A new coding scheme for cooperation in semi-deterministic channels

Haim Permuter

Ben-Gurion University

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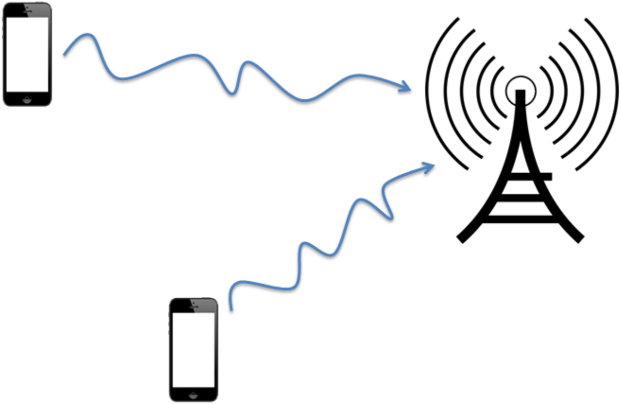
Uplink Communication



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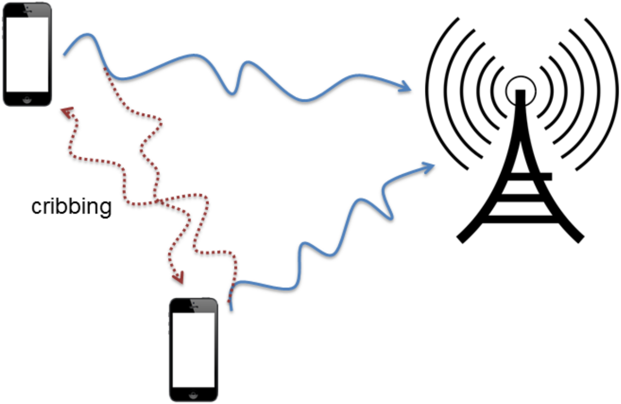
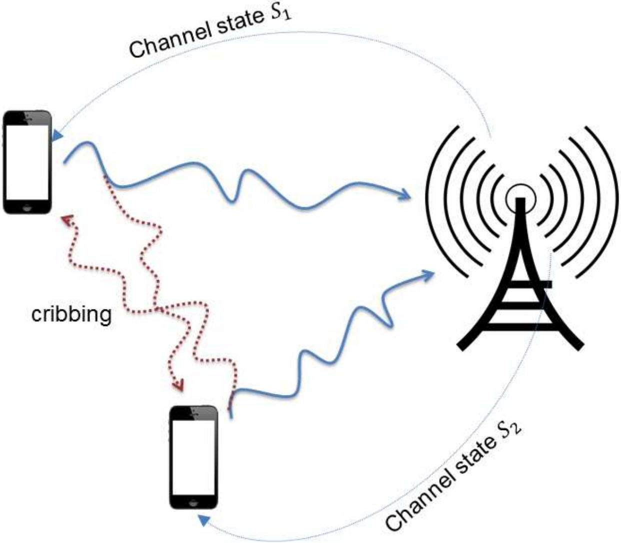
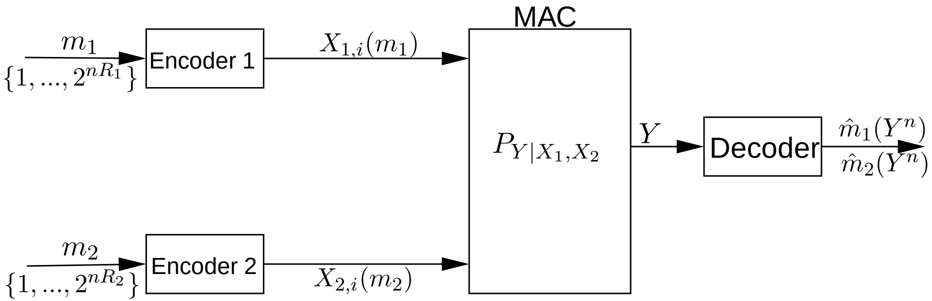


Uplink Communication



The communication setting considered in the talk



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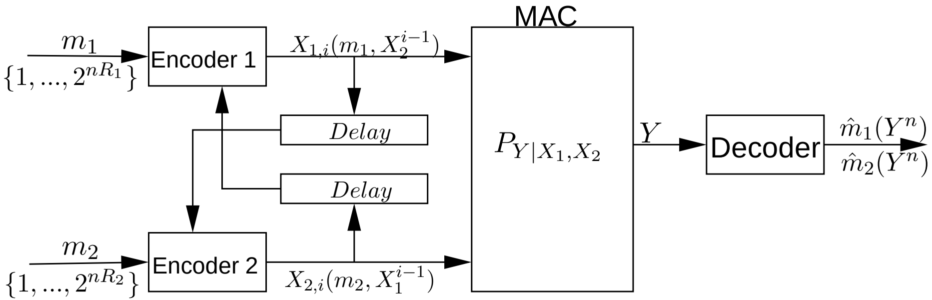
Perfect Cribbing [Willem82]



