The Quotient Image: Class-based Recognition and Synthesis Under Varying Illumination

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Example: 3 basis images



The movie is created by simply linearly combining the 3 basis images.

Image Synthesis:

Given a single image y =

And a database of other images of the same "class"

Bootstrap Set

We would like to generate new images from y simulating change of illumination.

Recognition

Given a database of images of the same "class",
under varying illumination conditions

and a novel image

Match between images of the same object.

Definition: Ideal Class of Objects

The images produced by an ideal class of objects are

$$\forall x, y \qquad I(x,y) = \rho_i(x,y)n(x,y)^T s_j$$

Where $\rho_i(x,y)$ is the albedo (surface texture) of object i of the class

$n(x,y)$ is the normal direction (shape) of the surface, i.e., all objects have the same shape.

s_j is the point light source direction.

Related Work

- Basic result :Shashua 91,97
- Application and related systems:Hallinan 94, Kriegman et. al 96,98
- Rendering under more general assumption: Dorsey et al. 93,94
- Work on “class-based” synthesis and recognition of images -mostly with varying viewing positions:
  Basri 96, Bymer & Poggio 95, Freeman & Tenenbaum 97,
  Vetter & Blanz 98, Vetter, Jones & Poggio 97, 98
  Edelman 95, Atick, Griffin & Redlich 97.
- Linear class : Vetter & Poggio 97,92
- Additive error term : Sali & Ulman 98
- Reflectance Ratio: Nayar & Bolle 96

Basic Assumptions

- Lambertian surface, linear model:

  no cast shadows , no highlights.

- Same view point (canonical view).

- Ideal class assumption

- Images have the same size and are (roughly) aligned.

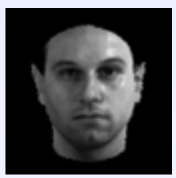
The Quotient image: Definition

Given images y_s , a_s

of objects ***y*** and ***a*** respectively, under illumination ***S***

$y_s = \rho_y N^T s$

$a_s = \rho_a N^T s$



$=$

$$Q_y = \frac{\rho_y}{\rho_a}$$

(pixel by pixel division)

Thus Q_y depends only on relative surface information

and is independent of illumination.

The Quotient image Method: Proposition

Let a_1, a_2, a_3 3 images of object a .

Let y_s Image of object y illuminated by light s .

Then, there exist x_1, x_2, x_3 that satisfy:

$$y_s = (\sum_j x_j a_j) \otimes Q_y$$

Moreover, the image space of y is spanned by varying the coefficients.

The Quotient image Method: Proof

a_1, a_2, a_3 Illuminated by: s_1, s_2, s_3

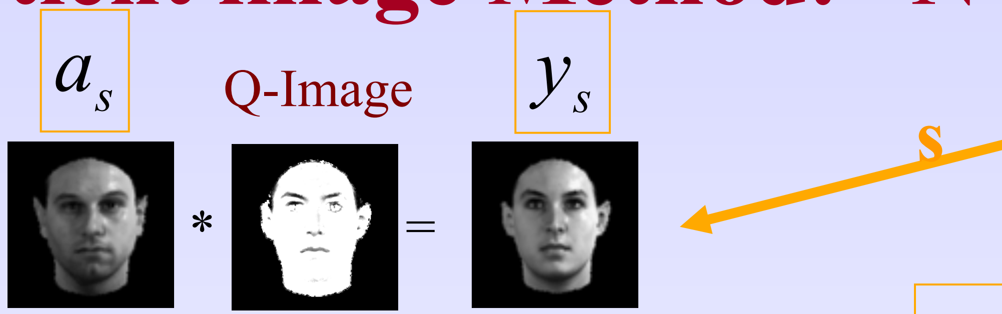
y_s Illuminated by: $s = \sum_j x_j s_j$

