A new estimation of the axial shielding factors for multishell cylindrical shields

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A new, more accurate formula describing double and multiple-shell axial magnetic shielding is obtained as a result of numerical verification of existing estimations. A standard ANSYS® software package was used. Parameters of the numerical model are as follows. Two concentric, closed cylinders of equal thickness and constant permeability are considered. The thickness-to-diameter ratio of the outer cylinder is $t/D_2=1/100$, its length-to-diameter ratio varies as $L_2/D_2=3$, 4, and 5, the ratio of the cylinders' outer diameters varies as $D_1/D_2=0.5$, 0.6, 0.7, 0.8, and 0.9, a range of the ratio of the cylinders' lengths is $L_1/L_2=0.1$–0.9, and a range of the relative permeability is $\mu=10^3$–10^6. A significant disagreement between the existing estimations and between each of them and the numerical model is found. One of the examined algorithms is modified to improve its precision. A remarkable improvement in the accuracy of the new algorithm compared to both existing methods is achieved. On a basis of the new algorithm, a new formula describing multishell axial magnetic shielding is suggested.
[S0021-8979(00)82008-3]

I. INTRODUCTION

Although conventional magnetic shielding is an old science, it is still difficult to express it mathematically and, as a result, it is not well developed analytically. The scientist or engineer faces a lack of reliable knowledge even when a relatively simple double-shell axial cylindrical assembly is designed. Existing algorithms[1–4] for calculating the axial shielding efficiency are based on two different approaches leading to contradictory results. The first approach[2–4] was originally developed by Mager,[4] and the second one was developed by Summner *et al.*[1] Reference 1 criticizes Mager's algorithm as taking into account only ratios of the shields' diameters while calculating the axial shielding factors and neglecting the ratio of their lengths. On the other hand, the second approach neglects the ratio of the shields' diameters. It is quite obvious that both the ratio of the shields' lengths and the ratio of their diameters should affect the shielding. Therefore, it was interesting to verify both existing estimations carefully with the aid of a standard numerical software package. Finally, the numerical verification helped us to obtain a new, more accurate estimation of axial shielding with closed double and multiple-shell shields.

II. THEORY

Mager[4] describes double-shell axial cylindrical shields (Fig. 1) as follows:

$$S_A \approx 1+4N_2^{\mathrm{ell}}(S_{T1}+S_{T2})+4N_2^{\mathrm{ell}}S_{T1}S_{T2} \times \frac{D_1+D_2-t}{2L_1+0.5(D_1+D_2-t)}\left[1-\left(\frac{D_1}{D_2-t}\right)^2\right], \quad (1)$$

where S_A is the axial shielding factor (defined as the ratio of the uniform external field to the field at the shield's center), N_2^{ell} is the demagnetizing factor of a general ellipsoid[5] of the same aspect ratio as that of the corresponding cylindrical shells, S_{Ti} is the transverse shielding factor, $S_{Ti}=\mu t/D_i$, μ is the relative permeability, t is the thickness, and L_i and D_i are the length and outer diameter of the corresponding shells, respectively (index 1 stands for the inner shell and index 2 stands for the outer shell). The axial shielding factor, S_{Ai}, for a single shell is given by Mager[4] as follows:

$$S_{Ai}\approx(1+4N_i^{\mathrm{ell}}S_{Ti})/[1+0.5/(L_i/D_i)]. \quad (2)$$

Summner *et al.*[1] describes double-shell axial cylindrical shields in a different way:

$$S_A\approx 1+S_{A1}+S_{A2}+S_{A1}S_{A2}(1-L_1/L_2). \quad (3)$$

Reference 1 also suggests an alternative treatment of single-shell axial shields, with results closely agreeing with Eq. (2).

III. NUMERICAL VERIFICATION

In order to verify the above theory, we used a standard ANSYS® 5.4 software package employing the finite-element method. Individual 2D axisymmetric models were built for 10 single-shell shields with an L/D ratio from 1 to 10 (t/D

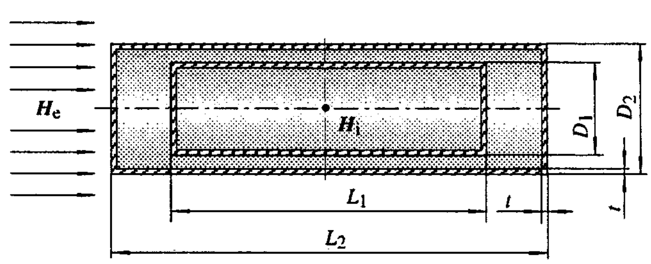

FIG. 1. Double-shell closed cylindrical magnetic shield.