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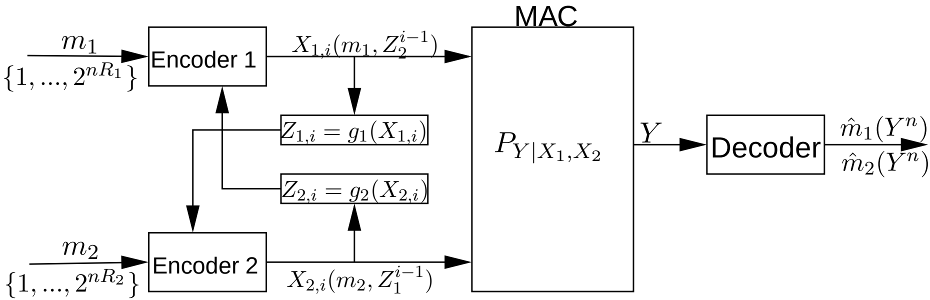
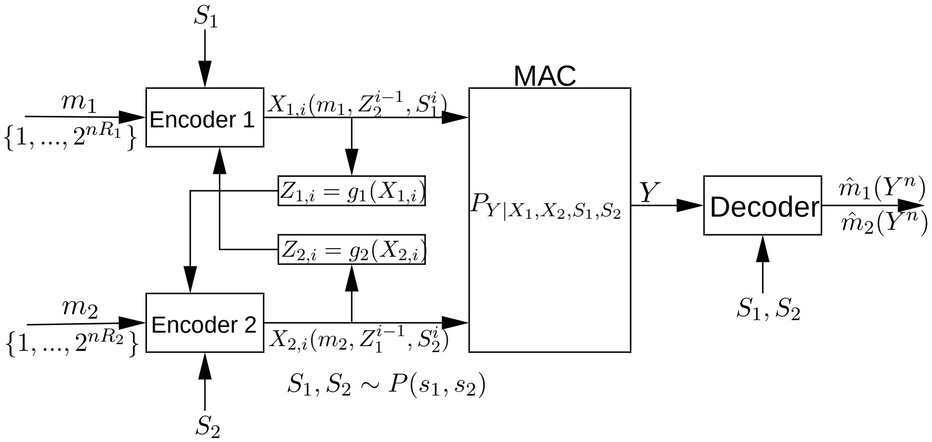
Partial (deterministic-function) cribbing [P./Asnani13]

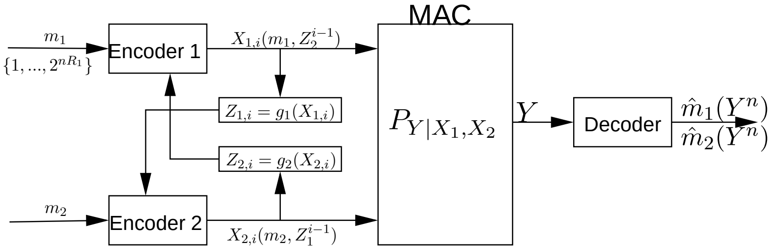


The communication setting considered in the talk

Partial (deterministic-function) cribbing with state [Kolte/Özgür/P.15]



Comments on MAC with cribbing



- Causal cribbing: $X_{1,i}(m_1, {Z_2}^{i})$
- Non-causal partial cribbing, $X_{1,i}(m_1, {Z_2}^{n})$, is open.
- Noisy cribbing [Bross/Steinberg/Tinguely10] is open.
- Perfect cribbing for Gaussian [Willems05] is trivial

Capacity Region

Theorem

Strictly causal: *[P./Asnani13]*

$$\mathcal{R} = \begin{cases} R_1 \leq I(X_1;Y|X_2,Z_1,U) + H(Z_1|U), \\ R_2 \leq I(X_2;Y|X_1,Z_2,U) + H(Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y|U,Z_1,Z_2) + H(Z_1,Z_2|U), \\ R_1 + R_2 \leq I(X_1,X_2;Y), \text{ for} \\ P(u)P(x_1,z_1|u)P(x_2,z_2|u)P(y|x_1,x_2). \end{cases}$$

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Causal: The same but

$$R_2 \leq I(X_2;Y|X_1,Z_2,U) + H(Z_2|Z_1,U),$$
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Two achievabilities:

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2. Cooperative binning

Achievability proof: Partial decoding

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- Encoder 2 decodes at the end of block b the message $m'_{1,b}$ from z_1^n. Hence $R_1' \leq H(Z_1|U)$.
- Block Markov code:
  x_1^n is determined by$(m'_{1,b}, m''_{1,b})$ conditioned on $(m'_{1,b-1}, m'_{2,b-1})$.

