3-D Magnetic Tracking of a Single Subminiature Coil With a Large 2-D Array of Uniaxial Transmitters

Anton Plotkin and Eugene Paperno

***Abstract*—A novel system and method for magnetic tracking of a single subminiature coil is described. The novelty of the method consists in employing a *large*, 8 × 8 array of coplanar transmitting coils. This allows us to always keep the receiving coil not far from the wide, flat transmitting array, to increase the signal-to-noise ratio, and to decrease the retransmitted interference. The whole transmitting array, 64 coils, is sequentially activated only at the initiation stage to compute the initial position of the receiving coil. The redundancy in the transmitters number provides fast and unambiguous convergence of the optimization algorithm. At the following tracking stages, a small (8 coils) transmitting subarray is activated. The relatively small subarray size allows us to keep a high update rate and resolution of tracking. For a 50-Hz update rate, the tracking resolution is not worse than 0.25 mm, 0.2° rms at a 200-mm height above the transmitting array's center. This resolution corresponds to an ∼1-mm, 0.6° tracking accuracy.**

The novelty of the method consists as well in optimizing the transmitting coils' geometry to substantially (down to 0.5 mm) reduce the systematic error caused by the inaccuracy of the dipole field approximation.

***Index Terms*—Coplanar transmitting coils, dipole field approximation error, large transmitting array, magnetic tracking, subminiature coil.**

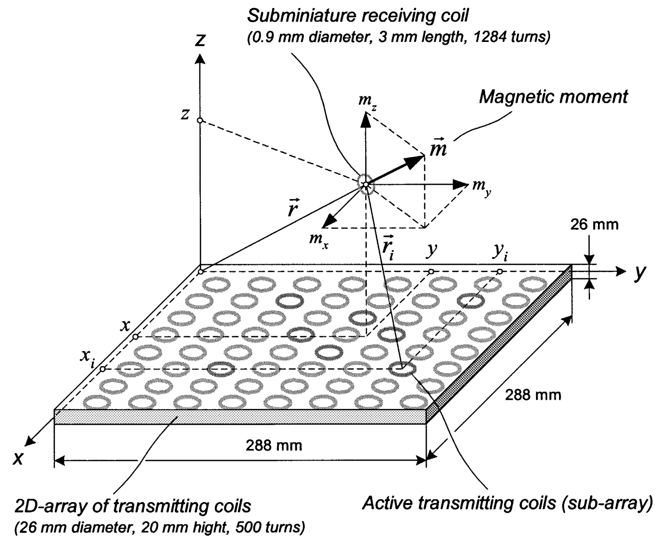

Fig. 1. Tracking a single subminiature receiving coil with a *large* coplanar array of transmitting coils. The whole transmitting array is sequentially activated only at the initiation stage to find the initial position ($\vec{r}$, $\vec{m}/m$) of the receiving coil. At the following tracking stages only a small number of transmitting coils (subarray) is activated.

I. Introduction

In this paper, we describe a novel system and method for magnetic tracking of a single subminiature coil, which can be used for intrabody navigation of medical instruments, eye tracking, computer three-dimensional (3-D) digitizing, virtual reality, motion tracking, etc.

The novelty of the method with respect to [1]–[5] consists in employing a *large* two-dimensional (2-D) array of uniaxial transmitting coils (see Fig. 1). This allows us to always keep the receiving coil not far from the wide, flat transmitting array, to increase the signal-to-noise ratio (SNR), and to decrease the interference from the signals retransmitted by nearby conducting objects.

Compared with a different, reciprocal method [5], where a 2-D array of uniaxial Hall sensors is employed for tracking a permanent magnet, the new method provides an up to *three* orders better SNR, is not sensitive to dc errors or $1/f$ noise, and allows simultaneous tracking of multiple receivers.

The novelty of the method consists as well in optimizing the transmitting coils' geometry to substantially reduce the systematic error caused by the inaccuracy of the dipole field approximation.