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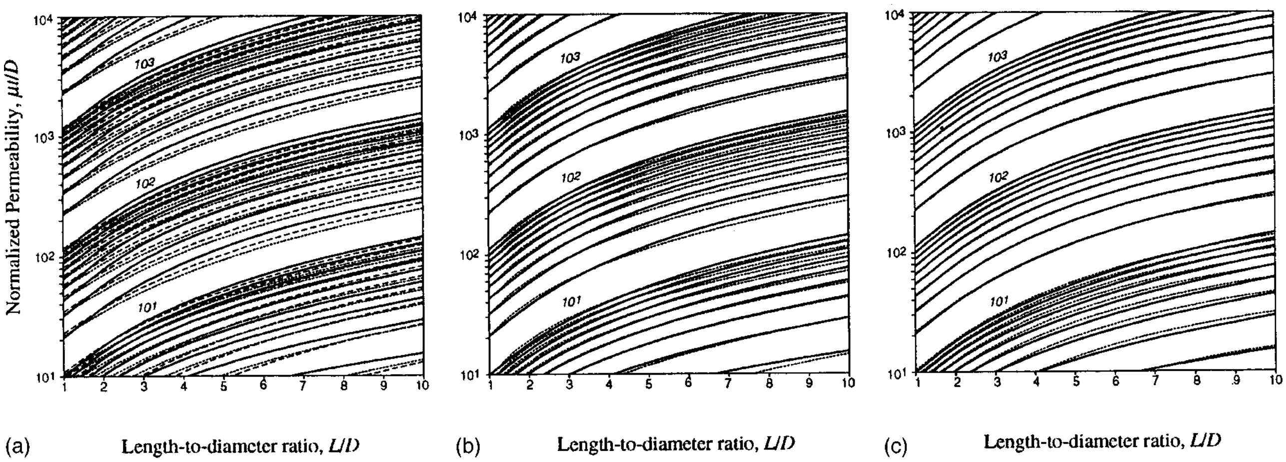

FIG. 2. The axial shielding factors for single-shell closed cylindrical shields: numerical results (solid lines) and analytical results predicted in (a) by Eq. (2) (dotted lines) and the theory in Ref. 1 (dashed lines), in (b) by Eq. (4) where $f=1$ (dotted lines), and in (c) by Eq. (4) where $f=1+(L/D)/100$ (dotted lines).

$=1/100$). 27 double-shell shield models were built with $t/D_2=1/100$, $L_2/D_2=3$, 4, and 5, $D_1/D_2=0.5$, 0.6, 0.7, 0.8, and 0.9, and a range of the L_1/L_2 ratio 0.1–0.9. The following permeability values were selected for the calculations: $\mu=10^3$, 10^4, 10^5, and 10^6, which correspond to the following values of the normalized permeability: $\mu t/D=10^1$, 10^2, 10^3, and 10^4. Other parameters of the numerical model were as follows, analysis type: linear static, element type: PLANE 53 (quadrangle shape, eight-nodes), mesh size: the finest, No. 1, standard smart sizing mesh adjusted manually to have more than two layers of elements filling cross sections of the cylinders. The aspect ratio of the elements was maintained smaller than approximately 1:2. A layer of air surrounding the model was about fourfold wider than the length of the outer cylinder. Finally, the numerical results are interpolated and represented as logarithmic-scale contour plots (Figs. 2–4).

First, we analyze single-shell shields. Figure 2(a) shows the axial shielding factors predicted by Mager[4] (dotted lines), by Summner *et al.*[1] (dashed lines), and by the numerical calculations (solid lines). To explain the difference between the numerical and analytical results in Fig. 2(a), we must notice that both Mager[4] and Summner *et al.*[1] assume a uniform distribution of the residual field within the shields. It allows Mager, for example, to estimate the residual field at the shields' center as the intrinsic field within the corresponding ellipsoid.[4] It seems more reasonable, however, to replace in Eq. (2) the demagnetizing factor calculated for ellipsoid,[5] N_i^{ell}, with that for a rod,[6] N_i^{rod}:

$$S_{Ai}\approx(1+4N_i^{\mathrm{rod}}S_{Ti})/f, \tag{4}$$

where N_i^{rod} can be approximated as follows:[7] $N_i^{\mathrm{rod}}\approx-0.048/\sqrt{L/D}+0.329/(L/D)-0.053/(L/D)^2$, f is a correction factor to take account of the shield's caps. Since the nonuniformity of the intrinsic field is taken into account while calculating the demagnetizing factor for rods, one can expect that Eq. (4) is more accurate than Eq. (2). Figure 2(b) shows a close agreement between the numerical results and the analytical expression Eq. (4) with $f=1$. If the effect of the caps is taken into account ($f<1$), the agreement is even closer [see Fig. 2(c)]. In Fig. 2(c), the axial shielding factors are estimated by using Eq. (4) where $f=1+(L/D)/100$. This simple expression for f is obtained intuitively as a result of efforts to match more closely the numerical and analytical results shown in Fig. 2(b).

Since numerical calculations provide us with a reasonable description of single-shell shields (see Fig. 2), we expect the same in the case of double-shell shields.

Our next task is to calculate the axial shielding factors for double-shell shields by using Eqs. (1) and (3) and then to compare the results with the corresponding numerical results. The results of Eq. (1) yield an enormous difference with respect to the results obtained numerically (see Fig. 3). From these observations and from careful consideration of Mager's work,[4] we came to the conclusion that N_2 in the

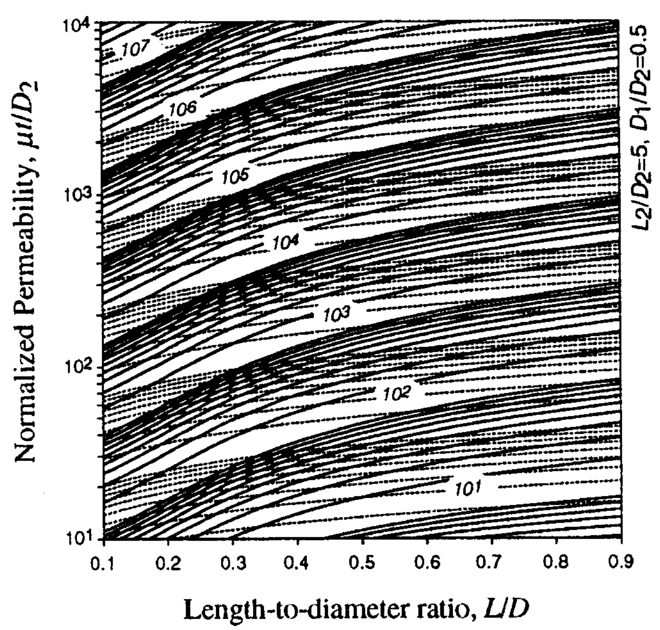

FIG. 3. The axial shielding factors for double-shell closed cylindrical shields: numerical results (solid lines) and analytical results (dotted lines) predicted by Eq. (1). An enormous difference is observed for all the numerical models (a representative example is shown).

