Imaging Transfer Function $\tau(\omega_r)$

If object irradiance $o(r_o) = \delta(r_o)$

then image irradiance $i(r_i) = s(r_i)$ = spread function

(intensity impulse response)

In general, for linear imaging system

$$i(r_i) = \int_{-\infty}^{\infty} s(r_i - r_o)\, o(r_o)\, dr_o.$$

If $\int_{-\infty}^{\infty} s(r_i) e^{-j\omega_r r_i} \equiv S(\omega_r)$

and $\int_{-\infty}^{\infty} o(r_o)\, e^{-j\omega_r r_o}\, dr_o \equiv 0(\omega_r)$

then $I(\omega_r) = S(\omega_r) = \int_{-\infty}^{\infty} i(r_i)\, e^{-j\omega_r r_i} dr_i$

where

ω_r = spatial frequency (cycles$\cdot$m^{-1})

r_o = radial coordinate in object plane

r_i = " " " " image plane

$$S(\omega_r) = \frac{I(\omega_r)}{0(\omega_r)} \rightarrow \tau(\omega_r) = \frac{S(\omega_r)}{S_{max}} = \text{MTF } e^{j\text{PTF}}$$

MTF = modulation transfer function

PTF = phase " "

Imaging Transfer Function $\tau(\omega_r)$

If object irradiance $o(r_o) = \delta(r_o)$

then image irradiance $i(r_i) = s(r_i)$ = spread function

(intensity impulse response)

In general, for linear imaging system

$$i(r_i) = \int_{-\infty}^{\infty} s(r_i - r_o)\, o(r_o)\, dr_o.$$

If $\int_{-\infty}^{\infty} s(r_i) e^{-j\omega_r r_i} \equiv S(\omega_r)$

and $\int_{-\infty}^{\infty} o(r_o)\, e^{-j\omega_r r_o}\, dr_o \equiv O(\omega_r)$

then $I(\omega_r) = S(\omega_r) = \int_{-\infty}^{\infty} i(r_i)\, e^{-j\omega_r r_i} dr_i$

where

ω_r = spatial frequency (cycles·m^{-1})