$$y_s = (\sum_j x_j a_j) \otimes Q_y$$

$$\rho_a n^T s \qquad \rho_y / \rho_a$$

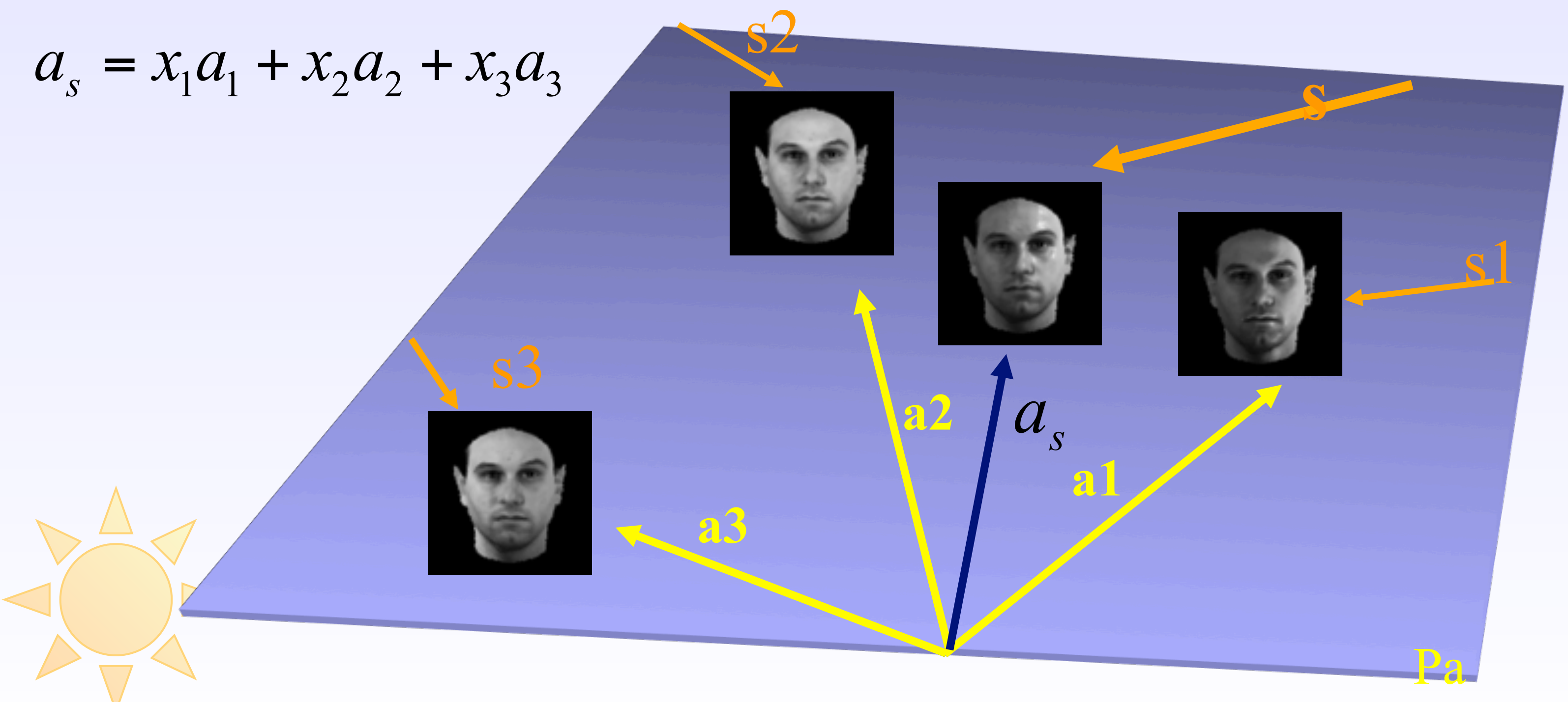
The Quotient image Method: N=1

N=1



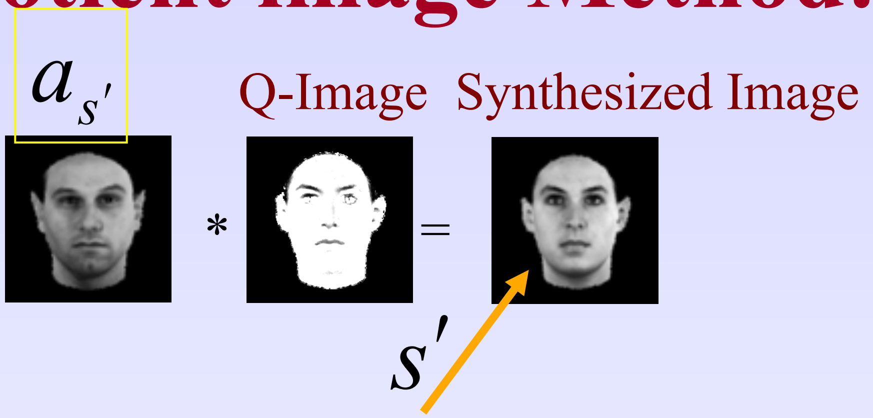
$$s = \sum_j x_j s_j$$

$$a_s = x_1 a_1 + x_2 a_2 + x_3 a_3$$

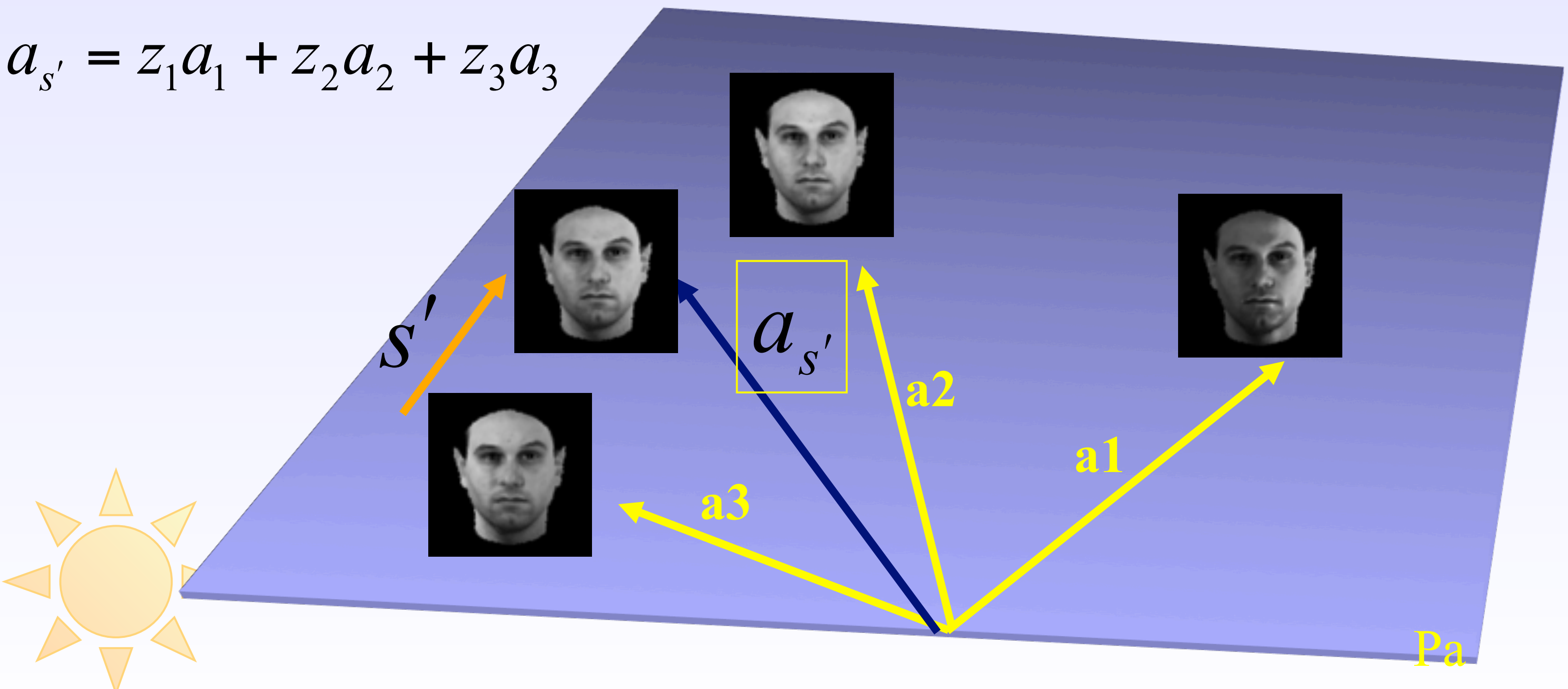


The Quotient image Method: N=1

N=1



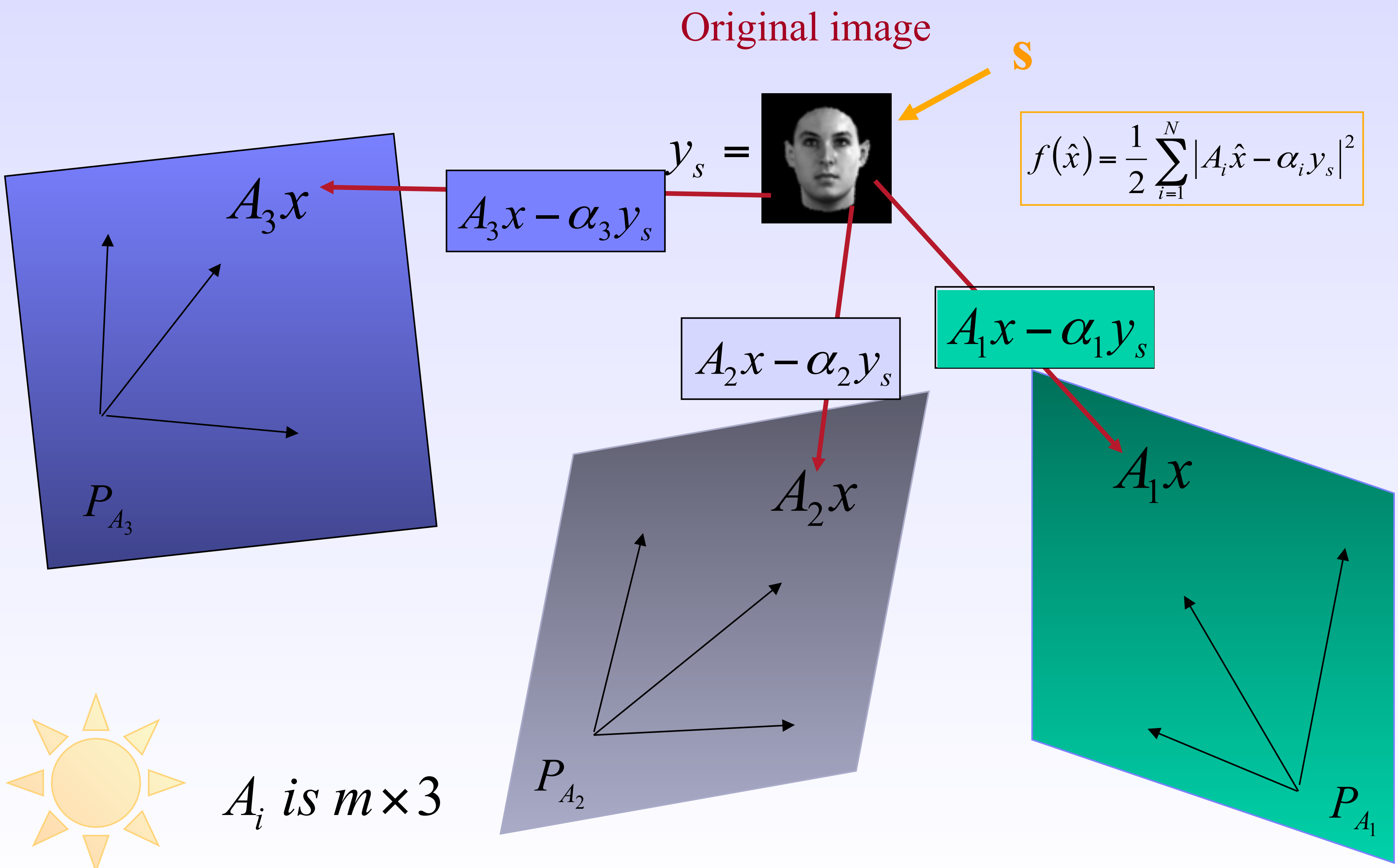
$$a_{s'} = z_1 a_1 + z_2 a_2 + z_3 a_3$$



The Quotient image Method: Conclusions

- Given Q_y one can generate y_s and all other images of the image space of $\boldsymbol{y}$.
- Given y_s and the coefficients x_j that satisfies $s = \sum_j x_j s_j$ then Q_y readily follows $Q_y = \dfrac{y_s}{\sum_j x_j a_j}$
- In order to obtain the correct coefficients x_j a bootstrap set of more than one object is needed .

