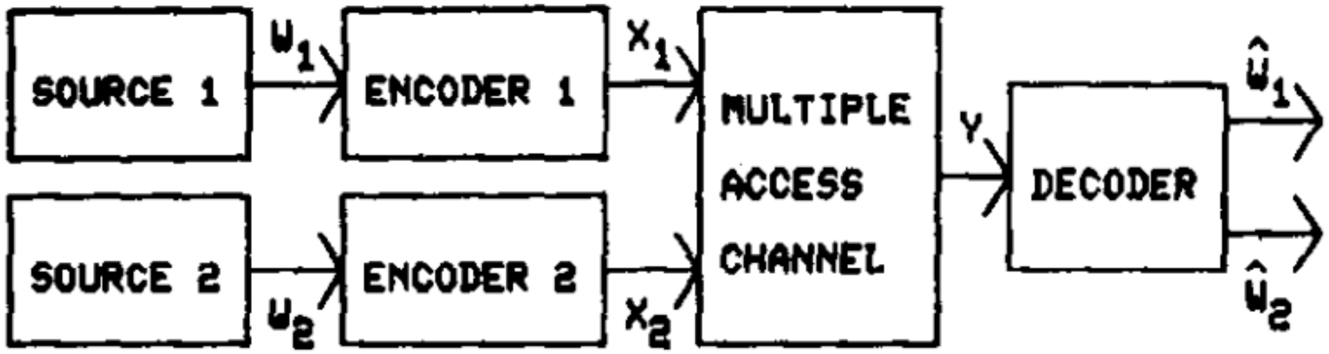
What is Cribbing ?

[Willems82]



Both encoders do not crib.

What is Cribbing ?

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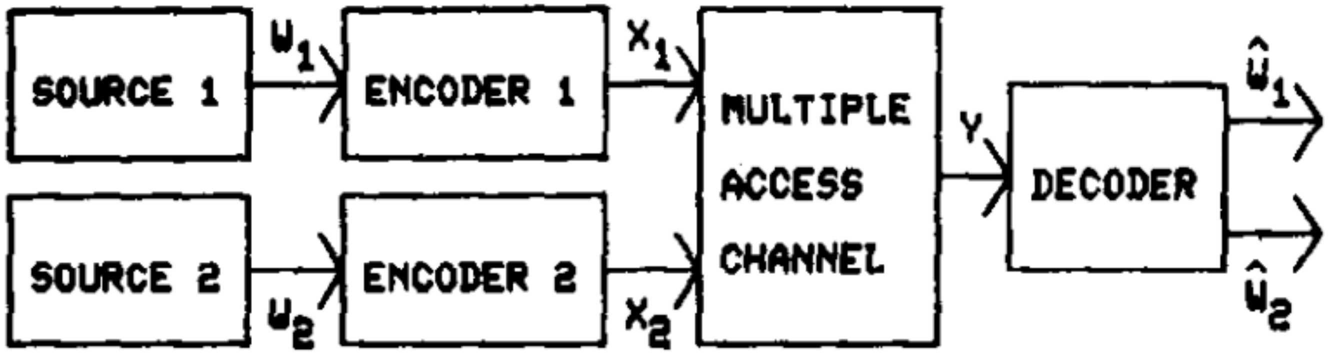

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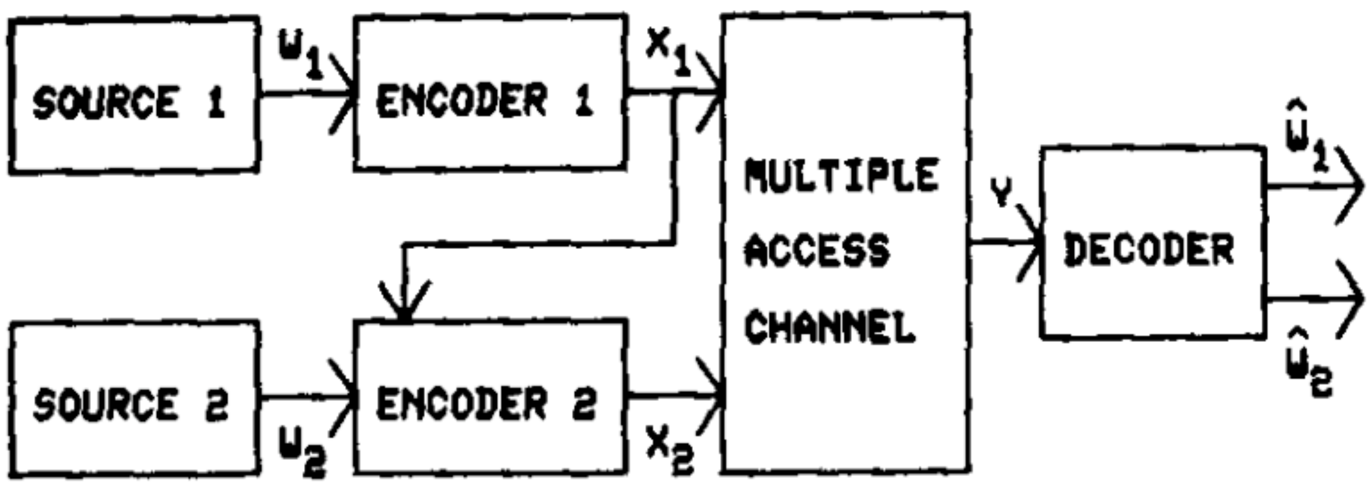

Only one encoder cribs.

What is Cribbing ?

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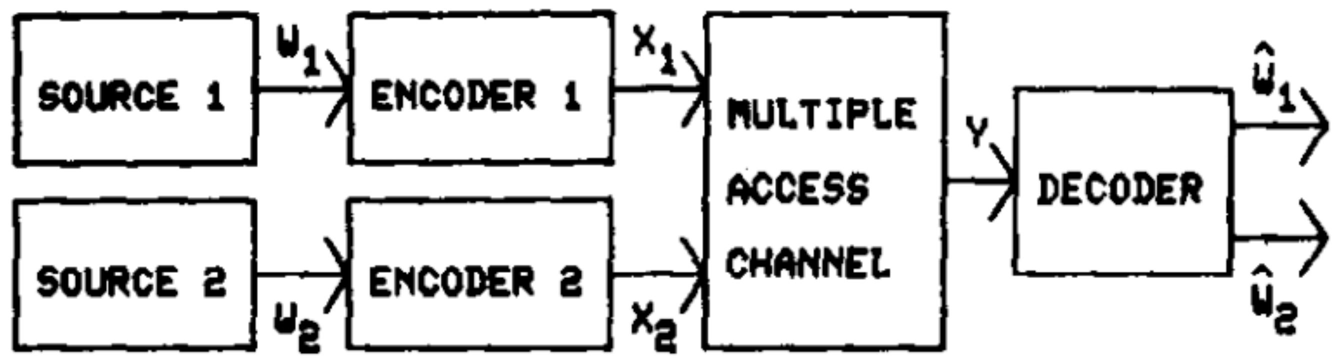

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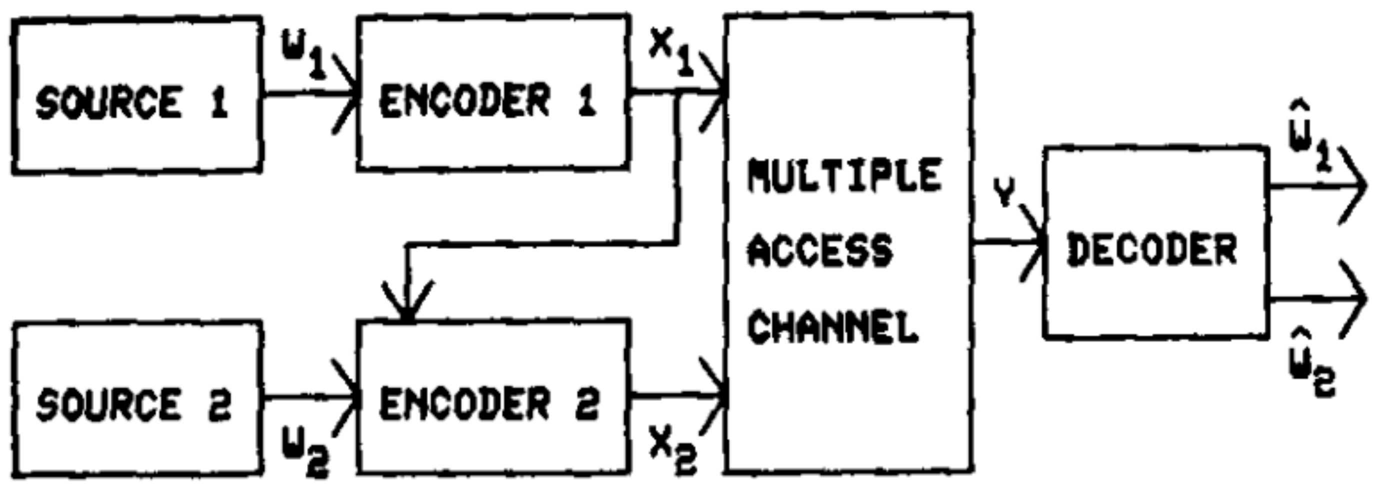