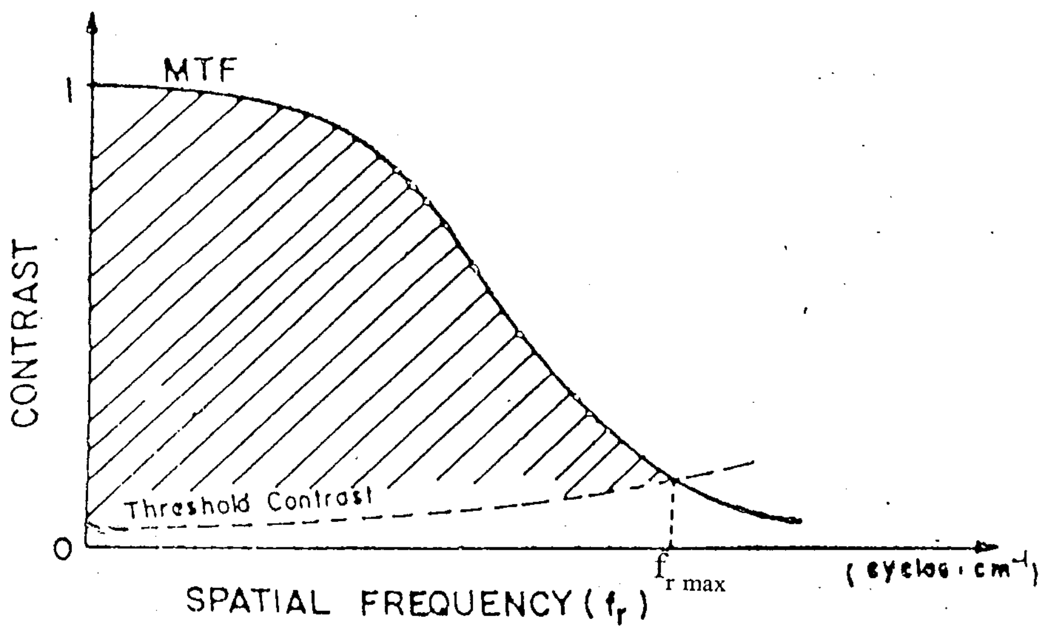
r_o = radial coordinate in object plane

r_i = " " " " image plane

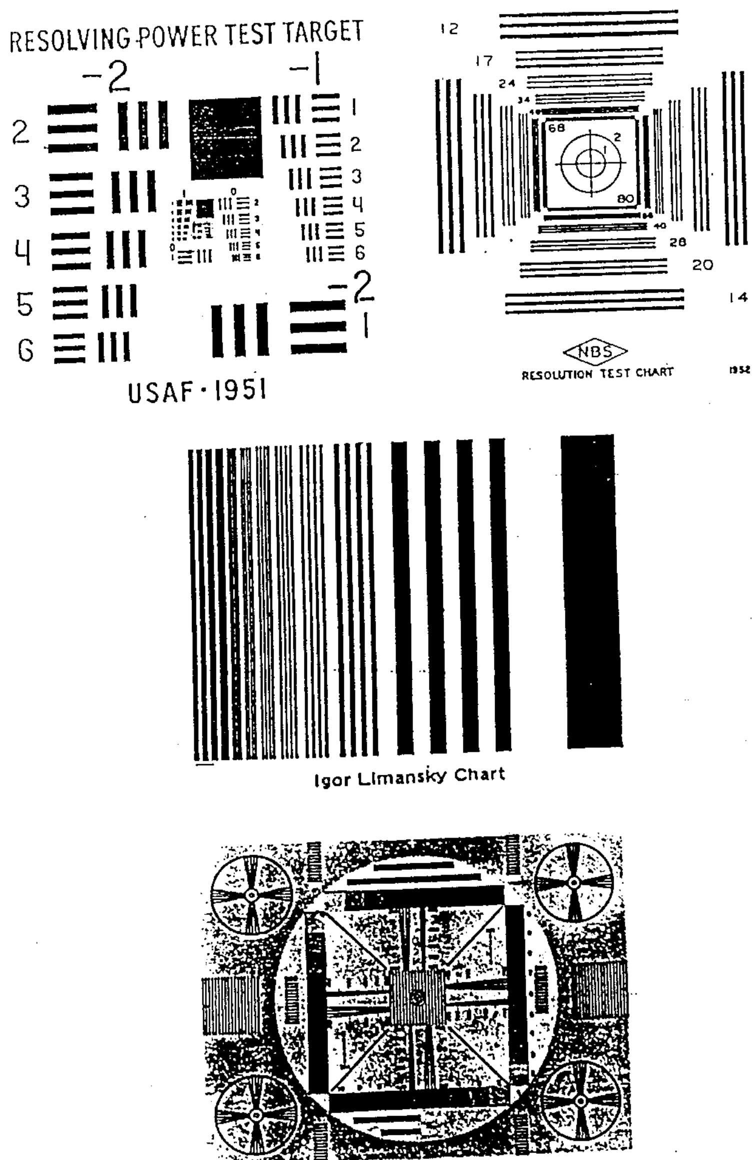
$$S(\omega_r) = \frac{I(\omega_r)}{O(\omega_r)} \rightarrow \tau(\omega_r) = \frac{S(\omega_r)}{S_{max}} = MTF\ e^{jPTF}$$

MTF = modulation transfer function

PTF = phase " "