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- Backward decoding: At block b, we assume that $(m_{1,b}', m_{2,b}')$ is known. Decode $m_{1,b-1}'$, $m_{2,b-1}'$, $m_{2,b}''$ and $m_{1,b}''$ from Y^n using joint typicality..

After error analysis we obtain

$$
\begin{aligned}
R_1' &\leq H(Z_1|U), \\
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Using Fourier$-$Motzkin elimination we obtain the region.

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After error analysis we obtain

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\begin{aligned}
R_1' &< H(Z_1|U,S_1), \\
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R_2'' &< I(X_2;Y|U,Z_2,X_1,S_1,S_2), \\
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Using Fourier−Motzkin elimination we obtain:

Theorem

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R_1 &\le I(X_1;Y|U,X_2,Z_1,S_1,S_2)+H(Z_1|U,S_1),\\
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for $P(u)P(x_1|u,s_1)P(x_2|u,s_2)$ *[Kolte/Özgür/P.15]*

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- FME: **each** lower bound less than **each** upper bound.

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- Very useful in multi-user problems
- A computer can do it, but inserts many redundant inequalities.

FME-IT [Gattegno/Goldfeld/P. 15]

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If $\rho^* \geq b_i$ then $\mathbf{a}_i^\top \mathbf{x} \geq b_i$ is a redundant constraint.

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$$\rho^* = \min_{\substack{\mathbf{h}:\ \mathrm{G}\mathbf{h} \geq \mathbf{0} \\ \mathrm{Q}\mathbf{h} = \mathbf{0}}} \mathbf{f}^\top \mathbf{h}$$

If $\rho^* = 0$ then $\mathbf{f}^\top \mathbf{h} \geq 0$.

  - $\mathbf{h}$ - Vector with joint entropies (canonical form).
  - $\mathrm{G}\mathbf{h} \geq \mathbf{0}$ - Elemental inequalities.
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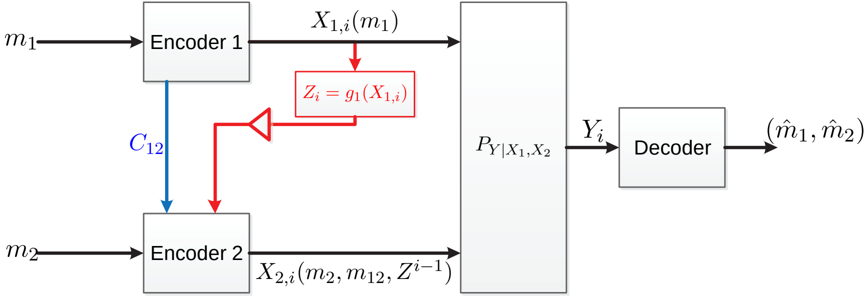
- FME-IT combines the two LPs in one problem.

MAC with Combined Cooperation & Partial Cribbing

[Kopetz/P./Shamai14]



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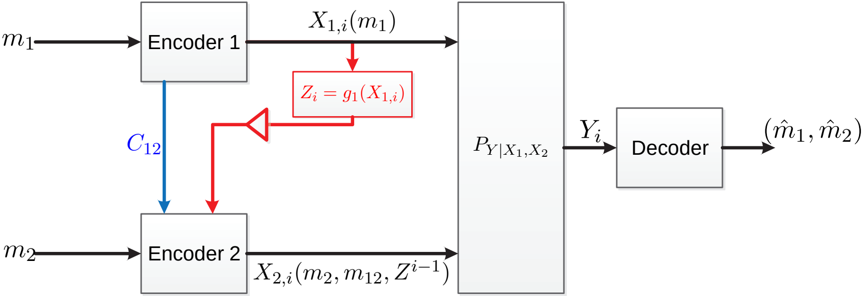

$$
\begin{aligned}
R_1 &\leq I(X_1;Y|X_2,Z,U) + H(Z|U) + \mathbf{C_{12}} \\
R_2 &\leq I(X_2;Y|X_1,U) \\
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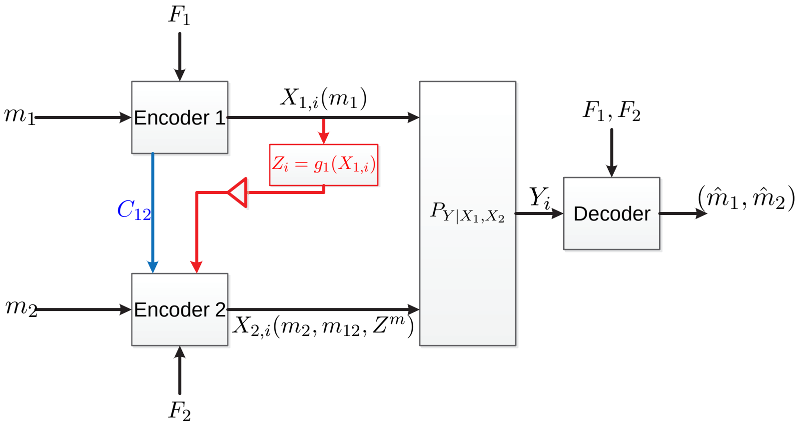
R_1 + R_2 &\leq I(X_1,X_2;Y)
\end{aligned}
$$