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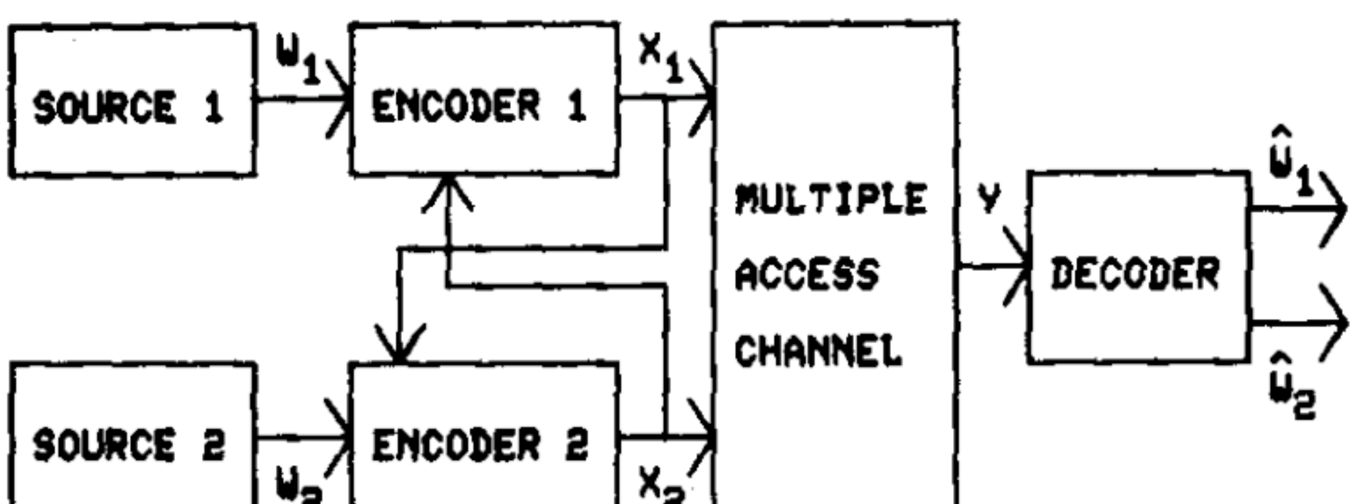
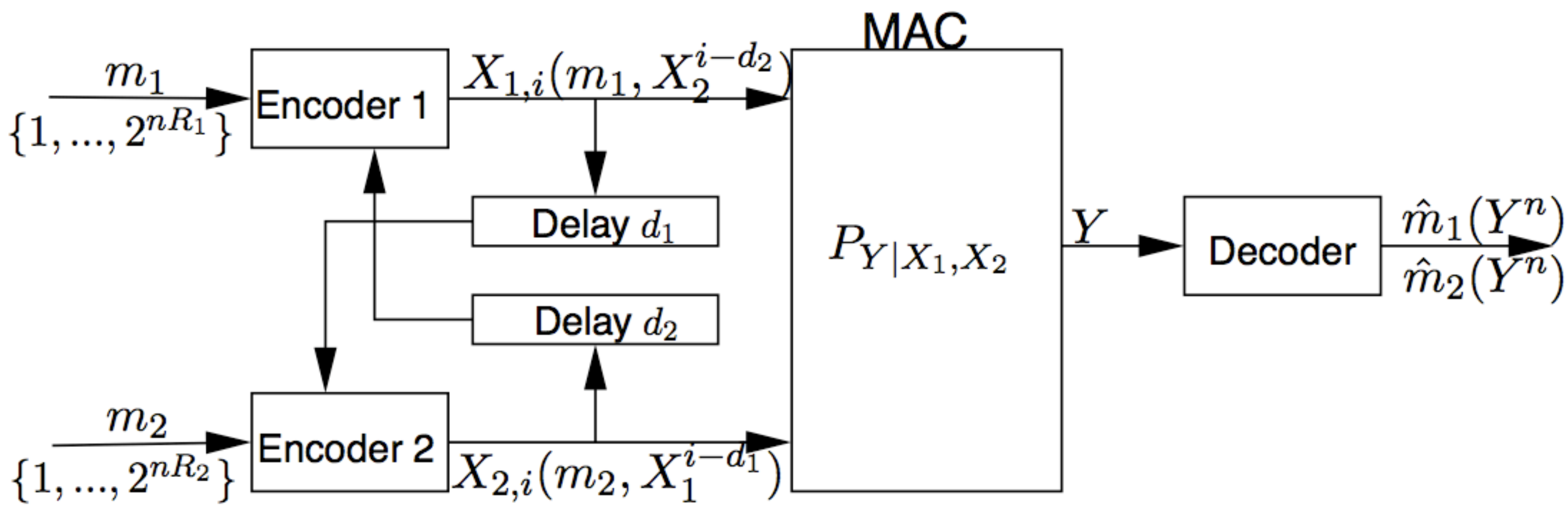


Both encoders crib.

What is Cribbing ?

Each encoder knows the output of other encoder with some fixed delay [Willems82]



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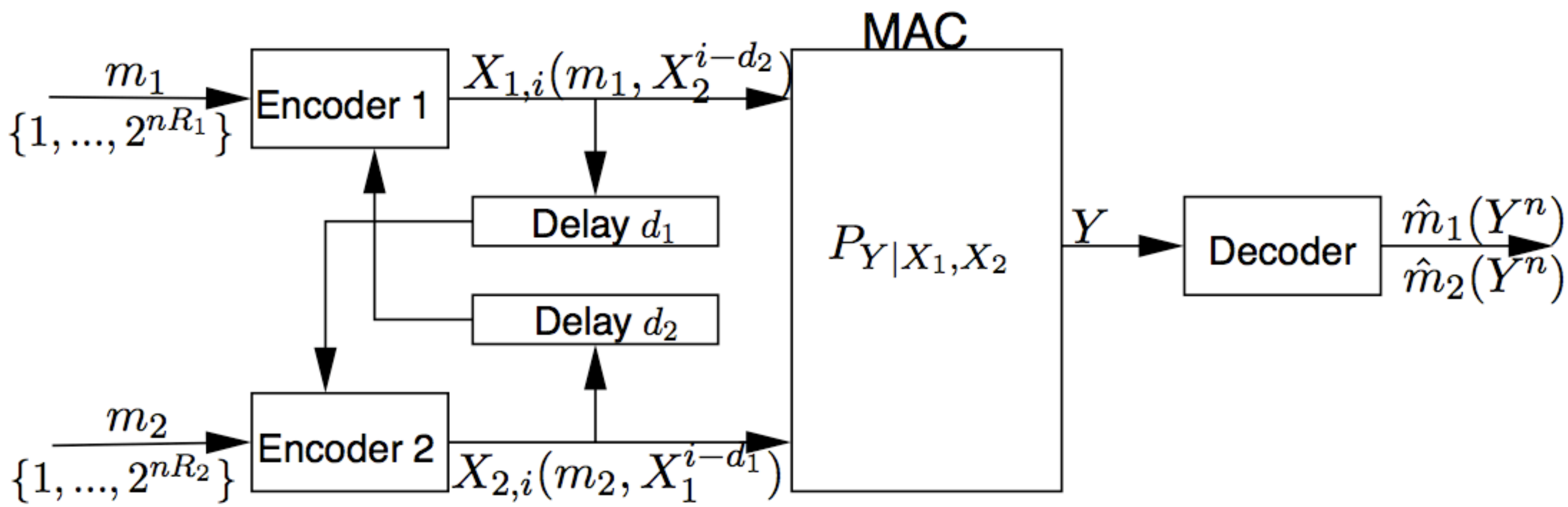
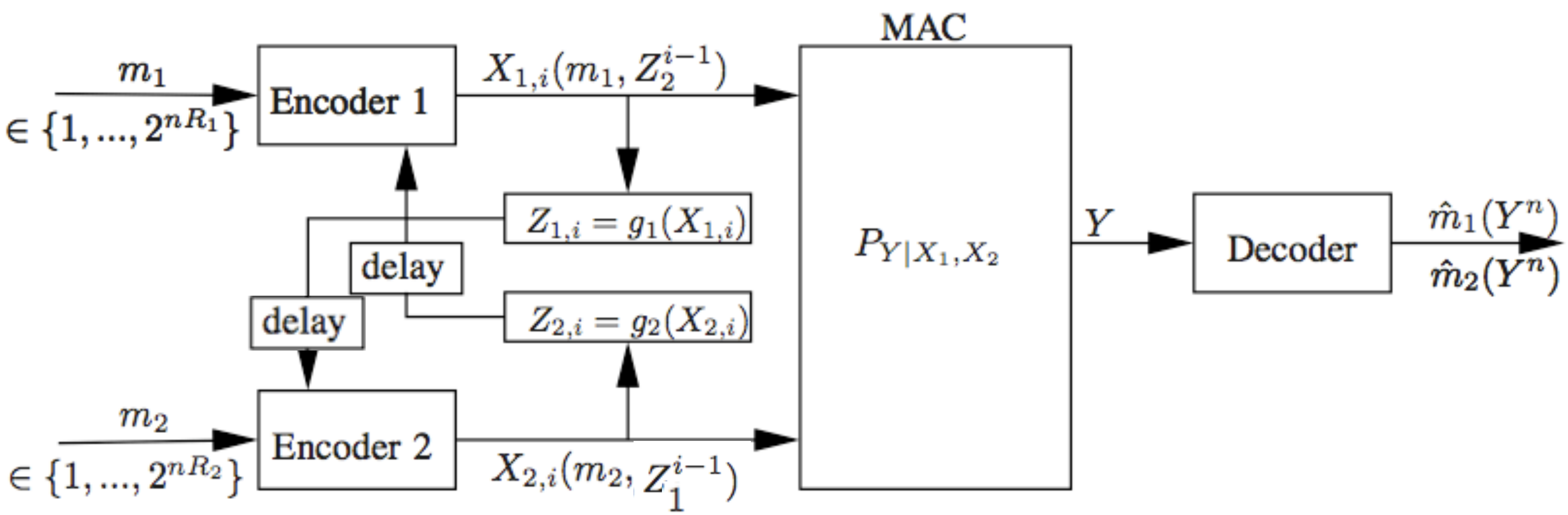


- Motivation : Cognitive Radio, Cooperation, Relay.
- Cribbing with State [BrossLapidoth10]
- Interference Channel with Cribbing Encoder [BrossSteinbergTinguely10]
- Trivial for Gaussian [Willems05]

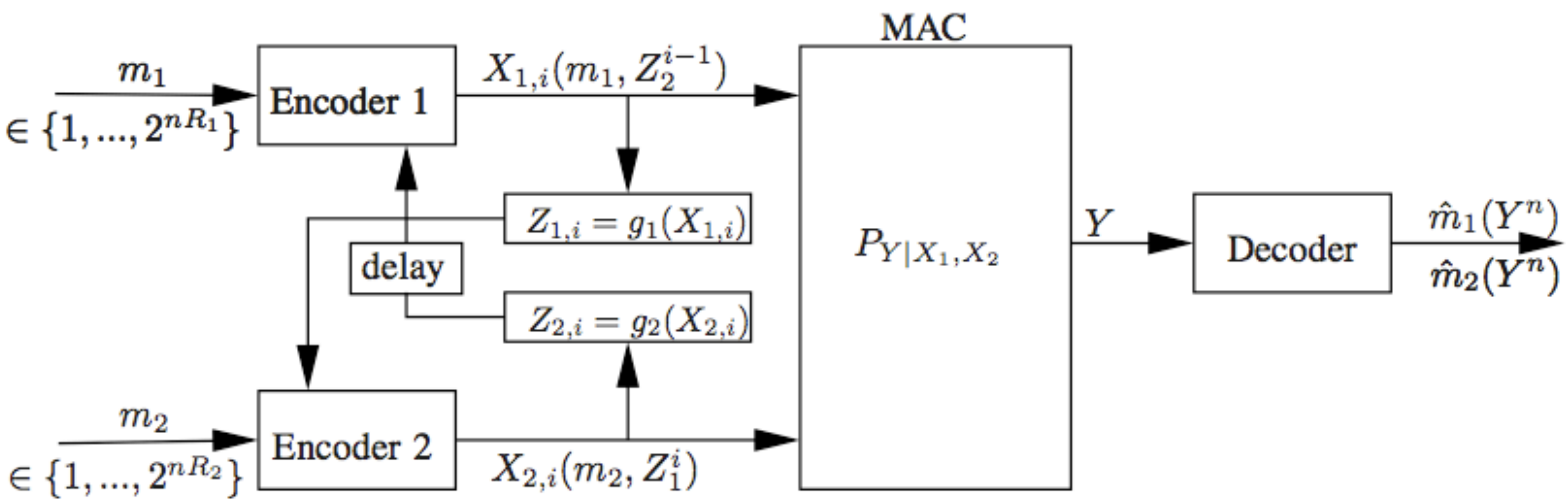
Partial (deterministic-function) Cribbing

Case A : The cribbing at both encoders is *strictly causal*.



Partial (deterministic-function) Cribbing

Case B : The cribbing at one encoder is *strictly causal* and at other encoder is *causal*.