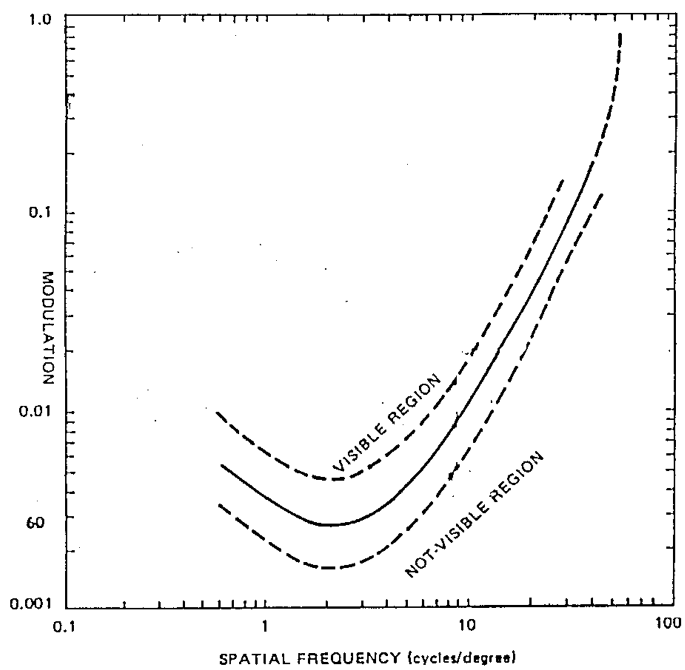


$$f_{r\,max} \approx \frac{1}{2\Delta x'}$$



USAF, NBS, Igor Limansky, and EIA resolution charts. [Note: the usual Igor Limansky resolution chart consists of 9 individual charts arranged in a 3 × 3 array.]

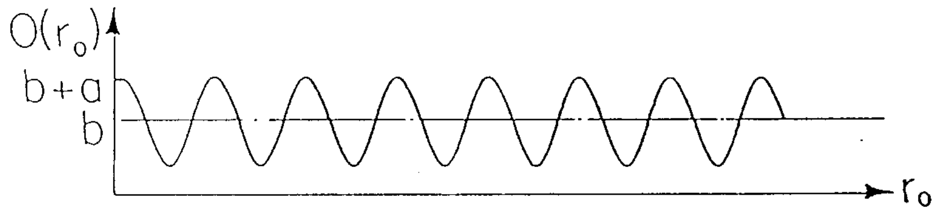
Threshold Contrast Curves for Human Visual System (90% of population)



$\Delta\alpha$ = angle subtended by target image

$\approx (2\ f_{a\ opt})^{-1}$

MODULATION CONTRAST



$$o(r_o) = b + a \cos \omega_0 r_o$$

$$i(r_i) = \int_{-\infty}^{\infty} o(r_o)\, s(r_o - r_i)\, dr_o$$

$$= \int_{-\infty}^{\infty} [b + a \cos \omega_0 r_o]\; s(r_o - r_i)\, d\, r_o$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \{b\, \delta(\omega_r) + \frac{a}{2} [\delta\, (\omega_r - \omega_o) + \delta(\omega_r + \omega_o)]\} S(\omega_r) e^{j\omega r_i}\, d\omega$$

$$= b\;\; S(o)\;\; + \frac{a}{2} [S(\omega_0)\, e^{j\omega_0 r_i} + S(-\omega_0) e^{-j\omega_0 r_i}]$$

If $S(\omega_r) = |S(\omega_r)|\, e^{-j\varphi(\omega_r)}$ · then

$$i\,(r_i) = bS(o) + a|S(\omega_0)|\cos[\omega_0\, r_i + \varphi(\omega_0)]$$

image of cosine is cosine of same frequency; only contrast and phase affected by linear system.

$$\text{Modulation Contrast} \equiv MC \equiv \frac{i_{max} - i_{min}}{i_{max} + i_{min}}$$

$$MC_o = \frac{(b + a) - (b - a)}{(b + a) + (b - a)} = \frac{a}{b} \;;\; MC_i = \frac{a}{b} \frac{|S(\omega_o)|}{S(o)}$$

$$\text{MODULATION CONTRAST FUNCTION} = MCF = \frac{MC_i}{MC_o}$$

For the cosine object here,

$$MCF = \frac{a}{b} \frac{|S(\omega_o)|}{S(o)} / (a/b) = \frac{|S(\omega_o)|}{S(o)} = MTF(\omega_o)$$

By measuring MCF at different frequencies (spatial), we can obtain spatial frequency plot of MTF. Overall system MTF is cascade of individual component MTFs.