The Quotient image Method: N>1



$$f(\hat{x}) = \frac{1}{2} \sum_{i=1}^{N} \left| A_i \hat{x} - \alpha_i y_s \right|^2$$

A_i *is* $m \times 3$

The Quotient image Method: Theorem-1

The energy function $$f(\hat{x}) = \frac{1}{2}\sum_{i=1}^{N}\left|A_i\hat{x} - \alpha_i y_s\right|^2$$

has a (global) minimum $x = \hat{x}$, if the albedo ρ_y

of object $\boldsymbol{y}$ is rationally spanned by the bootstrap set.

i.e if there exist coefficients $\alpha_1, \ldots, \alpha_N$ such that

$$\rho_y = \frac{\rho_1^2 + \ldots \rho_N^2}{\alpha_1\rho_1 + \ldots + \alpha_N\rho_N}$$

The Quotient Image Method: Solving For *X* and α_i

$$\underset{x,\alpha}{\mathrm{Min}}\, f(\hat{x}) = \frac{1}{2}\sum_{i=1}^{N} \left| A_i \hat{x} - \alpha_i y_s \right|^2$$

$$\hat{x} = \left(\sum_i A_i^T A_i \right)^{-1} \left(\sum_i \alpha_i A_i^T \right) y_s = \sum_i \alpha_i v_i$$

$$v_i = \left(\sum_{r=1}^{N} A_r^T A_r \right)^{-1} A_i^T y_s$$

We also have: $$0 = \frac{\partial f}{\partial \alpha_i} = \alpha_i y_s^T y_s - \hat{x}^T A_i^T y_s$$

The Quotient image Method: Solving For X and α_i

for $i = 1 \ldots N$ written explicitly

$$\alpha_1\left(v_1^T A_1^T y_s - y_s^T y_s\right) + \ldots + \alpha_N v_N^T A_1^T y_s = 0$$

$$\alpha_1 v_1^T A_2^T y_s + \ldots\ldots\ldots\ldots + \alpha_N v_N^T A_2^T y_s = 0$$

$$\vdots$$

$$\alpha_1 v_1^T A_{N\ldots}^T y_s + \ldots + \alpha_N\left(v_N^T A_N^T y_s - y_s^T y_s\right) = 0$$

The Algorithm

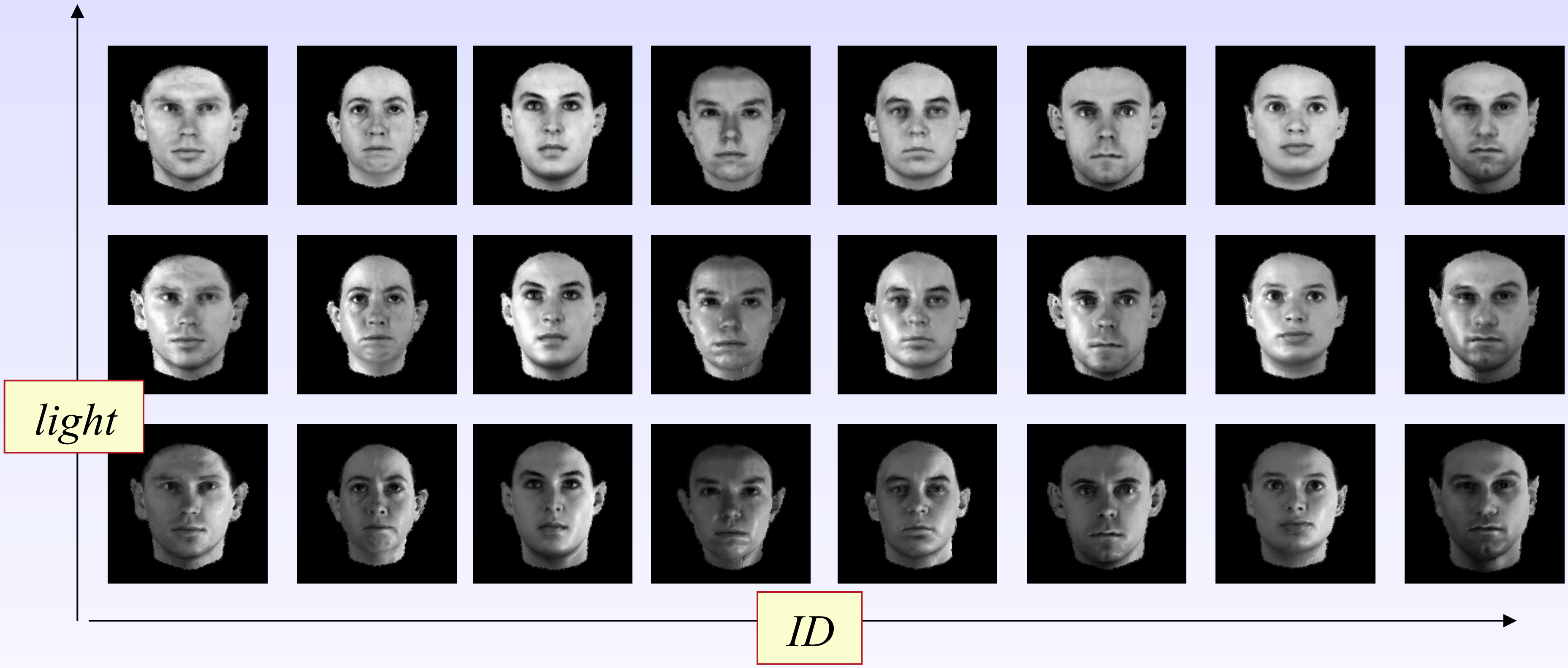
- Given : $A_1, \ldots, A_N$ a bootstrap set and a novel image y_s
- Use the minimization function:

$$\underset{x,\alpha}{\text{Min}} \quad f(\hat{x}) = \frac{1}{2}\sum_{i=1}^{N} \left| A_i \hat{x} - \alpha_i y_s \right|^2$$ to generate

homogenous system of linear equations in $\alpha_1, \ldots \alpha_N$

- Scale such that $\sum_i \alpha_i = N$
- Compute $x = \sum_i \alpha_i v_i$
- $Q_y = \frac{y_s}{Ax}$ Where A is the average of $A_1, \ldots, A_N$
- $y_{new}(Z) = Az \otimes Q_y$ For all choices of z

Frontal faces : Collection of objects all have the same shape but differ in their surface texture (albedo)...