for $P(u)P(x_1,z|u)P(x_2|u)P(y|x_1,x_2)$

Cooperative MAC with Oblivious Encoders[Kopetz/P./Shamai15]



Oblivious nodes:[Sanderovich/Shamai/Steinberg/Kramer08]

- Random codes
- Independent of the message and of each other.

Oblivious vs. Codebook-aware Encoding

Oblivious Encoding:

$$
\begin{aligned}
R_1 &\leq I(X_1;Y|X_2,Z,Q)+H(Z|Q)+C_{12},\\
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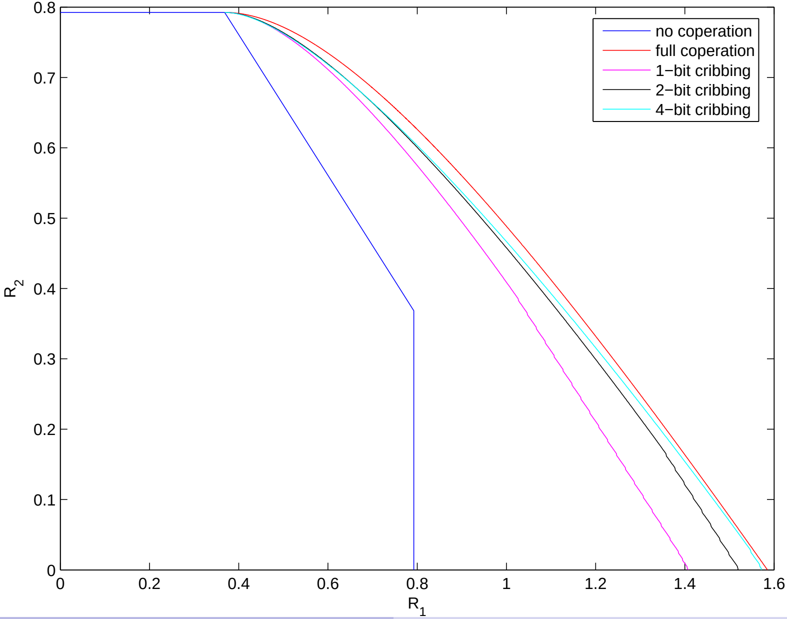
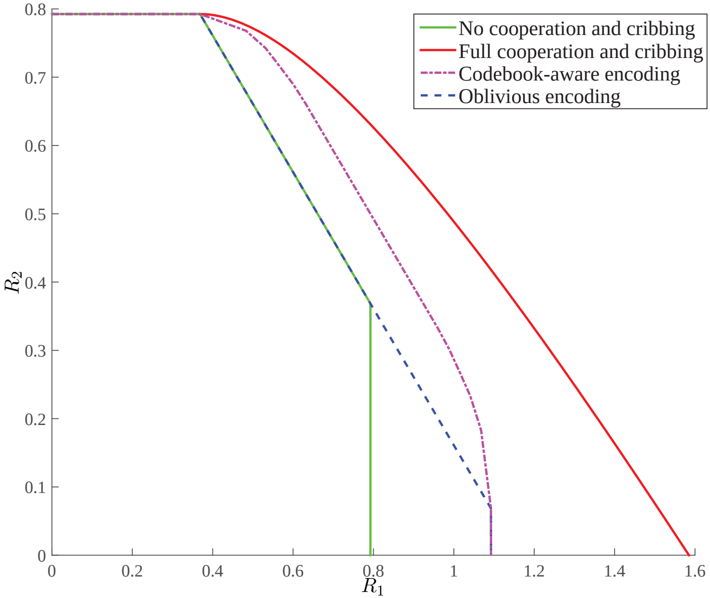
Bin-and-Forward vs Cooperative Binning

Additive Gaussian MAC with quantized cribbing



Additive Gaussian MAC, No cribbing, $C_{12} = 0.3$



Summary

- Two achievanilities scheme for MAC with partial (deterministic) cribbing:
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- Cooperative binning solves asymmetric state at the TXs.
- Cooperative binning solves semi-deterministic relay with state known to the source encoder and destination RX.
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Thank you very much !