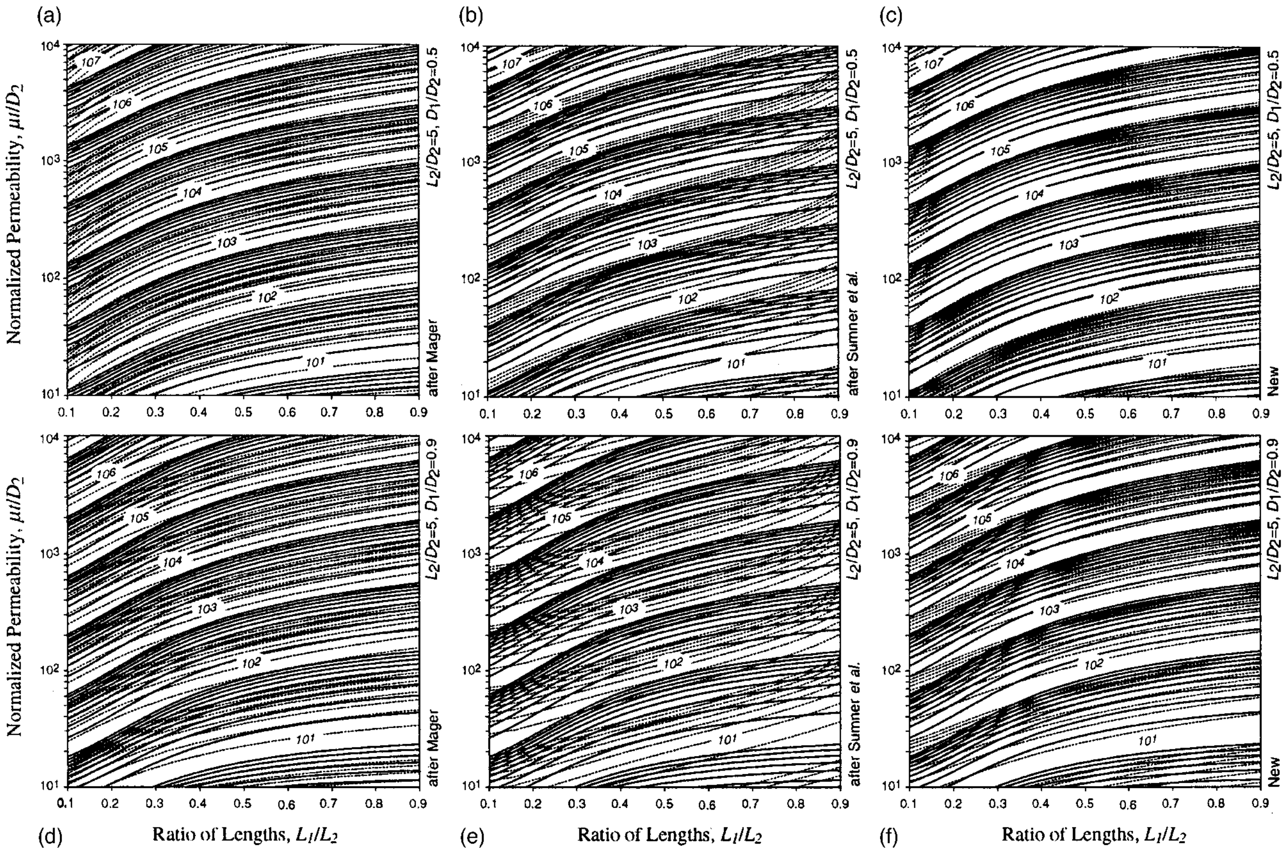

FIG. 4. The axial shielding factors for double-shell closed cylindrical shields: numerical results (solid lines) and analytical results (dotted lines) predicted in (a), (d) by Eq. (1), in (b), (e) by Eq. (3), and in (c), (f) by Eqs. (4) and (5). Equation (5) well approximates the numerical results for all the numerical models we tested (a representative example is shown).

third term of the right-hand side of Eq. (1) is a typographical error and should be changed to N_1. Our following results prove the above conclusion. After replacing N_2 with N_1, calculations were repeated and the results are presented in Figs. 4(a) and 4(d). The results in Figs. 4(a) and 4(d) lend support to the validity of the replacement of one of the two N_2 factors in Eq. (1) with N_1. Although Eq. (1) still predicts shielding factors that are greater than those obtained numerically [see Figs. 4(a) and 4(d)], the behavior of the analytical curves (dotted lines) is quite close now to that of the numerical data (solid lines). By contrast, Figs. 4(b) and 4(e) demonstrate a difference in behavior between the numerical and analytical results when Eq. (3) is used.

The similarity in the behavior of the analytical and numerical results observed in Figs. 4(a) and 4(d) led us to the belief that Eq. (1) should be modified to make it more precise. Trying to obtain a better matching between analytical and numerical results, we have found intuitively that the results of the following expression are in a good agreement with the numerical results for all of the numerical models of double-shell shields we tested:

$$S_A \approx 1 + S_{A1} + S_{A2} + S_{A1} S_{A2} \frac{f}{4N_2^{\rm rod}} \frac{5}{L_2/D_2} \times \frac{D_1 + D_2 - t}{4L_1 + 0.5(D_1 + D_2 - t)} \left[1 - \left(\frac{D_1}{D_2 - t} \right)^2 \right]. \tag{5}$$

Figures 4(c) and 4(f) show a representative example.

In general, for multishell shields with well separated shells, the axial shielding factor can be approximated by

$$S_A \approx \prod_{i=1}^{n-1} S_{Ai} S_{A(i+1)} \frac{f}{4N_{i+1}^{\rm rod}} \frac{5}{L_{i+1}/D_{i+1}} \times \frac{D_i + D_{i+1} - t}{4L_i + 0.5(D_i + D_{i+1} - t)} \left[1 - \left(\frac{D_i}{D_{i+1} - t} \right)^2 \right]. \tag{6}$$

IV. CONCLUSIONS

A new, more accurate estimation of the axial shielding factors for double and multiple-shell closed cylindrical shields is proposed as a result of numerical verification of existing algorithms.

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