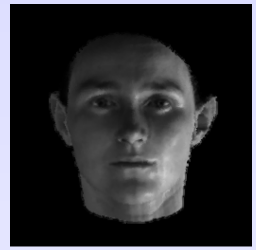


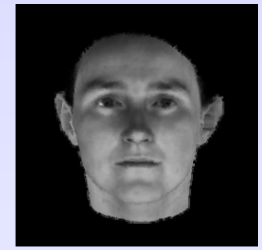
Samples of few faces out of 9*200 faces images taken from T. Vetter database which was mainly used as a bootstrap set and as a source for novel images in the further demonstration.

Quotient Method

Synthesis from Single Picture
And 10 faces from the bootstrap set
under 3 different light conditions

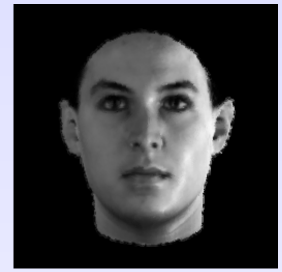
Linear Combination





Synthesis from 3 pictures

10
other faces
from the data
base, each under 3
light conditions



Synthesis from Single image
and the bootstrap set

Synthesis from 3 pictures

Bootstrap Set

Original image

Quotient Image

Synthesis from Single image and the bootstrap set

Synthesis from 3 pictures

Original Images Compared to Q-Image Synthesized Images

(0,0) (0,+20) (0,-35) (-50,0) (+50,0)

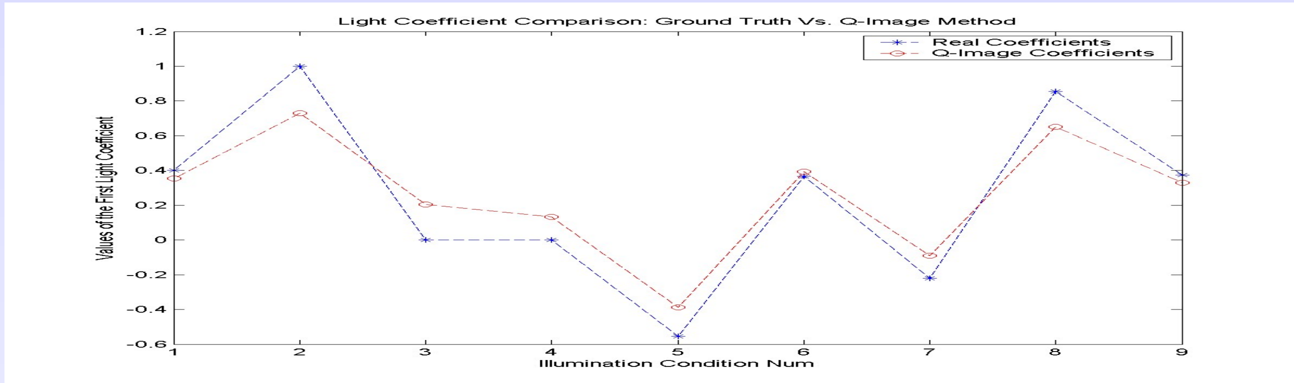
1st Row: Original Images

2nd Row: Q-Image Synthesized Images

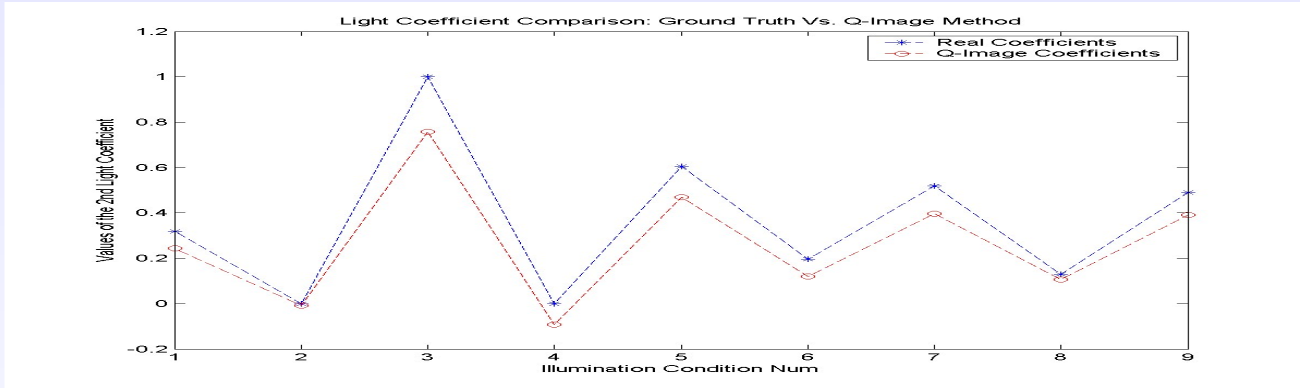
3rd Row: Exact Values of Light Direction: center, down, up, right, left