In practice, it is much easier to make square wave rather than sinusoidal objects. Square wave can be decomposed into Fourier series of sin waves. However, although MCF = MTF for single sin wave object, for Fourier series

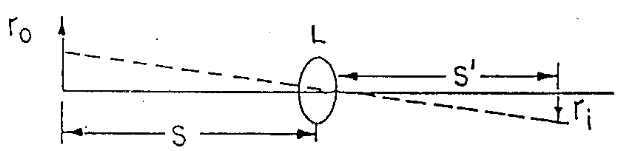
$$i_o(r_o) = b + \sum_{n=-\infty}^{\infty} a_n \cos(n\omega_o) r_o + \psi_n)$$

it turns out that $\text{MCF} = \frac{\sum a_n S(n\omega_o)}{S(o) \sum a_n} \neq \text{MTF} = \frac{\sum S(n\omega_o)}{S(o)}$.

Nevertheless, MCF of square wave object approximation to MTF. Thus, MTF is associated with contrast.
Resolution limit is determined by contrast.

From similar triangles

$$\frac{r_o}{s} = -\frac{r_i}{s'} \approx -\frac{r_i}{f_1} \Rightarrow \frac{\Delta r_o}{s} = -\frac{\Delta r_i}{f_1}$$



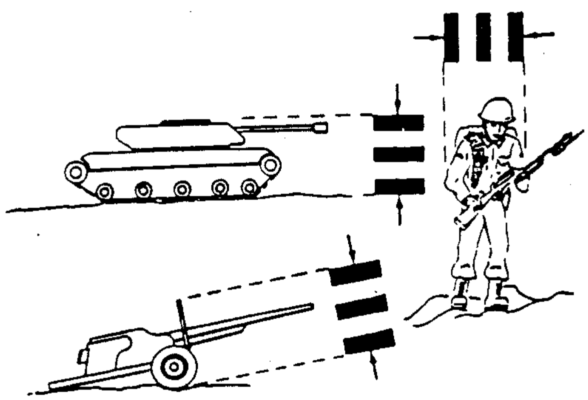
$$= -\frac{1}{2 f_{r\,max} f_1}$$

$$\therefore \Delta r_o = -\frac{1}{2\, f_{r\,max}\, M} \quad \text{detection}$$

$$= -\frac{n}{2\, f_{r\,max}\, M}$$

orientation
recognition
identification

(9)



TARGET	RESOLUTION PER MINIMUM DIMENSION IN LINE PAIRS (n)			
BROADSIDE VIEW	DETECTION	ORIENTATION	RECOGNITION	IDENTIFICATION
TRUCK	0.90	1.25	4.5	8.0
M-48 TANK	0.75	1.20	3.5	7.0
STALIN TANK	0.75	1.20	3.3	6.0
CENTURION TANK	0.75	1.20	3.5	6.0
HALF-TRACK	1.00	1.50	4.0	5.0
JEEP	1.20	1.50	4.5	5.5
COMMAND CAR	1.20	1.50	4.3	5.5
SOLDIER (STANDING)	1.50	1.80	3.8	8.0
105 HOWITZER	1.00	1.50	4.8	6.0
AVERAGE	1.0 ± 0.25	1.4 ± 0.35	4.0 ± 0.8	6.4 ± 1.5