Strictly Causal Cribbing (Case A)

Capacity Region $\mathcal{R}_A$

$$\mathcal{R}_A = \left\{ \begin{array}{l} R_1 \le H(Z_1|U) + I(X_1;Y|X_2,Z_1,U), \\ R_2 \le H(Z_2|U) + I(X_2;Y|X_1,Z_2,U), \\ R_1 + R_2 \le I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \le I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|u)P(y|x_1,x_2). \end{array} \right\}$$

Strictly Causal Cribbing (Case A)

Capacity Region $\mathcal{R}_A$

$$\mathcal{R}_A = \left\{ \begin{array}{l} R_1 \leq H(Z_1|U) + I(X_1;Y|X_2,Z_1,U), \\ R_2 \leq H(Z_2|U) + I(X_2;Y|X_1,Z_2,U), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|u)P(y|x_1,x_2). \end{array} \right\}$$

Mixed Strictly Causal and Causal Cribbing (Case B)

Capacity Region $\mathcal{R}_B$

$$\mathcal{R}_B = \left\{ \begin{array}{l} R_1 \leq H(Z_1|U) + I(X_1;Y|X_2,Z_1,U), \\ R_2 \leq H(Z_2|U) + I(X_2;Y|X_1,Z_2,U), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|z_1,u)P(y|x_1,x_2). \end{array} \right\}$$

Strictly Causal Cribbing (Case A)

Capacity Region

$$\mathcal{R}_A = \left\{ \begin{array}{l} R_1 \leq H(Z_1|U) + I(X_1;Y|X_2,Z_1,U), \\ R_2 \leq H(Z_2|U) + I(X_2;Y|X_1,Z_2,U), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|u)P(y|x_1,x_2). \end{array} \right\}$$

$Z_1 = \phi, Z_2 = \phi$

$$\mathcal{R}_{nc} = \left\{ \begin{array}{l} R_1 \leq I(X_1;Y|X_2), \\ R_2 \leq I(X_2;Y|X_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{for} \\ P(x_1)P(x_2)P(y|x_1,x_2). \end{array} \right\}$$

MAC with no cooperation

Strictly Causal Cribbing (Case A)

Capacity Region

$$\mathcal{R}_A = \left\{ \begin{array}{l} R_1 \leq H(Z_1|U) + I(X_1;Y|X_2,Z_1,U), \\ R_2 \leq H(Z_2|U) + I(X_2;Y|X_1,Z_2,U), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|u)P(y|x_1,x_2). \end{array} \right\}$$

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$$\mathcal{R}_{nc} = \left\{ \begin{array}{l} R_1 \leq I(X_1;Y|X_2), \\ R_2 \leq I(X_2;Y|X_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{for} \\ P(x_1)P(x_2)P(y|x_1,x_2). \end{array} \right\}$$

MAC with no cooperation

$Z_1 = X_1, Z_2 = X_2$

$$\mathcal{R}_c = \left\{ \begin{array}{l} R_1 \leq H(X_1|U), \\ R_2 \leq H(X_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1|u)P(x_2|u)P(y|x_1,x_2). \end{array} \right\}$$

MAC with perfect cribbing

Achievability Outline : Coding Techniques Used

- Block Markov Coding.
- Superposition Coding.
- Shannon Strategies.
- Backward Decoding.
- Rate Splitting.

Achievability Outline : Code Book Generation (Case A)

Divide a block of length Bn into B blocks of length n.

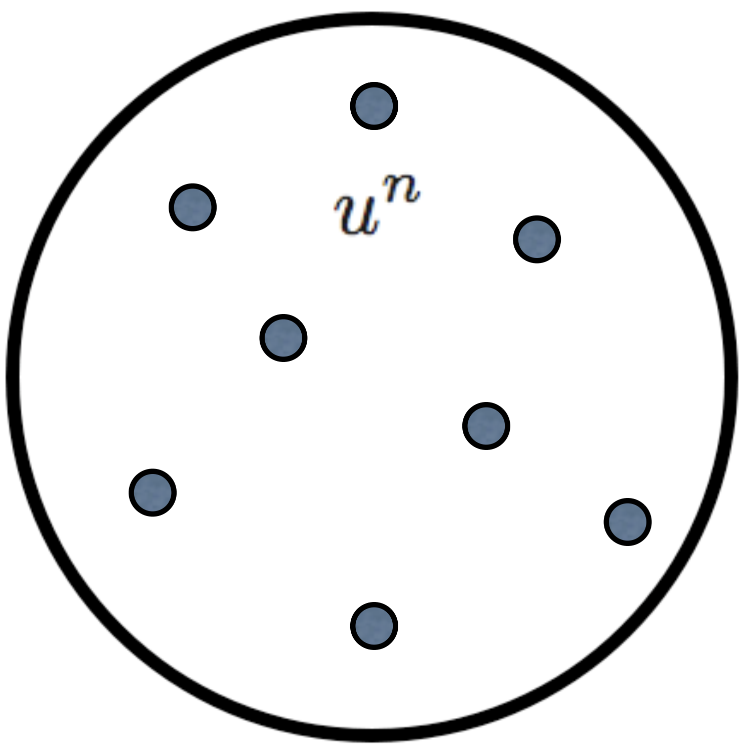
$R_1 = R_1' + R_1''$ $\quad$ $R_2 = R_2' + R_2''$

Achievability Outline : Code Book Generation (Case A)

Divide a block of length Bn into B blocks of length n.

$R_1 = R_1' + R_1''$ $\qquad$ $R_2 = R_2' + R_2''$

$2^{n(R_1'+R_2')}$
$\sim P(u)$



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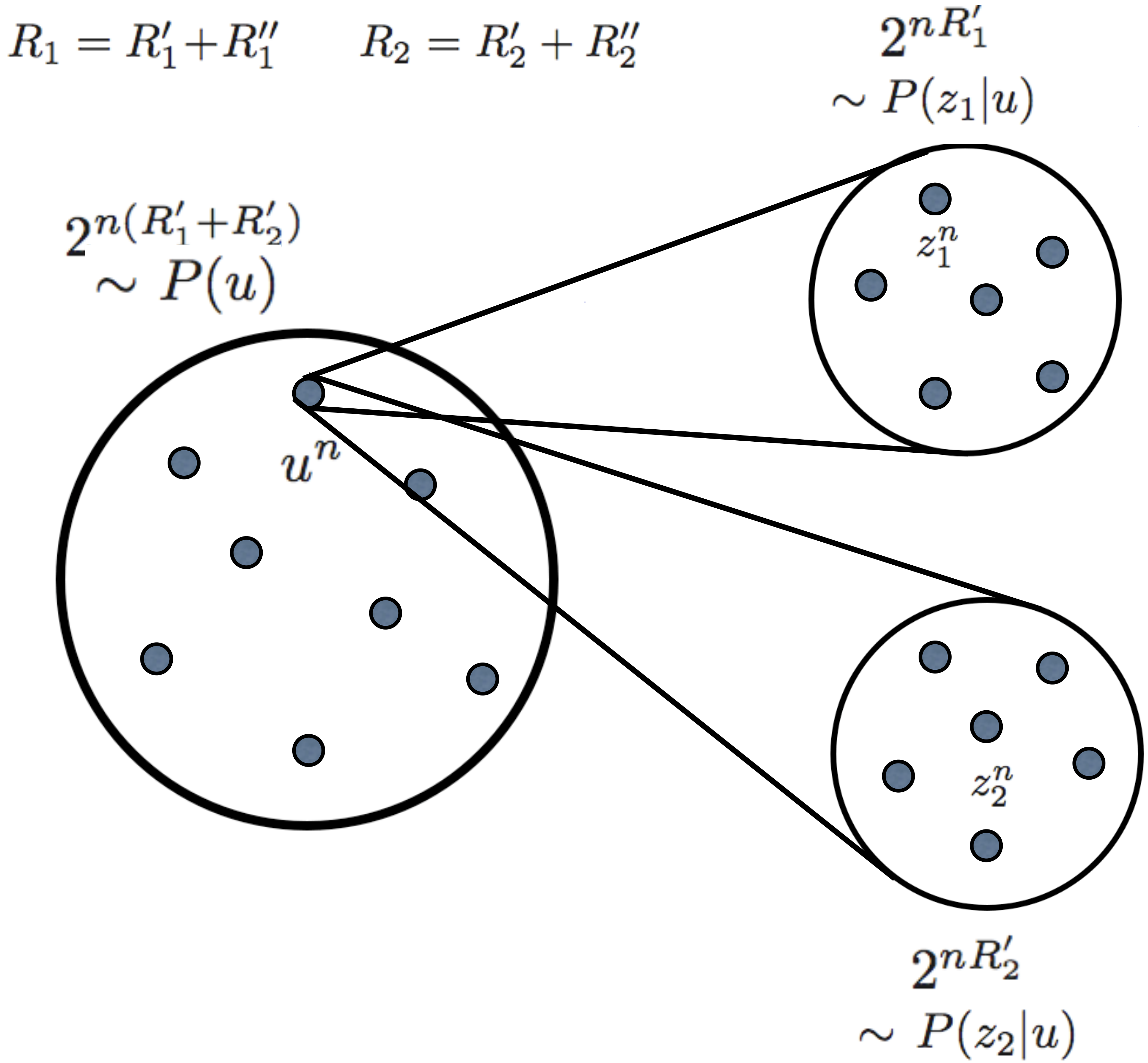