Light Coefficient Comparison Ground Truth Vs. Q-Image Coefficients

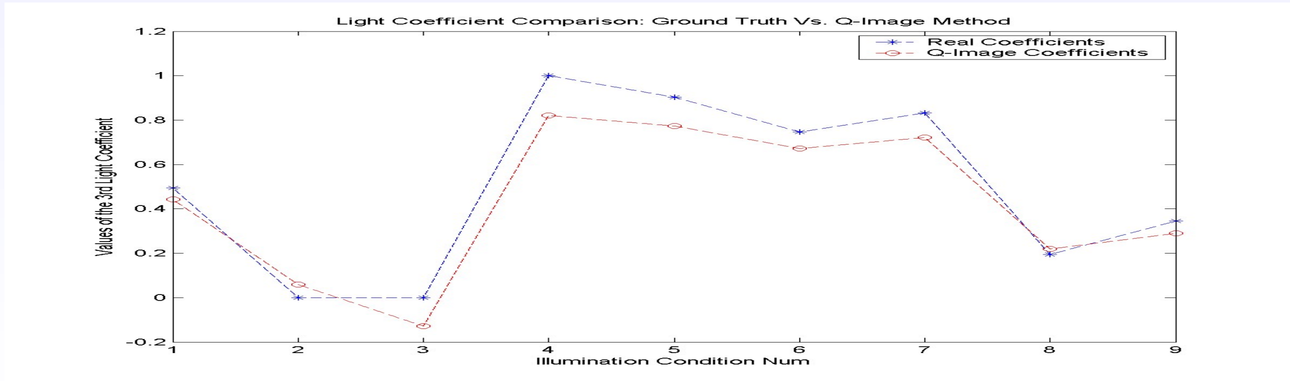
1st Coefficient



2nd Coefficient

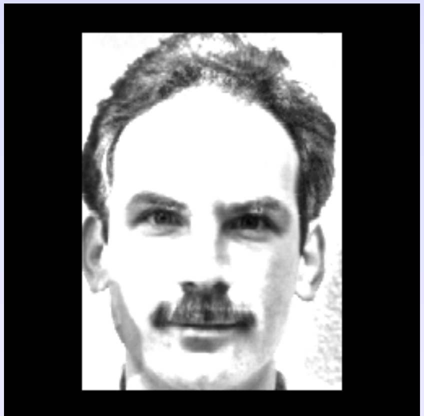


3rd Coefficient

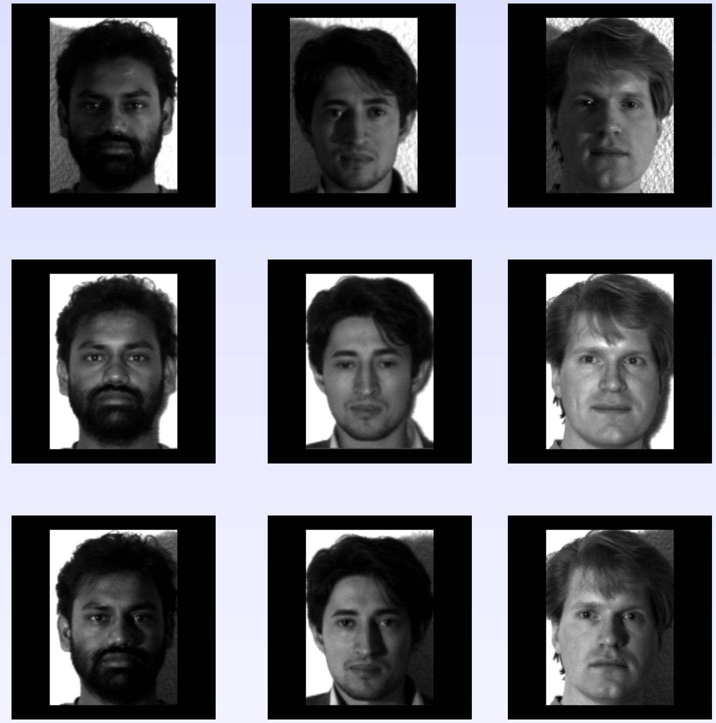


Using Different Database

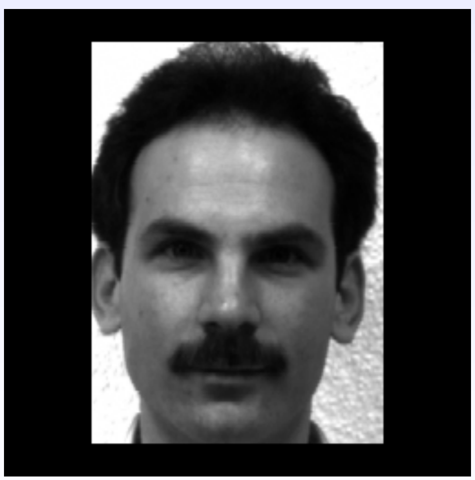
The Quotient Image



The original image



3x3 images' Database



Animation Using the database

The Original images

Q images , 1 object bootstrap set

Q images , 10 object bootstrap set

Handling Color Images

RGB ⟷ HSV Transformation

Original color image

R

G

B

H

S

V

Original Image

Quotient Image

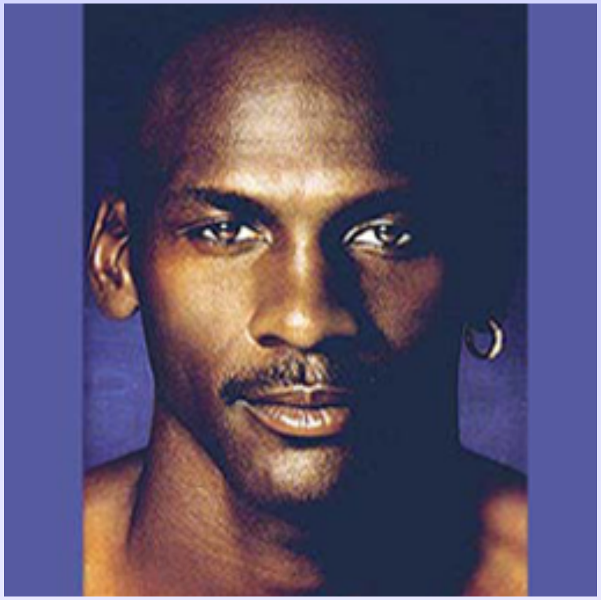
Synthesized Sequence

Monica and Bill Under a New Light

Original Images

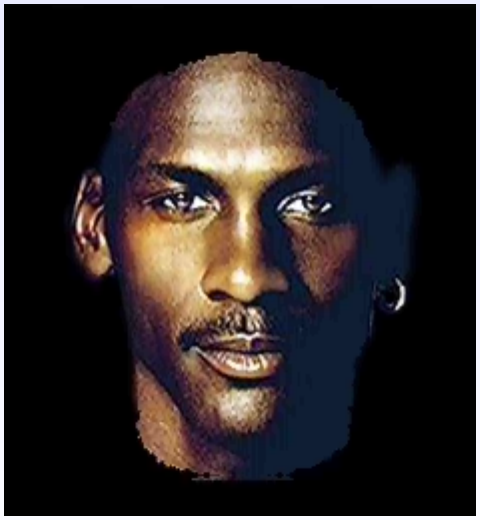
Quotient Images

Synthesized Sequences



Original Image

Quotient Image



