

Cylindrical induction coil to accurately imitate the ideal magnetic dipole

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Abstract

In this work, we optimize the geometry of a cylindrical induction coil in order to improve the imitation of the ideal dipole field. First, we analyze a *single-layer* coil and find that for an optimum length-to-diameter ratio (L/D) of 0.86:1 the imitation error, computed for a four radii distance from the coil's center, is reduced by a factor of ~ 12 (from 9.4 down to 0.73%) relative to a single-turn coil. Second, we analyze two different types of *multilayer* coils: one having for all the layers the same 0.86:1 optimum L/D (a coil with a trapezoidal cross section of the winding); and another having for each layer a different L/D (a coil with a rectangular cross section of the winding). The imitation error in this case is reduced by a factor of ~ 22 and ~ 33 , correspondingly. We finally show that optimizing the multilayer coil makes the imitation error practically negligible ($< 0.28\%$) even at as short as four radii distances from the coil's center. Employing such optimized coil allows one to use the simple dipole model with a high degree of accuracy. It also relieves the system of having to spend extra time for computing the exact values of the coil field.

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1. Introduction

Many applications in magnetics are based on generating and further measuring static and quasistatic dipole fields. Modern magnetic tracking systems employing arrays of transmitters or sensors [1–5] are one of the examples.

An ideal dipole field can be generated by a spherical coil [6]. This solution, however, is impractical because it is difficult to manufacture a precise spherical coil, and especially a multilayer coil having a large magnetic moment.

Much simpler cylindrical induction coils are traditionally used in practice [1–4] to imitate the ideal, infinitely-small dipole. Such imitation, however, being never perfect, causes inevitable systematic errors, which drastically increase at distances comparable with the coil dimensions.

There are obviously two ways to eliminate or reduce the above errors. The one is to compute the exact field of the coil [1] (what actually means to abandon the idea of imitation); and the other is to try and optimize the cylindrical coil geometry in order to achieve a better imitation of the ideal dipole.

In this work, we have found that optimizing the geometry of a simple cylindrical coil significantly reduces the imita-

tion error and makes it practically negligible even at relatively short distances from the coil's center. Employing such optimized coil allows one to use the simple dipole model with a high degree of accuracy. It also relieves the system of having to spend extra time for computing the exact values of the coil field.

In this work we optimize both the length of the coil and the geometry of its cross section. We start in Section 2 from optimizing the length of a thin, *single-layer* coil and then, in Section 3, optimize *multilayer* coils. We have to mention here that the idea of optimizing the length of a single-layer coil was actually triggered by [5], where an optimal aspect ratio of 0.87 has been reported for a permanent magnet. In Section 2, we confirm practically the same aspect ratio for an optimal single-layer coil. The principal novelty of the present work consists in optimizing *multilayer* coils. We have found that aspect ratios for optimal multilayer coils depend on the ratio of their inner and outer diameters and quite differ from the optimal aspect ratio for a bulk permanent magnet.

2. Optimal single-layer cylindrical coil

Let us consider a multi-turn, *single-layer* cylindrical induction coil (see Fig. 1) wound with a thin wire. Since the cylindrical coil field is axisymmetric, it can be described, without loss of generality, in the y - z plane only. The total

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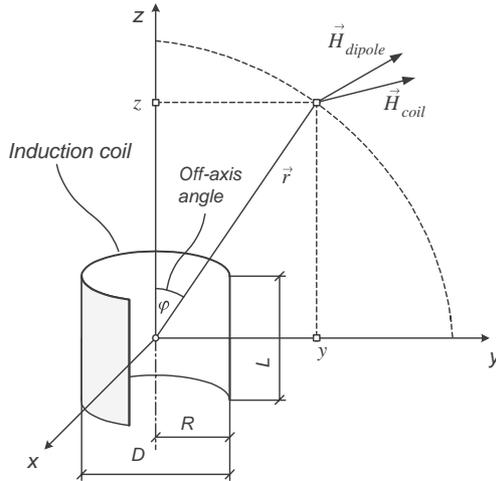


Fig. 1. Multi-turn, single-layer cylindrical induction coil wound with a thin wire.