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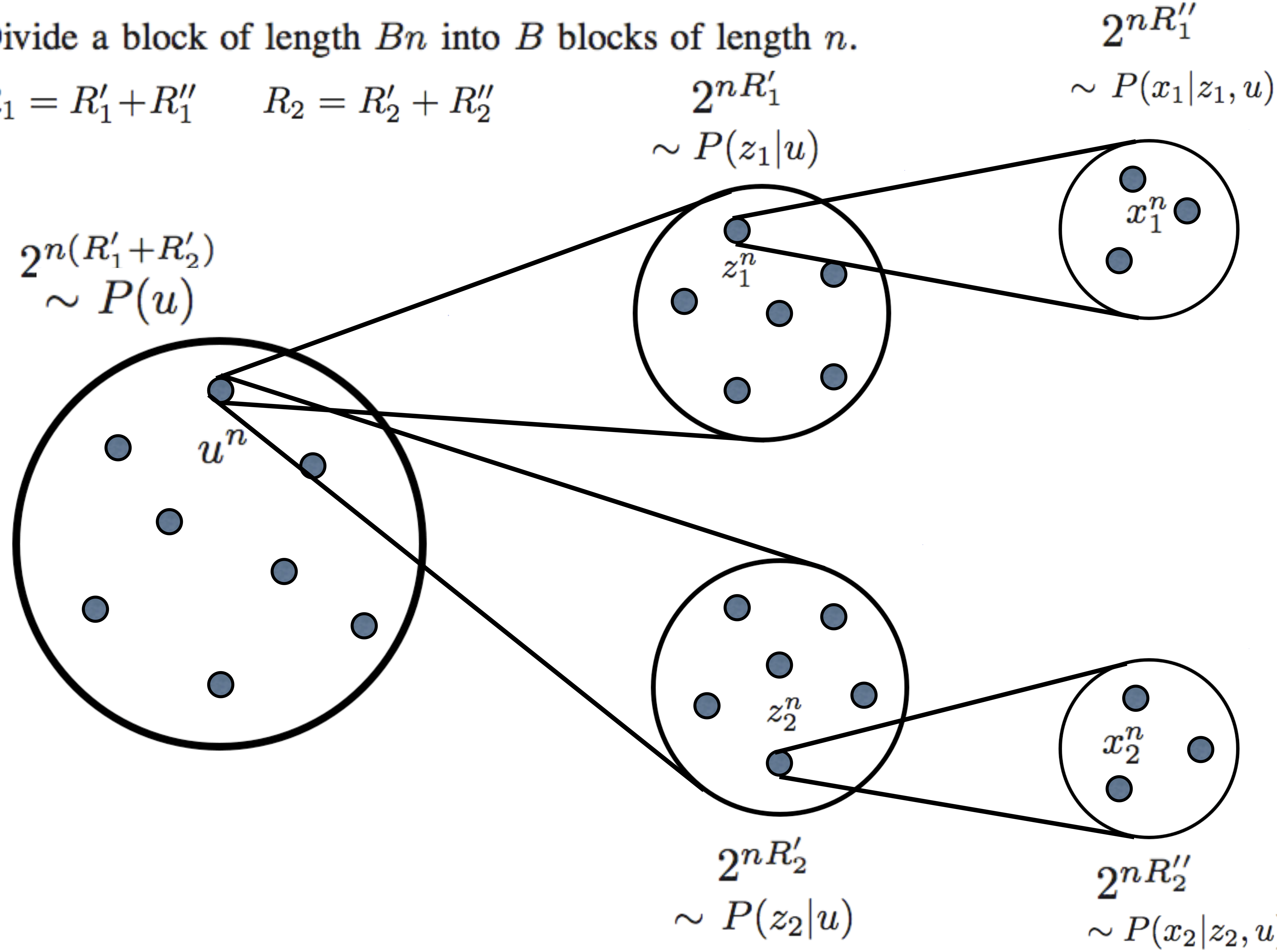
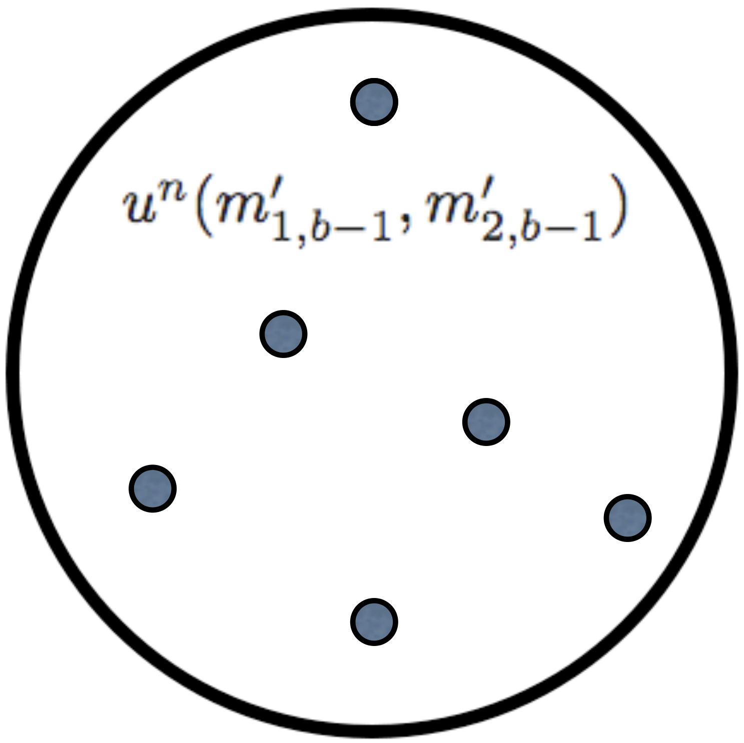


Achievability Outline : Encoding/Decoding (Case A)

Encoding : Block Markov Coding

$(m'_{1,b}, m''_{1,b})$

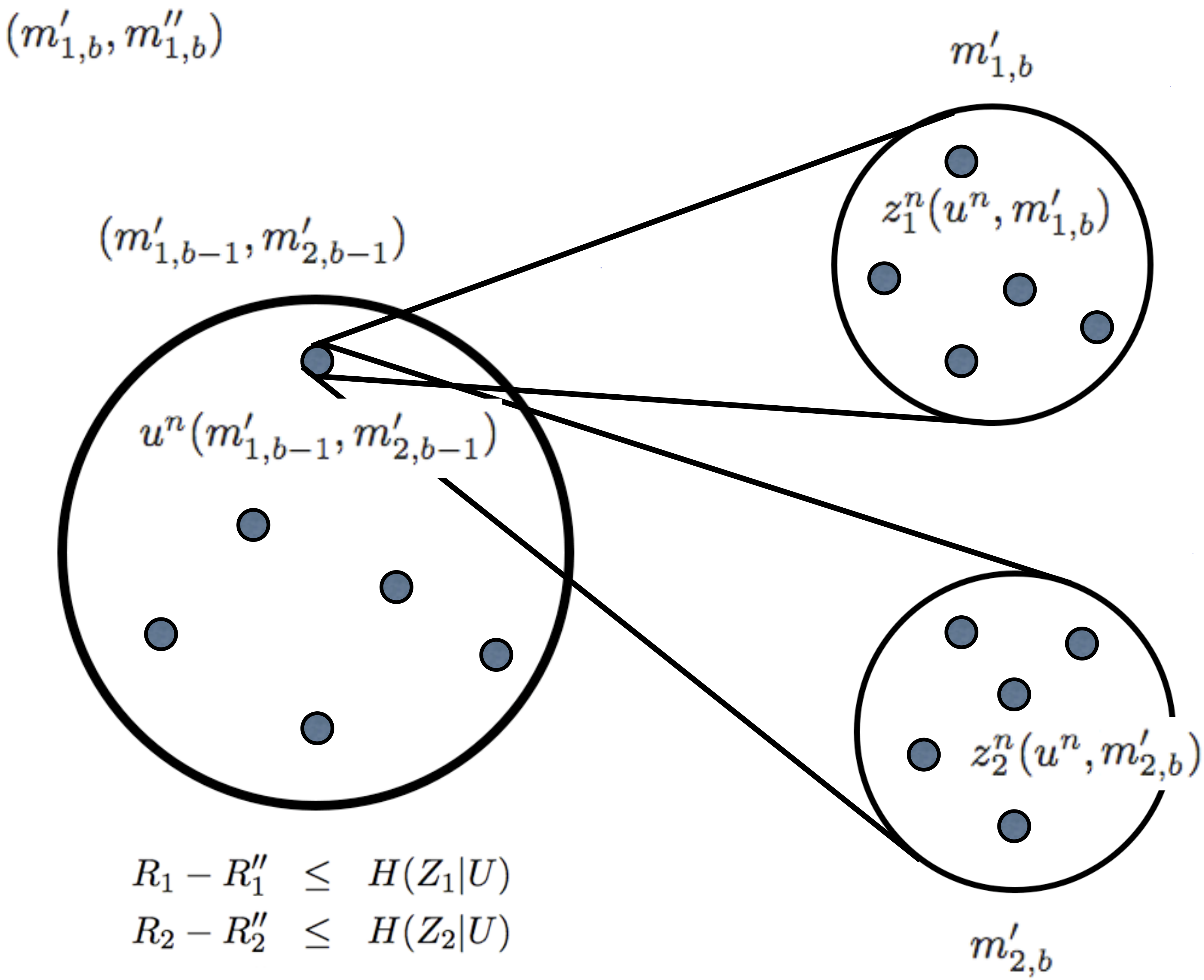
$(m'_{1,b-1}, m'_{2,b-1})$



$$
\begin{aligned}
R_1 - R_1'' &\leq H(Z_1|U) \\
R_2 - R_2'' &\leq H(Z_2|U)
\end{aligned}
$$

Achievability Outline : Encoding/Decoding (Case A)

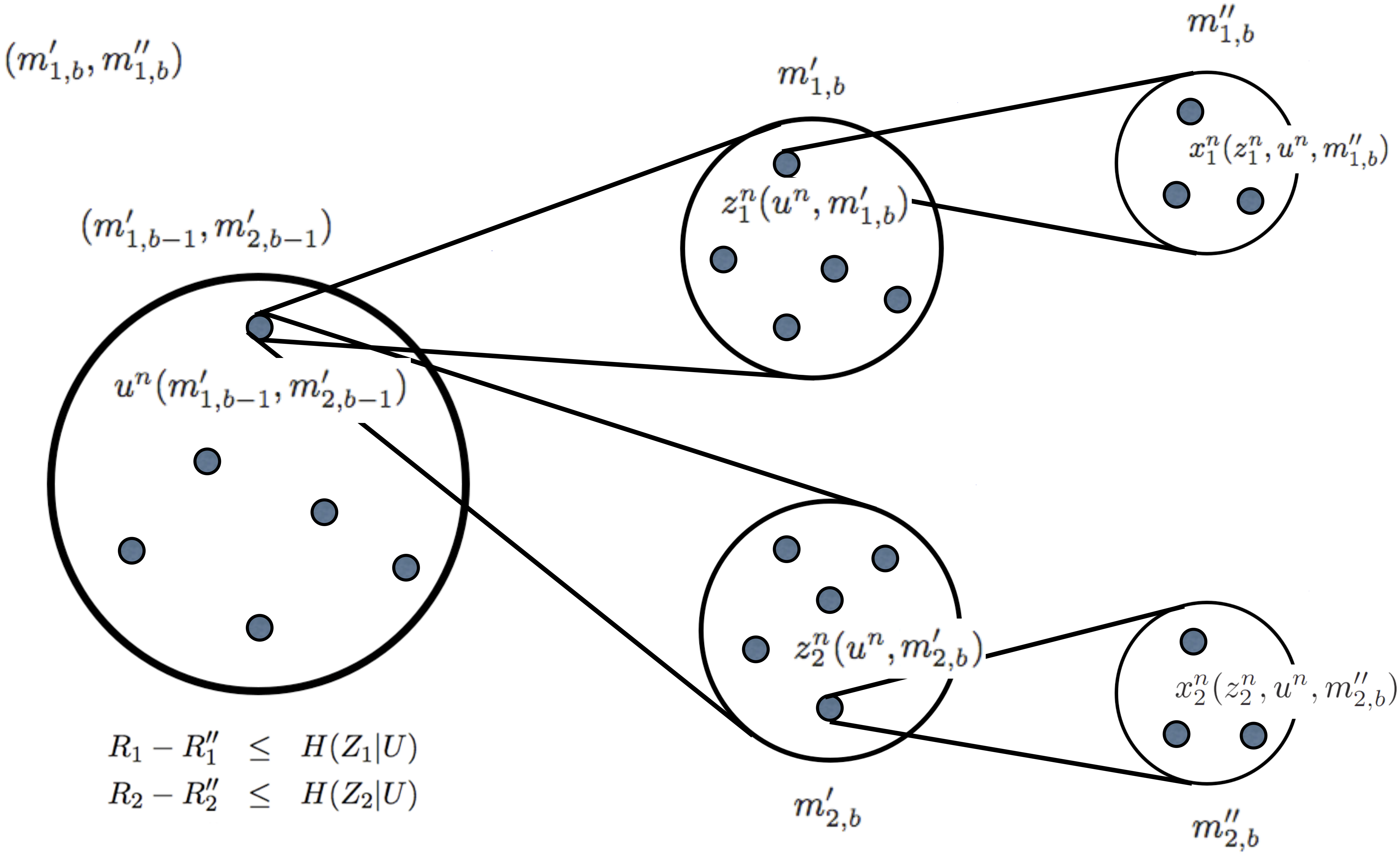
Encoding : Block Markov Coding



$$\begin{aligned} R_1 - R_1'' &\leq H(Z_1|U) \\ R_2 - R_2'' &\leq H(Z_2|U) \end{aligned}$$