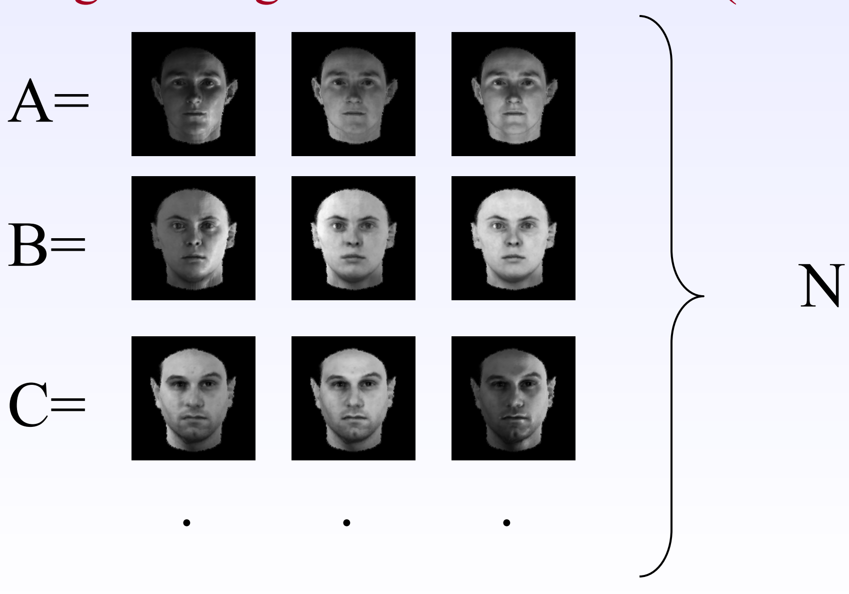
Synthesized Sequence

Recognition under varying illumination

Database generation

Each object in the database is represented by its quotient image only.
The quotients can be made of images with varying illuminations.

The quotient images was generated out of N*3 (N=20) base images.



Identification

Given a new image of an object appears in the data base under any light condition, it's quotient is computed from A,B,C …
(as was done in the database generation) . Then It is compared to the quotients in the data base.

Other methods used for comparison

1. Correlation

Database: Each object is represented in the database by it's image under any/certain lightening condition.

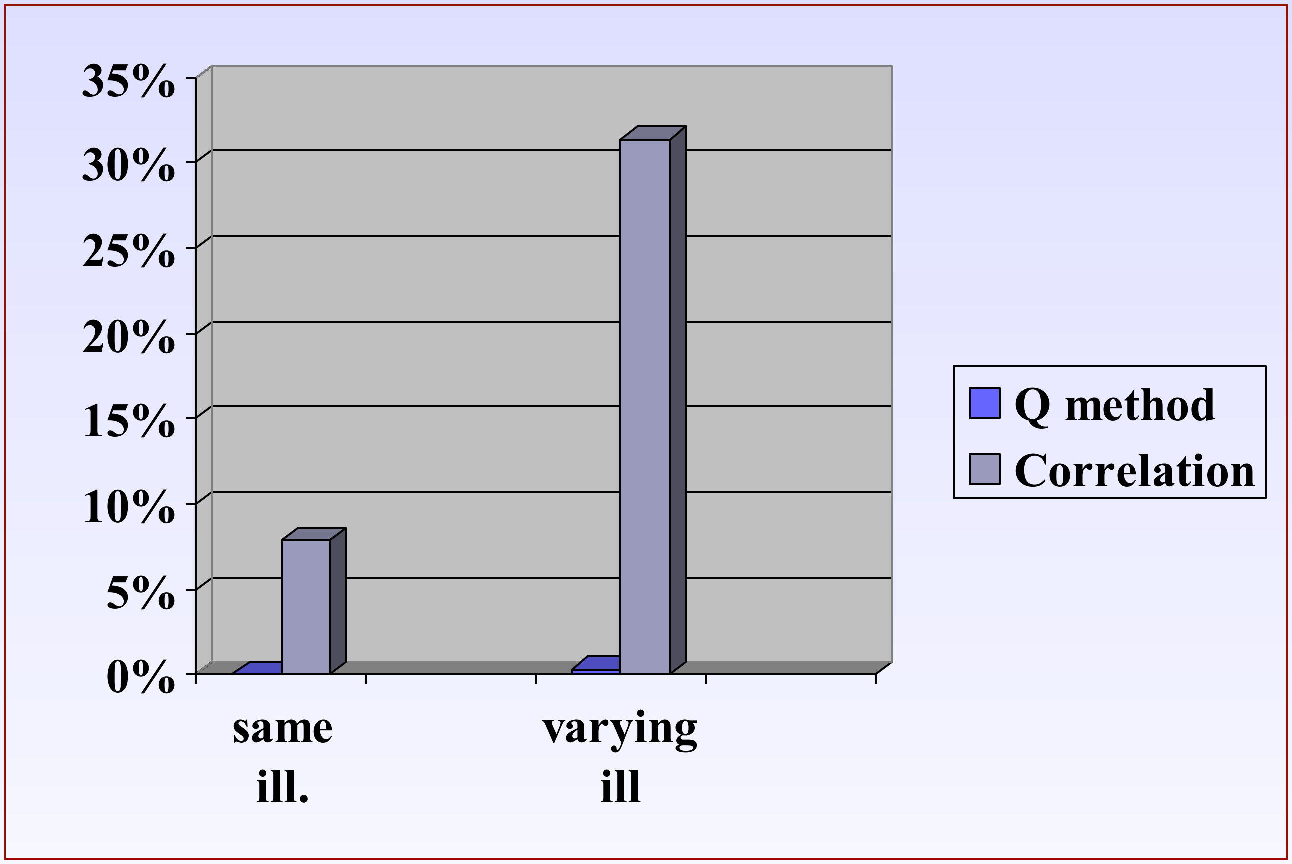
Identification: Correlation between the test image to the images stored in the database.