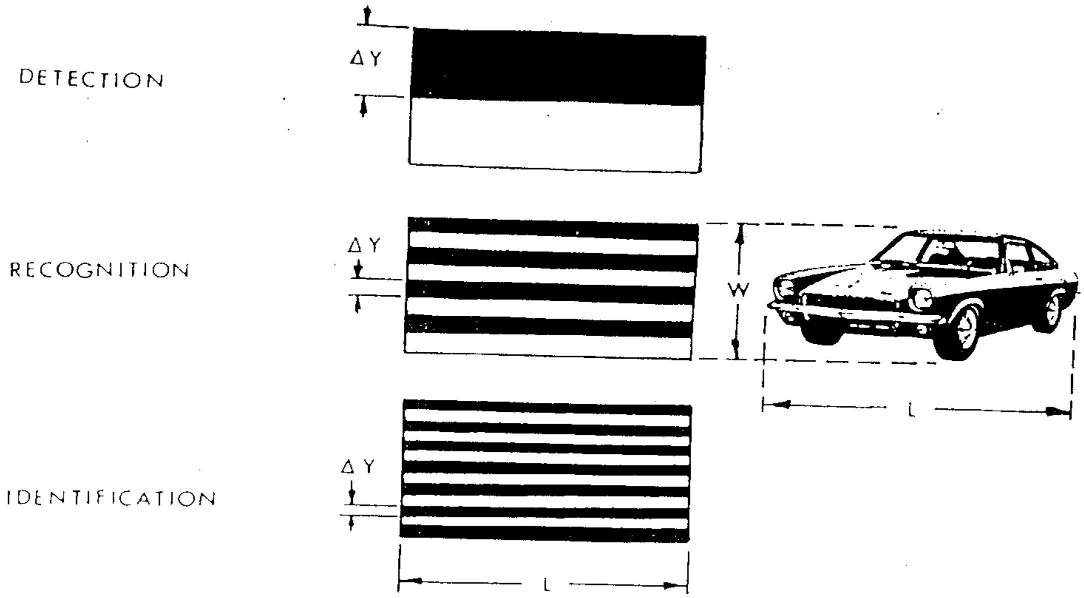
Fig. 1.1. Required resolution for detection, orientation, recognition, and identification.

television industry, which talks about "TV lines" (one "line pair" equals two "TV lines"). Further, Johnson's paper assumes a knowledgeable readership, and does not explain the implicit relationships between resolution, contrast, and their dependence upon the signal-to-noise ratio.

As a result, designers commonly misuse Johnson's data, quoting "lines on target" instead of line pairs in the minimum critical dimension. The "line pair" versus "TV lines" confusion often results in systems being

from RCA Electro-Optics Handbook or

M. Biberman, Perception of Displayed Information, Plenum, 1973,



Resolution required per minimum object dimension to achieve a given level of object discrimination expressed in terms of an equivalent bar pattern.

$$y' = 2n\Delta y' = -\frac{2n}{2f_{ymax}} = -\frac{n}{f_{ymax}}$$

$$y = 2n\Delta y = -\frac{n}{f_{ymax}M}$$

Navy Model

2-Dimensional Johnson Chart suggests would need

$2 \times 2 = 4$ pixels for detection

$2.8 \times 2.8 = 7.8$ pixels for orientation

$8 \times 8 = 64$ pixels for recognition

$12.8 \times 12.8 = 164$ pixels for identification.

However, U.S. Navy model suggests 66 pixels required for ship "classification", but 400 pixels required for identification. The Navy model implies that for a square object aout 10 line pairs required for each direction instead of the Johnson average of 6.4 ± 1.5 for identification.

Textual Resolution

Excellent reproduction of lower case "e" requires 8 line pairs per height, legible reproduction requires 5 line pairs per height, and decipherable reproduction requires 3 line pairs per height.