Achievability Outline : Encoding/Decoding (Case A)

Encoding : Block Markov Coding



$$
\begin{aligned}
R_1 - R_1'' &\leq H(Z_1|U) \\
R_2 - R_2'' &\leq H(Z_2|U)
\end{aligned}
$$

Achievability Outline : Encoding/Decoding (Case A)

Backward Decoding

At block b, $(m'_{1,b}, m'_{2,b})$ is already known

Achievability Outline : Encoding/Decoding (Case A)

Backward Decoding

At block b, $(m'_{1,b}, m'_{2,b})$ is already known

at block b it looks for $(\hat{m}'_{1,b-1}, \hat{m}'_{2,b-1})$, $\hat{m}''_{1,b}$ and $\hat{m}''_{2,b}$ for which

$$(u^n(\hat{m}'_{1,b-1}, \hat{m}'_{2,b-1}), z_1^n(u^n, m'_{1,b}), z_2^n(u^n, m'_{2,b}), x_1^n(z_1^n, u^n, \hat{m}''_{1,b}), x_2^n(z_2^n, u^n, \hat{m}''_{2,b})$$

$$\in T_\epsilon^{(n)}(U, Z_1, Z_2, X_1, X_2, Y)$$

Achievability Outline : Error Analysis (Case A)

$$
\begin{aligned}
R_1 - R_1'' &\leq H(Z_1|U), \\
R_2 - R_2'' &\leq H(Z_2|U), \\
R_1'' &\leq I(X_1;Y|X_2,Z_1,U), \\
R_2'' &\leq I(X_2;Y|X_1,Z_2,U), \\
R_1'' + R_2'' &\leq I(X_1,X_2;Y|Z_1,Z_2,U), \\
R_1 + R_2 &\leq I(X_2,X_1;Y),
\end{aligned}
$$

Achievability Outline : Error Analysis (Case A)

$$
\begin{aligned}
R_1 - R_1'' &\leq H(Z_1|U), \\
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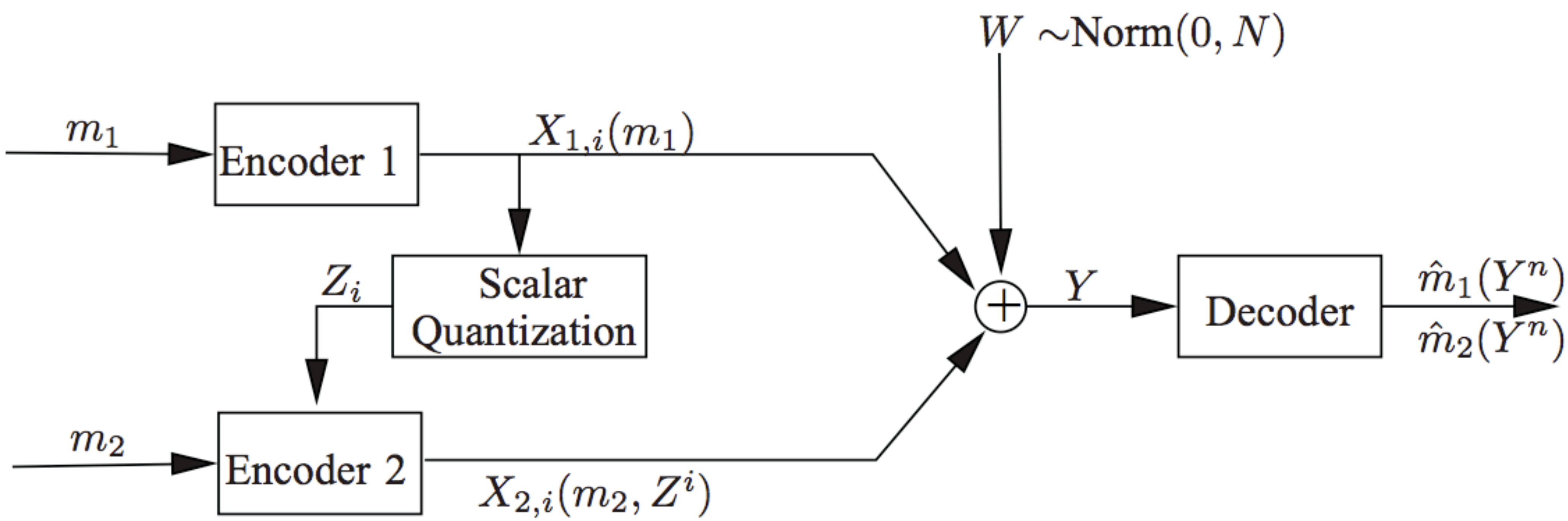
R_1'' + R_2'' &\leq I(X_1,X_2;Y|Z_1,Z_2,U), \\
R_1 + R_2 &\leq I(X_2,X_1;Y),
\end{aligned}
$$

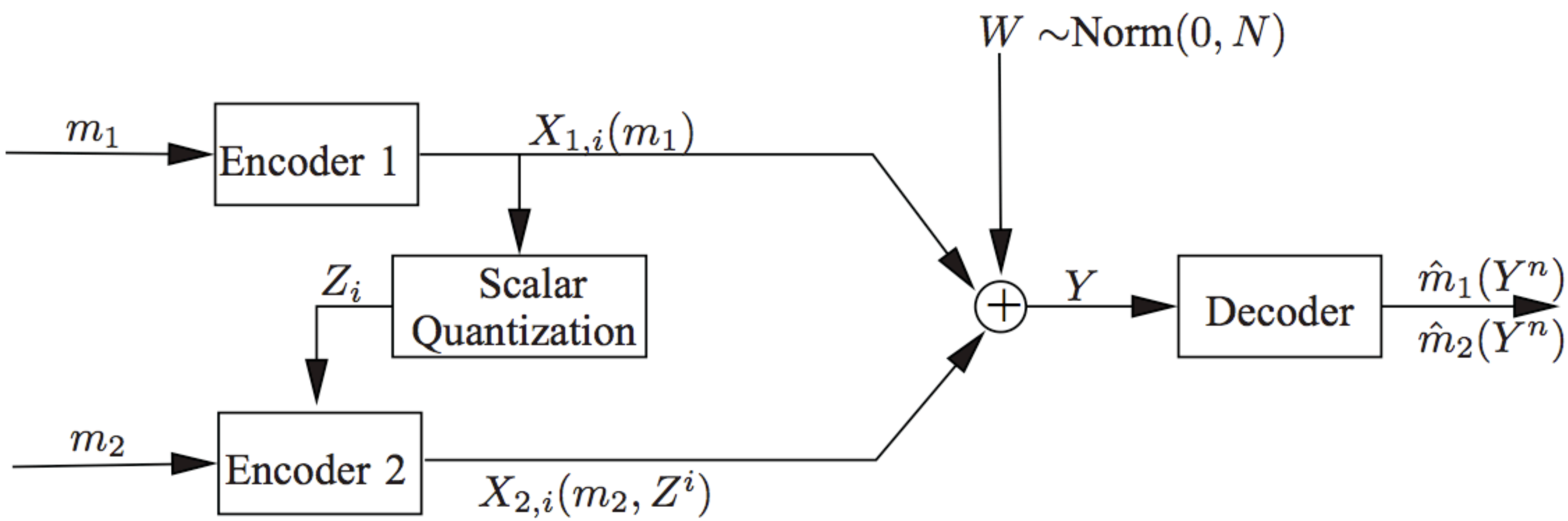
Using Fourier–Motzkin elimination

$$
\begin{aligned}
R_1 - H(Z_1|U) &\leq I(X_1;Y|X_2,Z_1,U), \\
R_2 - H(Z_2|U) &\leq I(X_2;Y|X_1,Z_2,U), \\
R_1 - H(Z_1|U) + R_2 - H(Z_2|U) &\leq I(X_1,X_2;Y|Z_1,Z_2,U), \\
R_1 + R_2 &\leq I(X_2,X_1;Y),
\end{aligned}
$$

Additive White Gaussian MAC with Quantized Cribbing



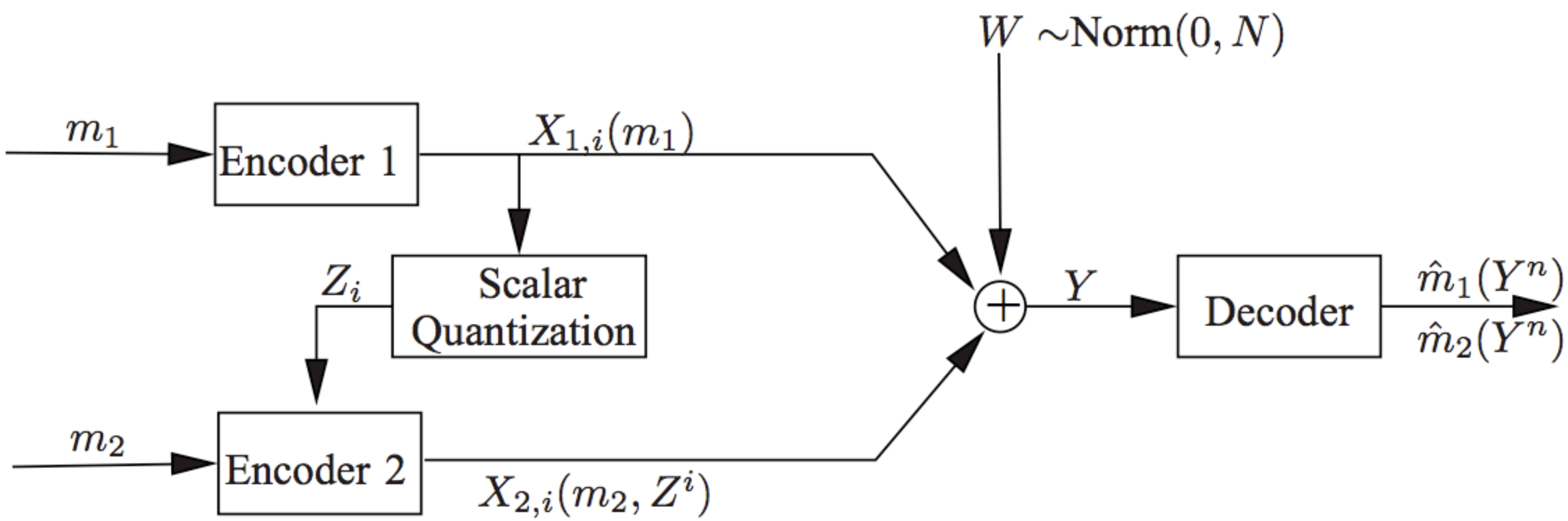
Additive White Gaussian MAC with Quantized Cribbing