II. Description of the Tracking System

The tracking system (see Fig. 1) is composed of a single subminiature (0.9-mm diameter, 3-mm length) induction coil wound with 1284 turns of 28-μm copper wire and an 8 × 8-array of transmitting coils (26-mm outer diameter, 18-mm inner diameter, 20-mm height) wound with 500 turns of a 0.4-mm litz wire. The distance between the transmitting coil centers is 36 mm and the total array size is 288 mm × 288 mm × 26 mm. A number of such arrays can be combined to spread out the operating region (a region within 50- and 200-mm heights above the transmitting array).

The receiving coil output is amplified and sampled by a precise electronic circuit (500 gain, less than 3-nV/$\sqrt{\text{Hz}}$ input noise) and fed into a computer for further processing. The computer sequentially activates the transmitting coils through a buffer and implements an optimization algorithm to calculate the location and orientation of the receiving coil with respect to the transmitting array. The activation electric current is of a 1-A amplitude and 50-kHz frequency. The active power dissipated by each transmitting coil does not exceed 1 W.

Manuscript received January 2, 2003. This work was supported in part by the Ivanier Center for Robotics Research and Production Management.

The authors are with the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel (e-mail: antonp@ee.bgu.ac.il; paperno@ee.bgu.ac.il).

Digital Object Identifier 10.1109/TMAG.2003.816750

III. Method

A. Theory

The near field generated by a transmitting coil can be approximated by the equivalent dipole field [6]

$$\vec{B}_i^t = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{M}\cdot\vec{r}_i)\vec{r}_i}{r_i^5} - \frac{\vec{M}}{r_i^3}\right) \quad (1)$$

where μ_0 is the permeability of free space, $\vec{M}$ is the magnetic moment of the transmitting coil, i is the transmitting coil number, and $\vec{r}_i$ is the distance between the corresponding transmitting coil and the receiving coil.

The field seen by the receiving coil is

$$B_i^s = \vec{B}_i^t \cdot \vec{m} \quad (2)$$

where $\vec{m}$ is the magnetic moment of the receiving coil.

Considering (1), we can rewrite (2) as follows:

$$B_i^s = \frac{\mu_0}{4\pi}\, mM\, \frac{1}{r_i^5}\left[3z(\vec{r}_i\cdot\vec{n}) - n_z r_i^2\right] \quad (3)$$

where $\vec{r}_i = (x - x_i,\, y - y_i,\, z)$, $x_i,\, y_i$ are the transmitting coil coordinates, $\vec{n} = \vec{m}/m = (n_x,\, n_y,\, n_z)$ describes the receiving coil orientation, and $\vec{r} = (x,\, y,\, z)$ describes the receiving coil location (see Fig. 1).

A system of equations (3) can be expressed in the homogeneous form

$$[F_1(\vec{r},\,\vec{n}) = 0,\, \ldots,\, F_i(\vec{r},\,\vec{n}) = 0,\, \ldots,\, F_k(\vec{r},\,\vec{n}) = 0], \quad (i = 1,\, 2\cdots k). \quad (4)$$

For $k \geq 6$, (4) can be solved numerically by a nonlinear least-square optimization algorithm [7]. The algorithm convergence can be estimated by the objective function

$$\Phi = \sum_{i=1}^{k} F_i(\hat{\vec{r}},\, \hat{\vec{n}})^2 \quad (5)$$

where $\hat{\vec{r}}$ and $\hat{\vec{n}}$ are estimates of the vectors $\vec{r}$ and $\vec{n}$. The complete algorithm of computing $\vec{r}$ and $\vec{n}$ can be divided into the initiation and tracking stages.

B. Initiation Stage

At the initiation stage, the whole transmitting array is activated to estimate the initial position of the receiving coil.