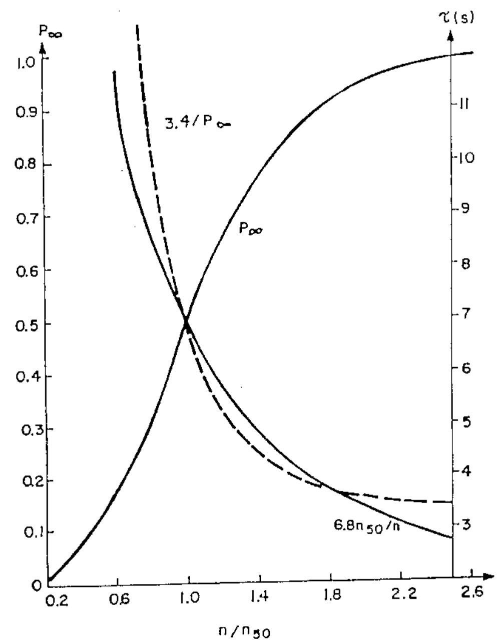
In general,

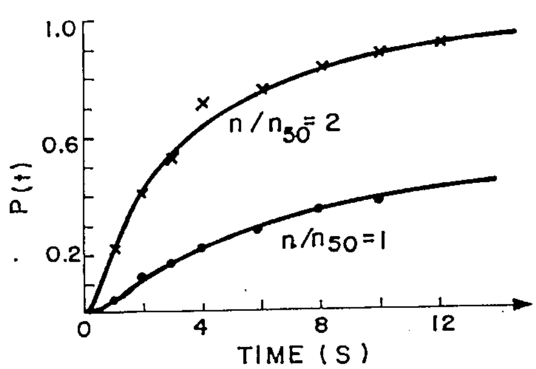
$$P(t) = P_\infty(1 - e^{-t/\tau})$$

where

$$P_\infty = \frac{(n/n_{50})^{x_o}}{1 + (n/n_{50})^{x_o}}$$

$$x_o = 2.7 + 0.7\,(n/n_{50})$$
$$\tau = 6.8\, n_{50}/n \approx 3.4\, P_\infty \quad \Big| \quad 1 \le n/n_{50} \le 2$$





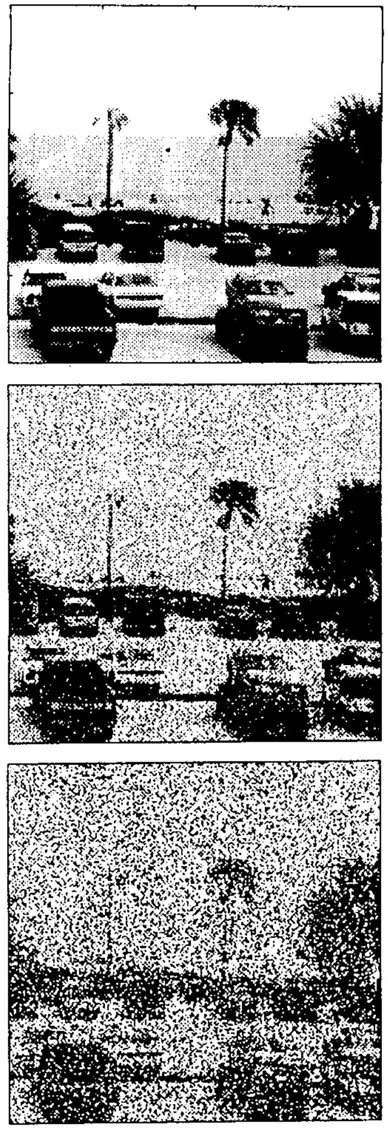
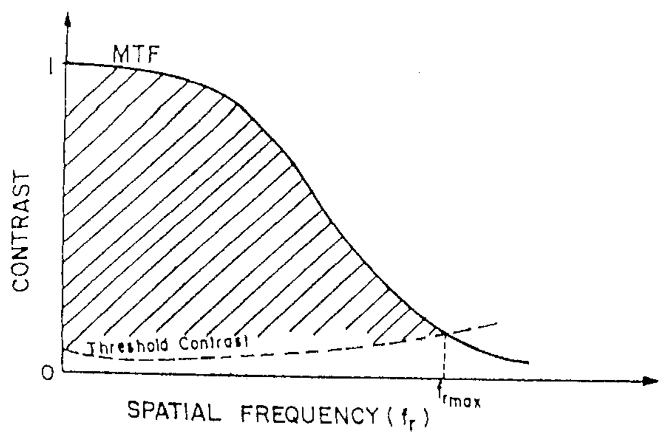