$$\frac{1}{n} \sum_{i=1}^{n} E[X_{1,i}^2] \le P_1$$

$$\frac{1}{n} \sum_{i=1}^{n} E[X_{2,i}^2] \le P_2$$

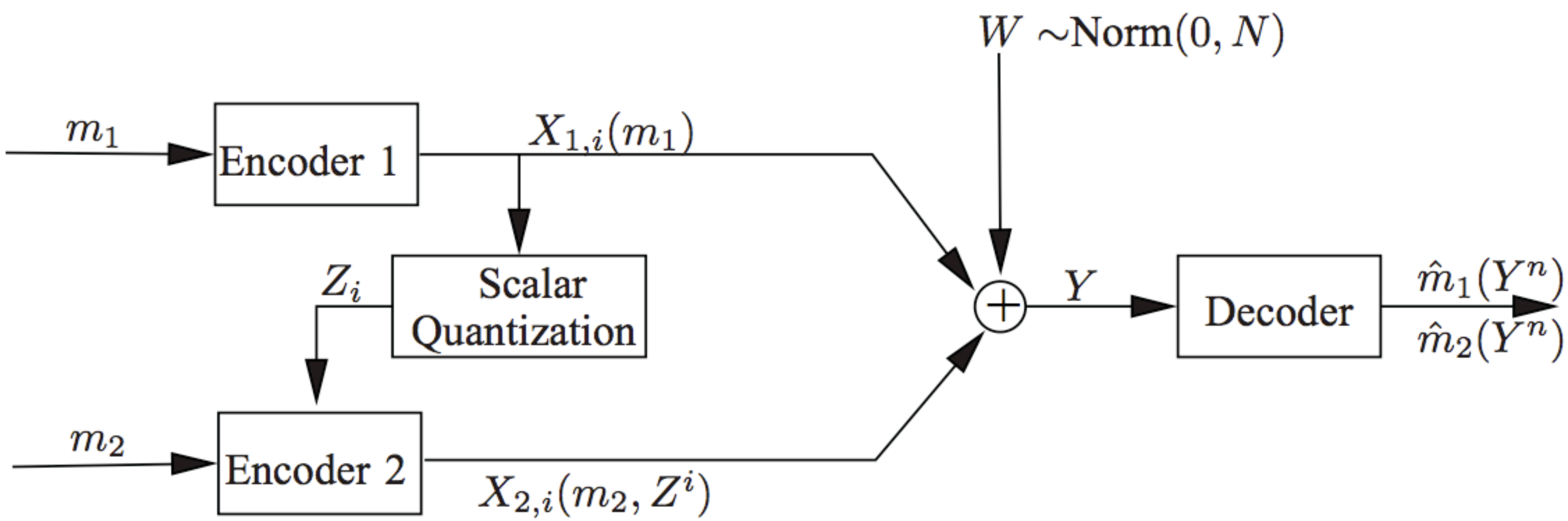
Additive White Gaussian MAC with Quantized Cribbing



No Cooperation

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+\frac{P_1}{N}\right) \\ R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_2}{N}\right) \\ R_1+R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_1+P_2}{N}\right) \end{aligned}$$

Additive White Gaussian MAC with Quantized Cribbing

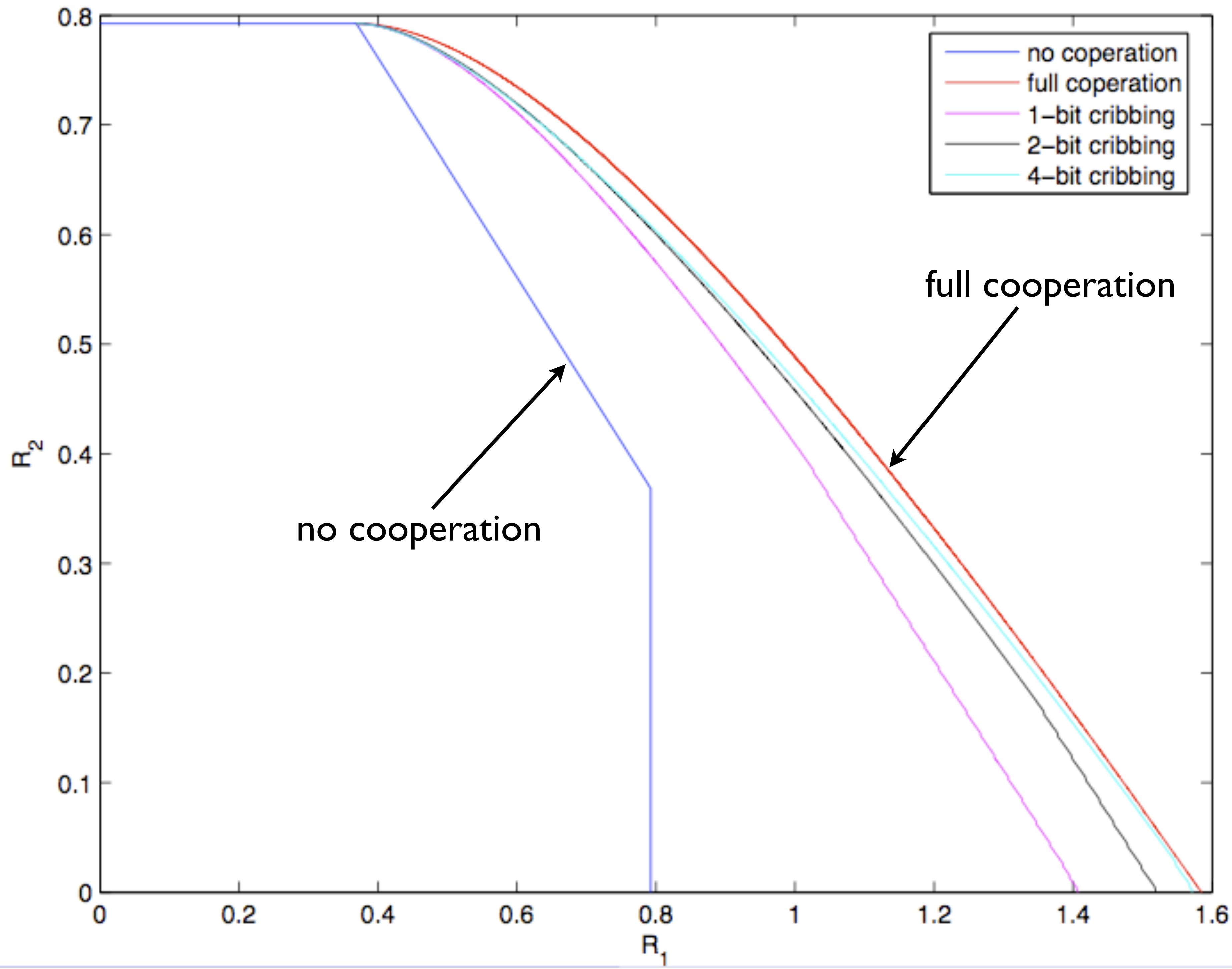


Full Cooperation

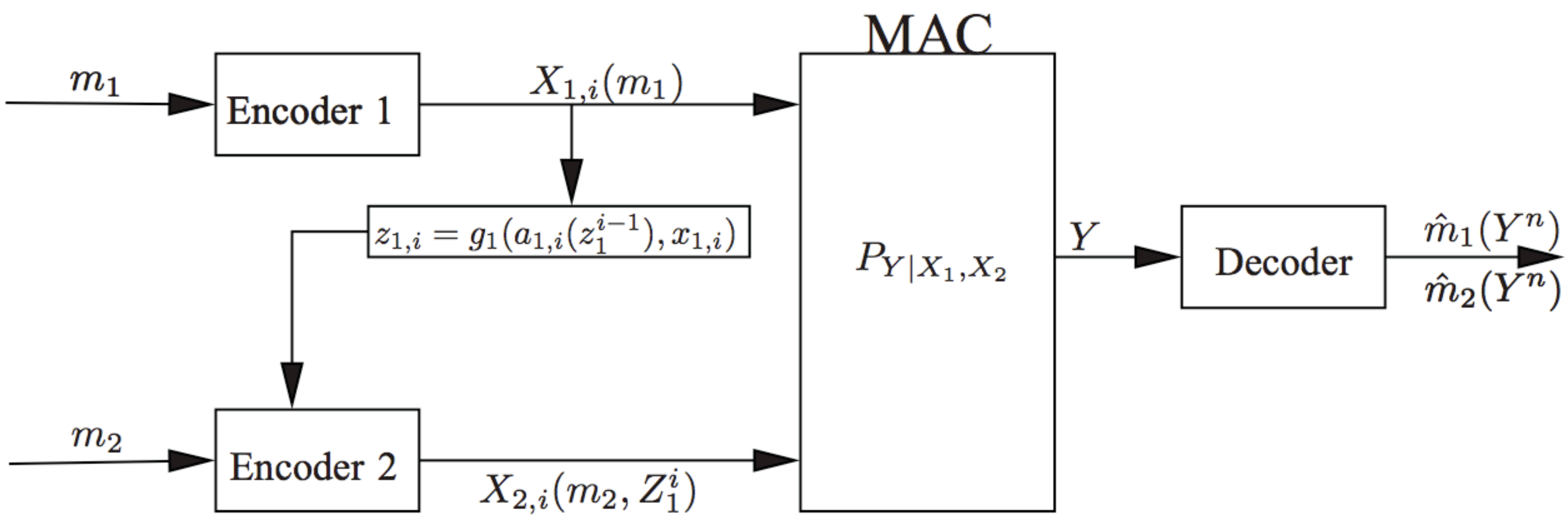
the capacity is the union over $0 \leq \rho \leq 1$ of the regions

$$
\begin{aligned}
R_2 &\leq \frac{1}{2}\log\left(1 + \frac{P_2}{N}(1-\rho^2)\right) \\
R_1 + R_2 &\leq \frac{1}{2}\log\left(1 + \frac{P_1 + 2\rho\sqrt{P_1 P_2} + P_2}{N}\right)
\end{aligned}
$$

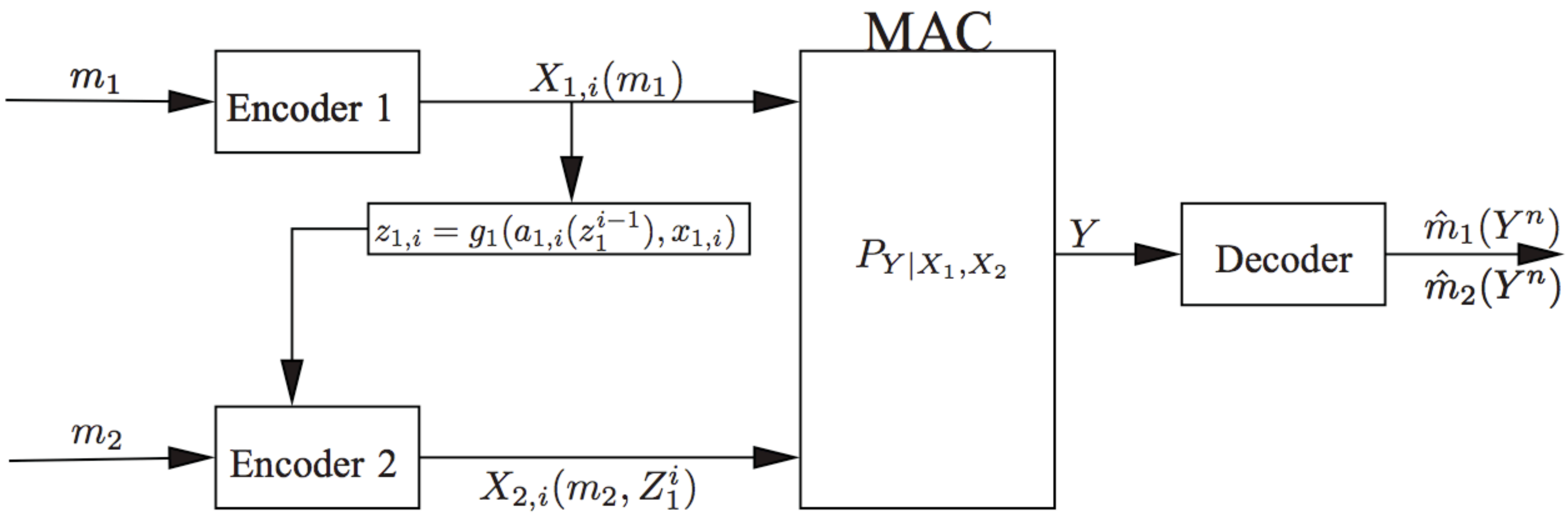
Additive White Gaussian MAC with Quantized Cribbing