Activation of the large number of transmitting coils provides redundancy in (4). This helps avoid the local minima problem (see Fig. 2). If only six transmitters are activated, then the objective function Φ in Fig. 2(a) shows a large number of local minima. Thus, a large number of starting points should be tested to ensure correct convergence of the algorithm. Activating the *whole* transmitting array at the initiation stage smoothes the objective function [see Fig. 2(b)]. This provides unambiguous convergence to the global minimum from an arbitrary chosen starting point. Thus, for any initial position of the receiving coil, we can choose the center of the transmitting array as a fixed starting point for the iteration process.

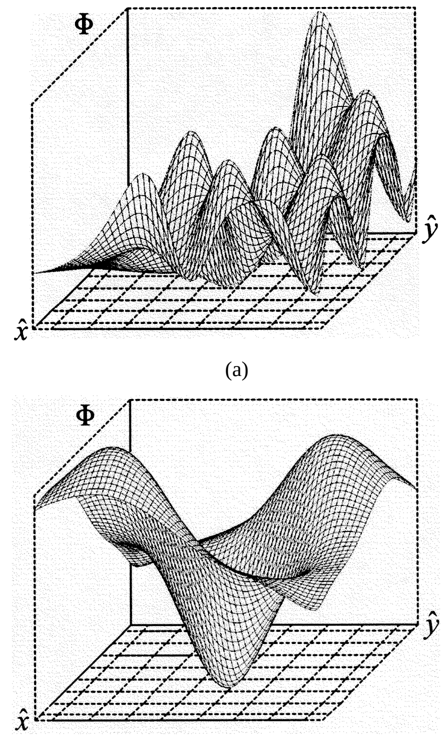

Fig. 2. Dependence of the objective function Φ on estimates $\hat{x}$ and $\hat{y}$ of the corresponding receiving coil coordinates. $\vec{r} =$ (170 mm, 170 mm, 70 mm) and $\vec{n} = (\sqrt{0.5},\, \sqrt{0.5},\, 0)$. (a) Only six transmitters are activated. A large number of local minima causes an ambiguous convergence. (b) The whole transmitting array is activated. The objective function is much smoother in this case, and the convergence becomes unambiguous.

At both the initiation and the following tracking stages, we employ a modified Levenberg–Marquardt optimization algorithm [7]. This algorithm comprises advantages of both the Newton and the steepest descent iterative methods. It is fast and provides a large convergence area.

C. Tracking Stage

Once the initiation process has converged correctly, the solution found can be used as the starting point for the next tracking process. Assuming that the actual receiving coil position is not changing drastically, a much smaller number of transmitting coils (subarray) can be activated. Decreasing the number of activated transmitters helps increase the system update rate. We have found that the subarray consisting of eight transmitting coils provides correct tracking when the change in actual receiving coil location is less than ~5 mm, no matter how large the change in orientation is.

The transmitting coils in the subarray are chosen in accordance with the receiving coil coordinates computed previously. The subarray pattern is fixed (see Fig. 1) while its location in the array is dynamic. Its center tends to be located as close as possible to the receiving coil projection on the transmitting array plane.

It may be shown that for the chosen subarray pattern the number of degraded equations in (4) cannot be greater than two [(2) and (3) are degraded when the orientation vector $\vec{n}$ becomes orthogonal to the vector $\vec{B}_i^t$]. As a result, the full rank of (4) is always assured, $k \geq 6$, for the chosen number of transmitters.