2. PCA

Database: Applying PCA on the objects' images + 3*20 additional images of 20 objects under 3 illumination
(to compare conditions to the quotient method). Having eigen vectors, each object is represented by it's eigen vectors' coefficients.

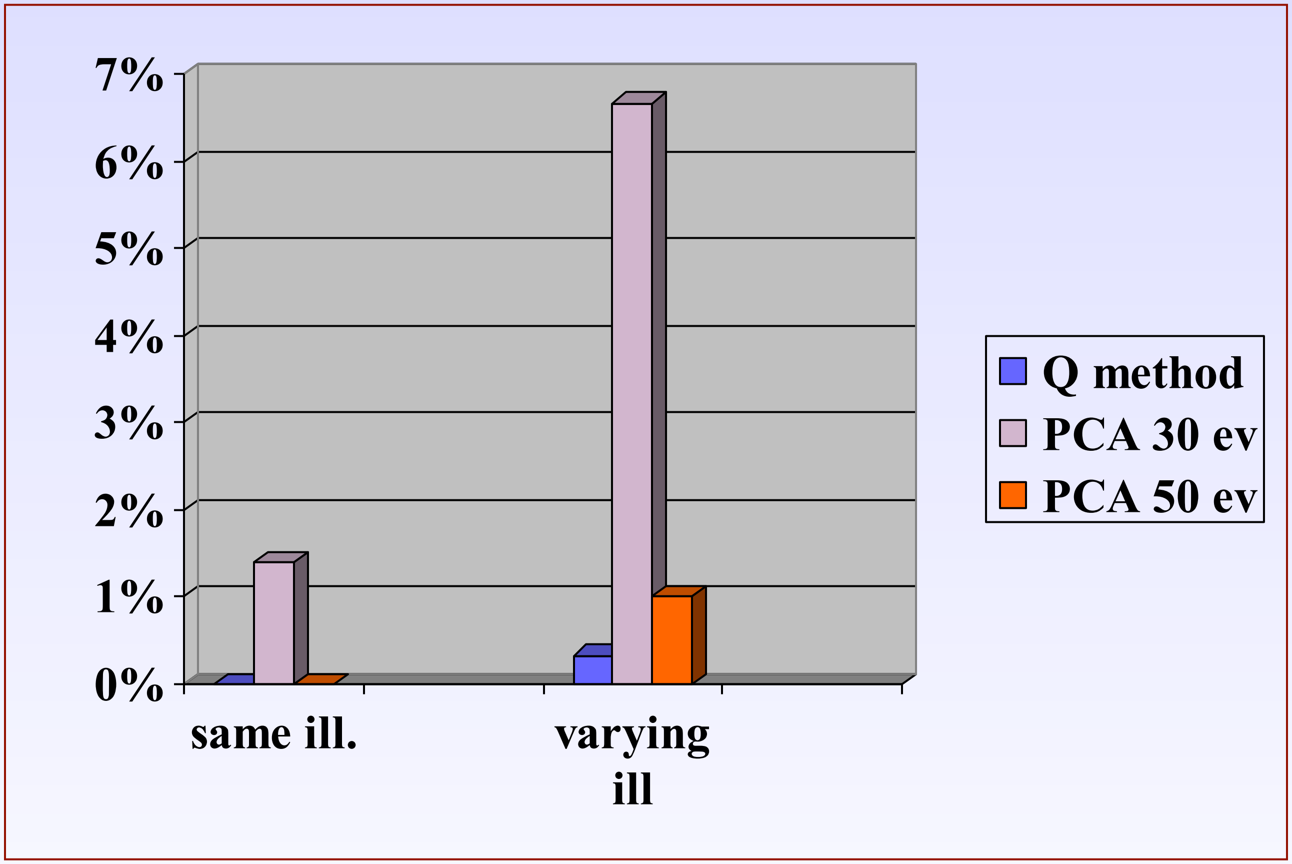
Identification: Comparison (LSE) between the test image coefficients (generated the same way as the database) and the database.

Recognition Results



Quotient method comparing to correlation

Recognition Results - cont



Quotient Method Vs. PCA

The End