Controlled Cribbing



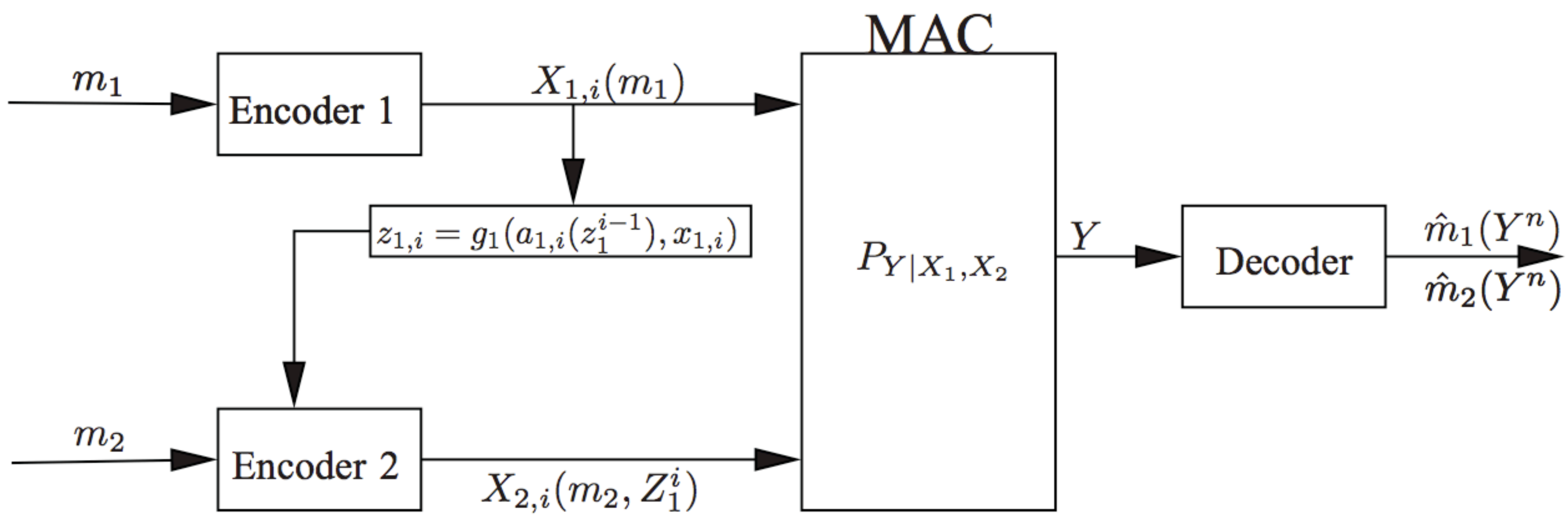
Controlled Cribbing



Case A $\quad f_{2,i} : \{1, ..., 2^{nR_2}\} \times \mathcal{Z}_1^{i-1} \mapsto X_{2,i},$

Case B $\quad f_{2,i} : \{1, ..., 2^{nR_2}\} \times \mathcal{Z}_1^{i} \mapsto X_{2,i},$

Controlled Cribbing



$$\frac{1}{n}\sum_{i=1}^{n} E[\Lambda_1(A_{1,i})] \leq \Gamma_1$$

Case A : Strictly Causal Controlled Cribbing

$$\mathcal{R}_A^a =$$

$$\begin{cases} R_1 \leq H(Z_1|U, A_1) + I(X_1; Y|X_2, Z_1, U, A_1), \\ R_2 \leq I(X_2; Y|X_1, U, A_1), \\ R_1 + R_2 \leq I(X_1, X_2; Y|U, A_1, Z_1) + H(Z_1|U, A_1), \\ R_1 + R_2 \leq I(X_1, X_2; Y), \text{ for} \\ P(u, a_1)P(x_1, z_1|u, a_1)P(x_2|u, a_1)P(y|x_1, x_2) \\ \text{ s.t. } E[\Lambda_1(A_1)] \leq \Gamma_1. \end{cases}$$

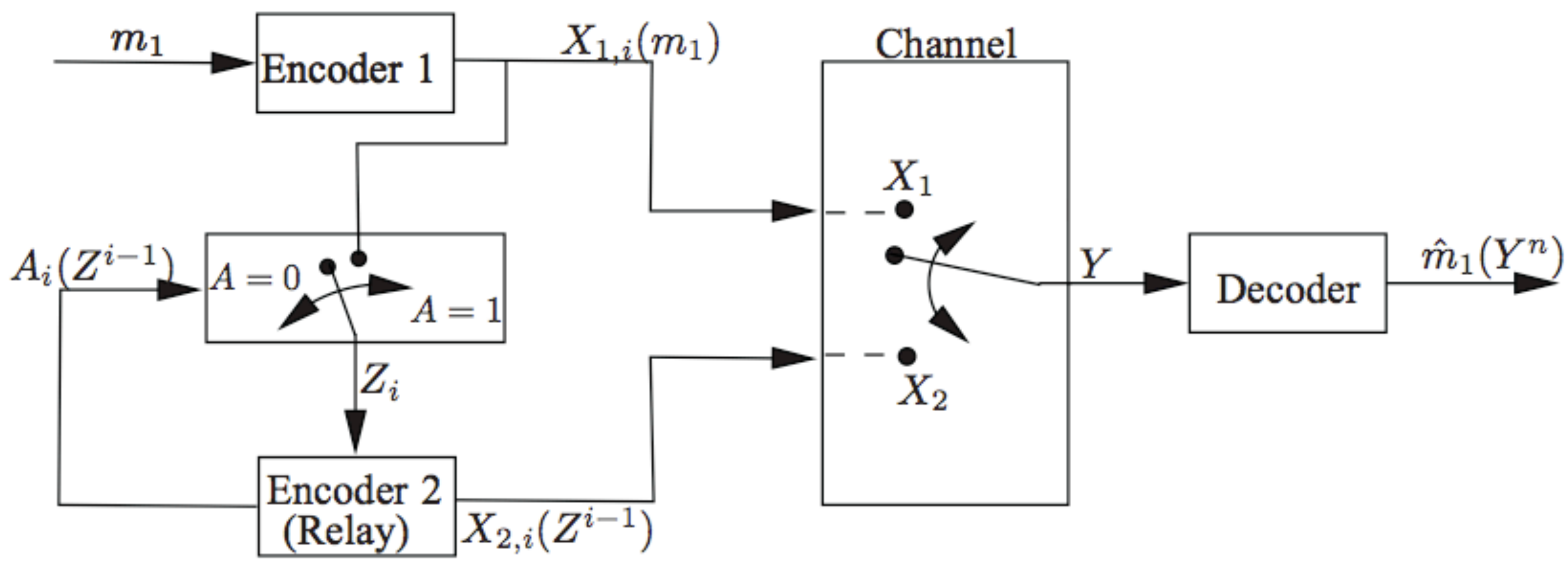
Case A : Strictly Causal Controlled Cribbing

$$\mathcal{R}_A^a = \begin{cases} R_1 \leq H(Z_1|U,A_1) + I(X_1;Y|X_2,Z_1,U,A_1), \\ R_2 \leq I(X_2;Y|X_1,U,A_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,A_1,Z_1) + H(Z_1|U,A_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u,a_1)P(x_1,z_1|u,a_1)P(x_2|u,a_1)P(y|x_1,x_2) \\ \text{ s.t. } E[\Lambda_1(A_1)] \leq \Gamma_1. \end{cases}$$

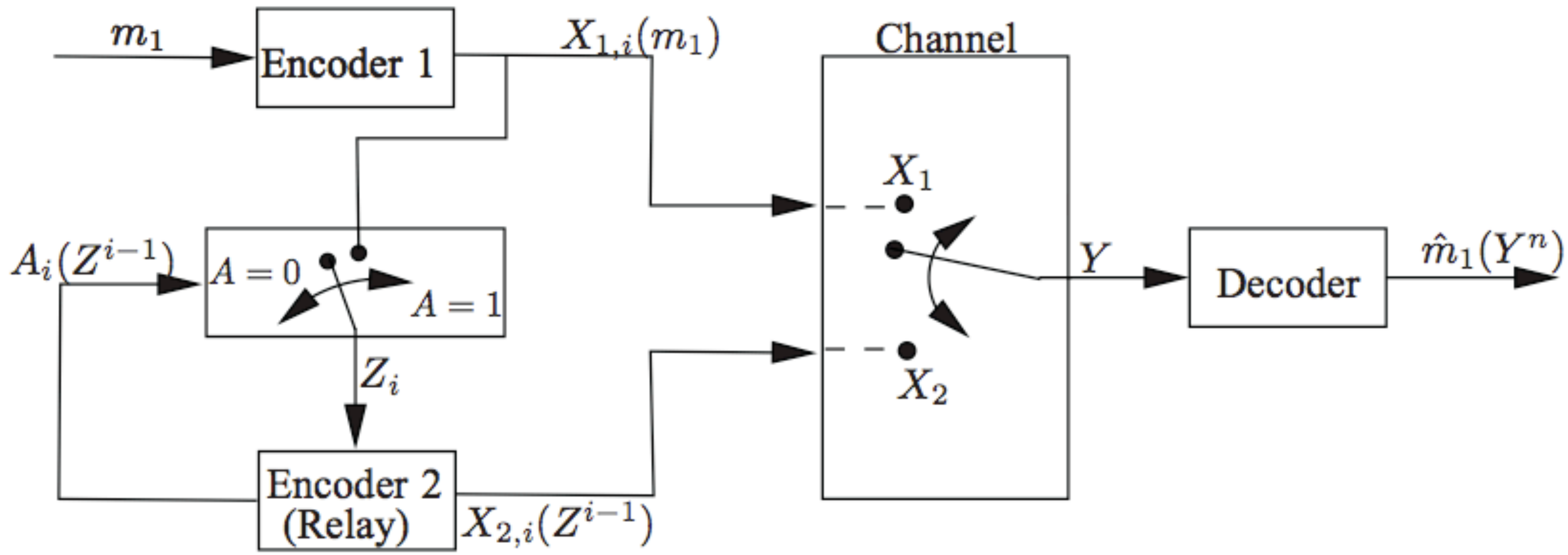
Case B : Causal Controlled Cribbing

$$\mathcal{R}_B^a = \begin{cases} R_1 \leq H(Z_1|U,A_1) + I(X_1;Y|X_2,Z_1,U,A_1), \\ R_2 \leq I(X_2;Y|X_1,U,A_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,A_1,Z_1) + H(Z_1|U,A_1), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u,a_1)P(x_1,z_1|u,a_1)P(x_2|z_1,u,a_1)P(y|x_1,x_2) \\ \text{ s.t. } E[\Lambda_1(A_1)] \leq \Gamma_1 \end{cases}$$