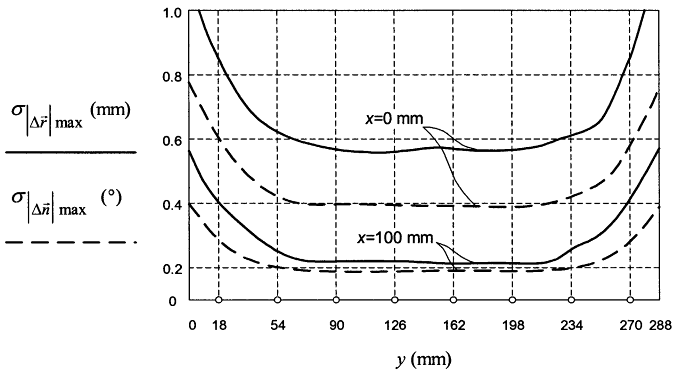

Fig. 3. Location and orientation tracking errors $\sigma_{|\Delta\vec{r}|\,\max}$ and $\sigma_{|\Delta\vec{n}|\,\max}$, where σ stands for the standard deviation of the vectors $\Delta\vec{r} = \hat{\vec{r}} - \vec{r}$ and $\Delta\vec{n} = \hat{\vec{n}} - \vec{n}$, obtained for the worst-case orientations at a 200-mm height above the transmitting array. (The small circles in the figure correspond to y coordinates of the transmitting coils.)

For the sake of simplicity and consistency of the tracking algorithm, we neglect the $\vec{m}_z = \vec{m}_x \times \vec{m}_y$ relationship that reduces the number of independent variables in (3) from six to five. For the same reason, we do not consider any additional equations other than (3), despite the fact [3] that this may reduce the size of the subarray from eight transmitters to six.

IV. Resolution

A. Temporal Resolution

A fast and unambiguous convergence allows us to reduce the computing time at the initiation stage down to 50 ms. The relatively small subarray size allows us to keep a high update rate of tracking. The typical computing time at tracking stages does not exceed 10 ms.

B. Spatial Resolution

The spatial resolution strongly depends on the SNR. The SNR measured for a 200-mm distance between the coaxial receiving and transmitting coils is as large as 86 dB/$\sqrt{\text{Hz}}$. For a 50-Hz update rate, the tracking resolution is not worse than 0.25 mm, 0.2° rms measured at a 200-mm height above the transmitting array's center for any orientation of the receiving coil (see Fig. 3). (This resolution corresponds to an ~0.75-mm 0.6° tracking accuracy).

The tracking resolution remains not worse than 1.1 mm, 0.8° (see Fig. 3), even at the borders of the operating region. Both the SNR and tracking resolution increase proportionally to $1/z^3$ when the receiving coil approaches the transmitting array.

C. Approximation Error

The special relationship chosen for the transmitting coil dimensions is optimal in terms of generating an almost ideal dipole field. It reduces the relative error of the dipole field approximation down to 0.3% at distances of four radii from the coil's center. As a result, the systematic tracking error caused by the inaccuracy of the approximation is below 0.5 mm even at the shortest, 50-mm distances from the transmitting array. The small systematic error relieves the tracking algorithm of having to compute the elliptic integrals to find the exact values of the transmitted fields.

V. Conclusion

A novel method and system are developed for tracking a single subminiature coil with a large coplanar transmitting array. The method's advantage compared with a reciprocal approach is in substantial increase in the SNR and resolution, and eliminating dc errors. Due to optimizing the transmitting coils' geometry, we have also improved the dipole field approximation. This relieves the system of having to spend extra time for computing the exact values of the coil field.

References

[1] A. Govari, "Electromagnetic position single axis system," U.S. Patent 6 484 118 B1, Nov. 19, 2002.
[2] N. Amorai-Moriya, M. Itzkovich, and B. Spivak, "System for three dimensional positioning and tracking," U.S. Patent 6 316 934, Nov. 13, 2001.
[3] M. Shneider, "Measuring position and orientation using magnetic fields," U.S. Patent 6 073 043, June 6, 2000.
[4] J. S. Bladen and A. P. Anderson, "Position location system," U.S. Patent 5 913 820, June 22, 1999.
[5] V. Schlageter, P.-A. Besse, R. S. Popovich, and P. Kucera, "Tracking system with five degrees of freedom using a 2-D array of Hall sensors and a permanent magnet," *Sensors Actuators A*, vol. 2951, pp. 1–6, Jan. 2001.
[6] A. Matveyev, *Principles of Electrodynamics*. New York: Reinhold, 1966.
[7] M. A. Wolfe, *Numerical Methods for Unconstrained Optimization, an Introduction*. New York: Van Nostrand Reinhold, 1978.