Fig. 11.2 Transition from contrast-limited image (top) to noise-limited image (bottom) of same scene. (Courtesy of Nir Corse and Ofer Hadar.)


MTF
1
CONTRAST
Threshold Contrast
0
f_{rmax}
SPATIAL FREQUENCY (f_r)

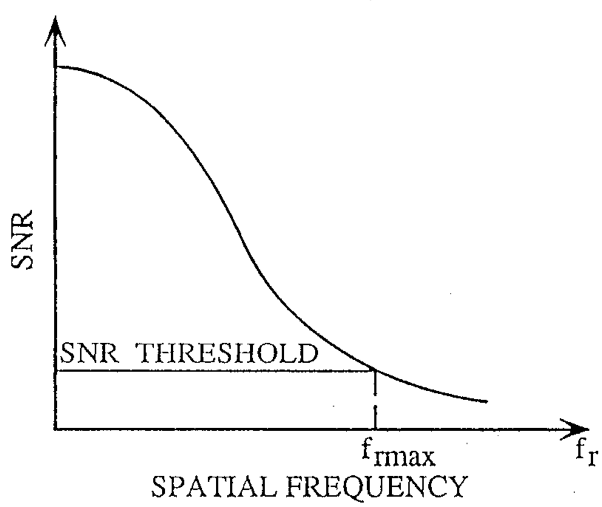

SNR
SNR THRESHOLD
f_{rmax}
f_r
SPATIAL FREQUENCY

Fig.11-1

Therefore, as spatial frequency content of image increases,

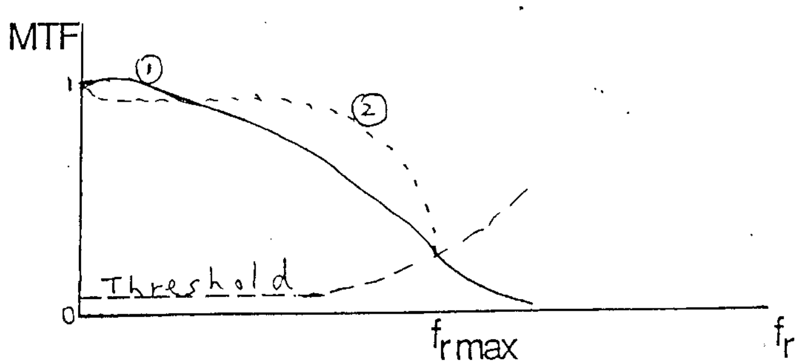
Δx decreases $\Rightarrow$ improved resolution.

$$M \equiv \frac{s'}{s} = \text{lateral magnification} \approx \frac{f_l}{s}\Big|_{s\ \text{large}}$$

M is often determined by considerations of desired image size on display system.

In this way, $f_{r\,max}$ can be determined from system resolution requirements.

MTF roll-off to $f_{r\,max}$ also important.



System (2) can resolve same size objects as (1), but with poorer contrast for larger objects and better contrast for smaller objects.

Resolution limited by system component whose MTF is worst. For reconnaissance systems that component is usually the atmosphere.

We want to determine at which wavelengths atmosphere least degrades image quality and resolution.

Obtaining the maximum spatial frequency in Optical system