To Crib or Not to Crib



To Crib or Not to Crib



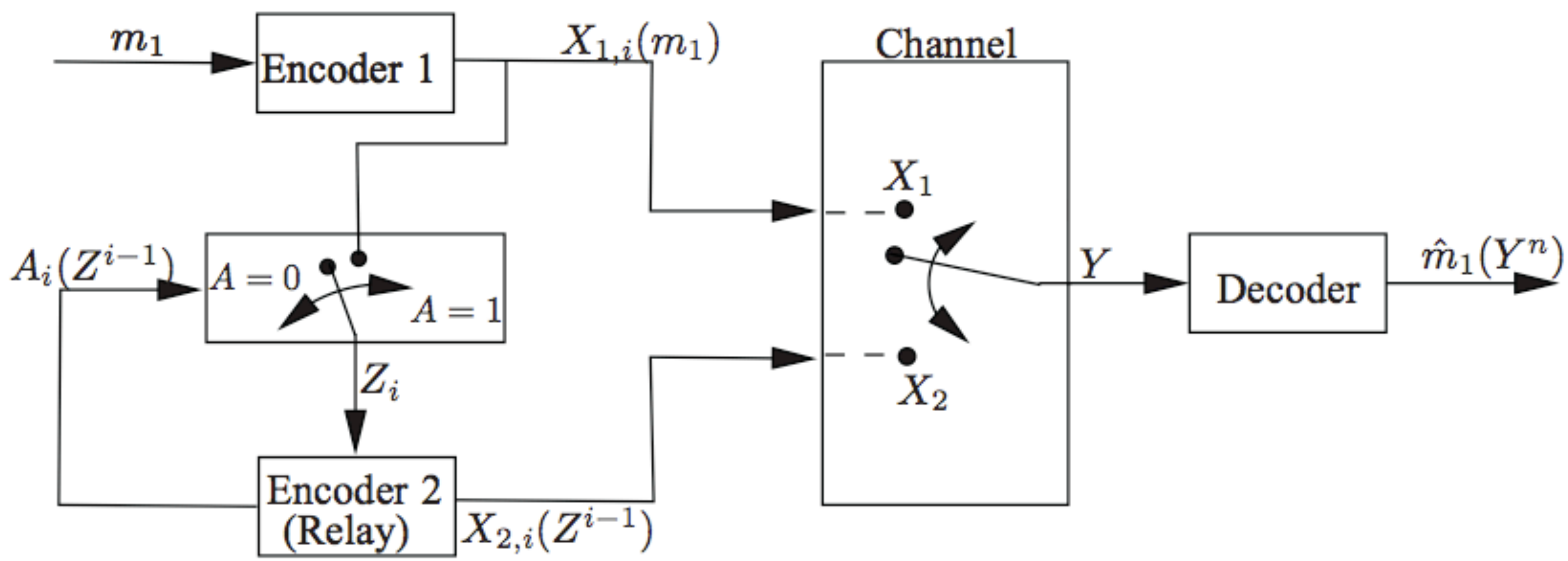
If $A_i = 1$, then $Z_i = X_i$, and otherwise Z_i is a constant.

$$\frac{1}{n}\sum_{i=1}^{n} E[A_i] \leq \Gamma$$

$$R_2 = 0$$

To Crib or Not to Crib

Capacity Region

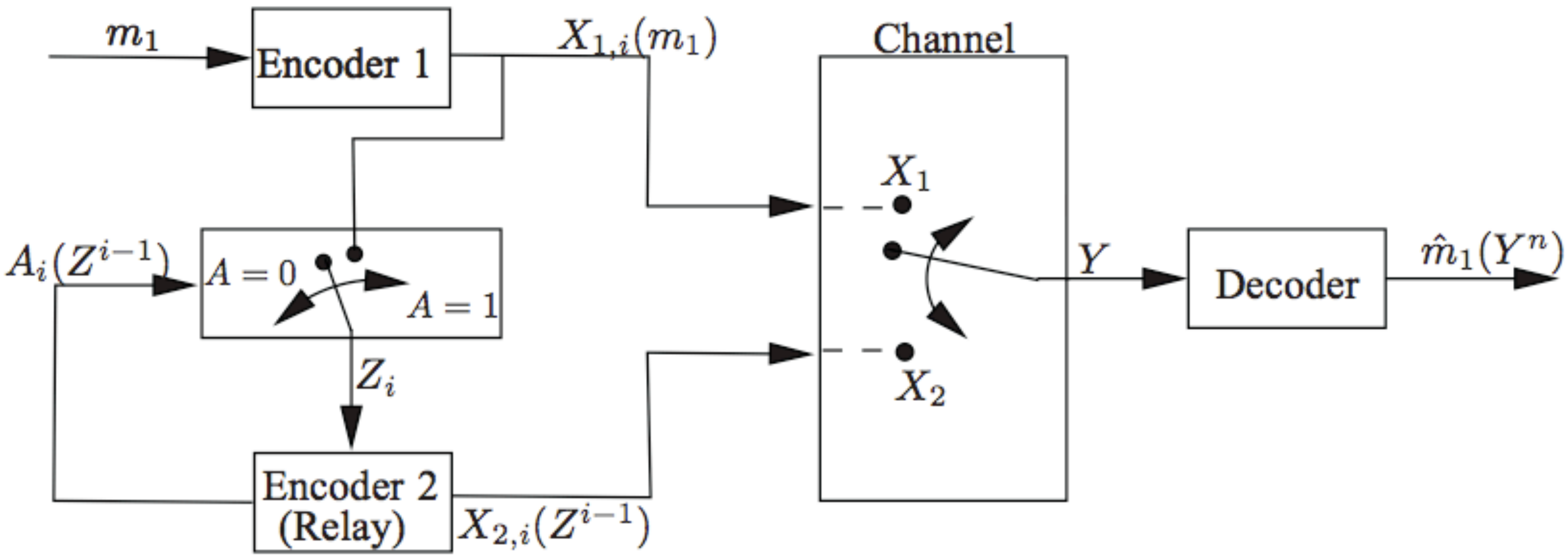


delay in the cribbing (Case A) [ElGamalAref82]

$$R_1 = \max_{P_{X_1,X_2,A}:E[c(A)]\leq\Gamma} \min\{H(Z|X_2,A) + I(X_1;Y|X_2,Z_1,A), I(Y;X_1,X_2)\}.$$

To Crib or Not to Crib

Capacity Region



delay in the cribbing (Case A) [ElGamalAref82]

$$R_1 = \max_{P_{X_1,X_2,A}:E[c(A)]\leq\Gamma} \min\{H(Z|X_2,A) + I(X_1;Y|X_2,Z_1,A), I(Y;X_1,X_2)\}.$$

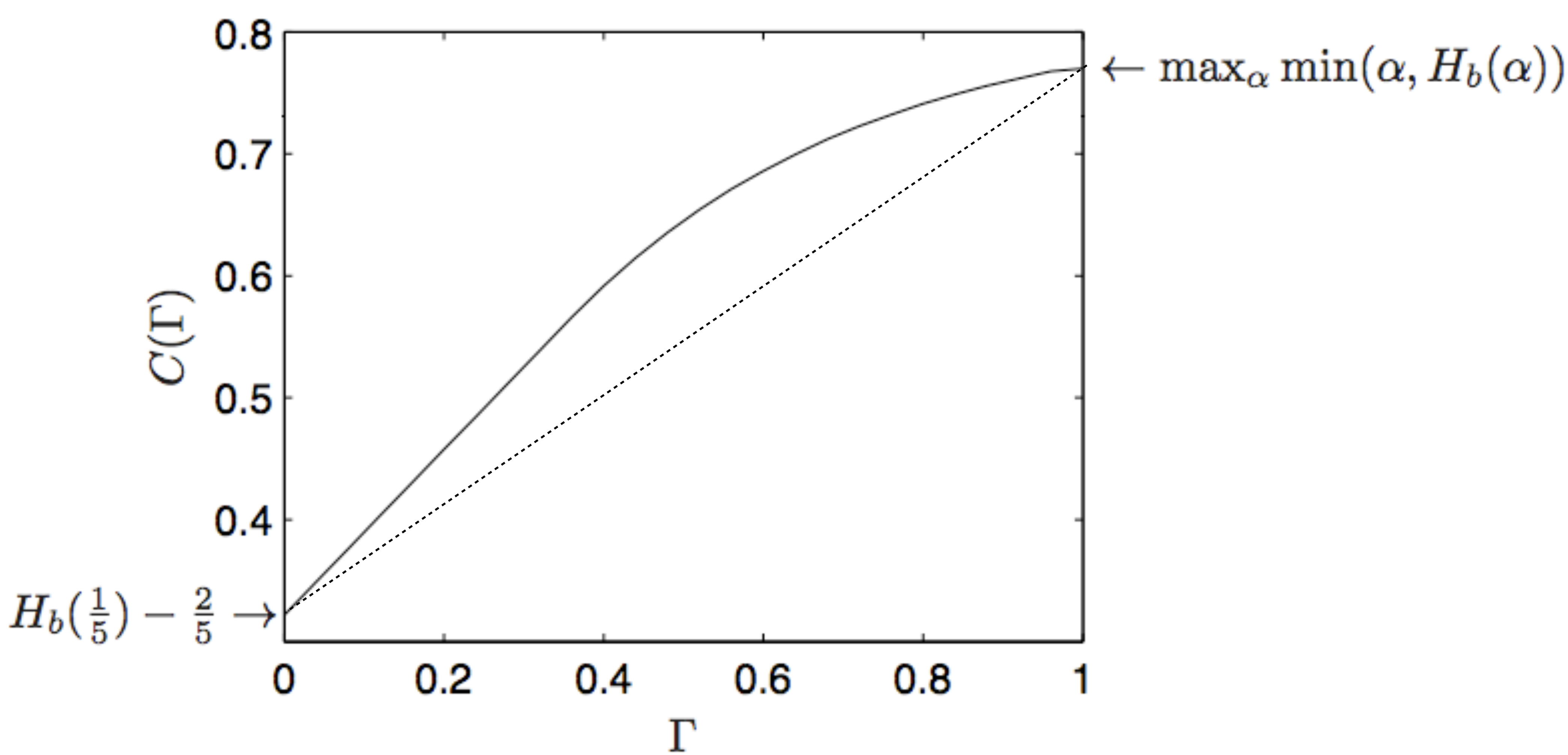
no delay in the cribbing (Case B), i.e., $X_{2,i}(Z^i)$ [ElGamalHassanpourMammen07]

$$R_1 = \max_{P_{U,X_1,A}P_{X_2|U,Z,A}:E[c(A)]\leq\Gamma} \min\{H(Z|U,A) + I(X_1;Y|X_2,Z,U,A), I(Y;X_1,X_2)\}.$$

To Crib or Not to Crib

Capacity Region

Case A



Summary

- Cribbing is an important concept in cognitive radios, cooperation and relaying.
- Capacity region of deterministic using : block markov codes, strategies, superposition coding, backward decoding and rate splitting
- We see that for AWGN MAC with causal quantized cribbing, few bits are enough.
- Controlled Cribbing.

Future Work

- Non-causal Partial (deterministic) cribbing.
- Controlled Cribbing with actions dependent on messages, i.e., $a_{1,i}(z_1^{i-1}, m_1)$

Thanks !!!

Questions ?