field components can be found by superposition of the fields generated by the coil turns:

$$H_{y\text{ coil}} = \frac{I}{2\pi} \sum_{i=1}^N \frac{yz}{y^2 \sqrt{(R+y)^2 + z^2}} \times \left(-K(\alpha) + E(\alpha)\alpha^2 + y^2 + \frac{z^2}{(R-y)^2 + z^2} \right), \quad (1)$$

$$H_{z\text{ coil}} = \frac{I}{2\pi} \sum_{i=1}^N \frac{1}{\sqrt{(R+y)^2 + z^2}} \times \left(K(\alpha) + E(\alpha)\alpha^2 - y^2 - \frac{z^2}{(R-y)^2 + z^2} \right), \quad (2)$$

where

$$K(\alpha) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \alpha^2 \sin^2 \theta}}$$

$$E(\alpha) = \int_0^{\pi/2} \sqrt{1 - \alpha^2 \sin^2 \theta} d\theta, \quad (3)$$

$$\alpha = \sqrt{\frac{4Rr_i}{(R+r_i)^2 + z^2}}$$

I is the electric current flowing through the coil, N , the number of turns, R , the radius of the coil, r_i , the distances between the center of the corresponding turn and a position (y, z) in the y - z plane, $r = \sqrt{y^2 + z^2}$ is the distance between the coil's center and the position (y, z) , $K(\alpha)$, and $E(\alpha)$ are the complete elliptic integrals of the first and second kind.

The equivalent dipole field can be found as follows:

$$\begin{cases} H_{y\text{ dipole}} = \frac{M}{4\pi} \frac{3 \cos \varphi \sin \varphi}{r^3} \\ H_{z\text{ dipole}} = \frac{M}{4\pi} \frac{3 \cos^2 \varphi - 1}{r^3} \end{cases}, \quad (4)$$

where M is the magnetic moment of the coil, $M = N I \pi R^2$ and φ , the of-axis angle shown in Fig. 1.

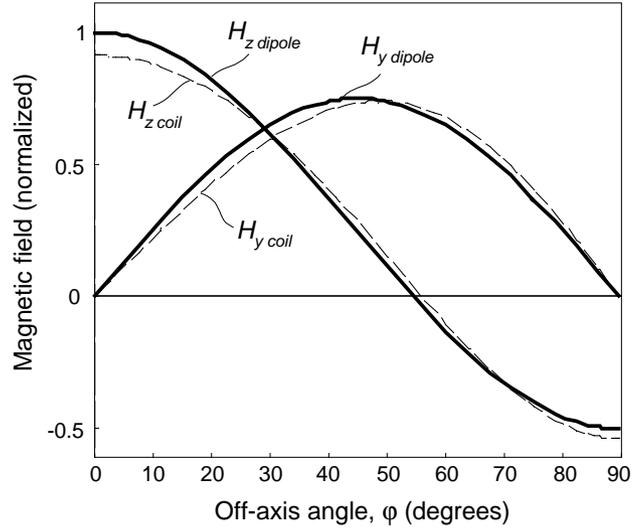


Fig. 2. Comparison between the dipole field and that of a single-turn coil. The aspect ratio of the coil is $L/D = 0$. The field components are computed at a four radii distance from the coil's center.

According to (1)–(4), the imitation error:

$$\delta = \frac{|\vec{H}_{\text{coil}} - \vec{H}_{\text{dipole}}|}{H_{\text{coil}}} \times 100\%, \quad (5)$$

can be found as a function of the off-axis angle φ for a given length-to-diameter ratio, (L/D) of the coil (see Fig. 1).

Let us first analyze a relatively short cylindrical induction coil that is traditionally used in magnetic-tracking transmitters. Such a coil is practically identical to a single-turn coil having an $L/D = 0$. Fig. 2 shows that the magnetic field of a single-turn coil much differs from the equivalent dipole field: the imitation error reaches 9.4% for $r = 4R$ distances.

In order to minimize the imitation error, we have examined in Fig. 3 the dependence of the δ on the aspect ratio, L/D , for a fixed $r = 4R$. Fig. 3 shows that the imitation error decreases with increasing the L/D ratio and is minimized ($\delta < 0.73\%$) for an optimum $(L/D)_{\text{opt}} = 0.86 : 1$.

One can see from Fig. 3 that the optimal-length single-layer cylindrical coil much better imitates the ideal dipole than a single-turn coil.

One can expect a further decreasing of the imitation error for a multilayer cylindrical coil having the same outer diameter as that of the optimal single-layer coil. It is so because the inner turns in the multilayer coil have smaller radii and are relatively more distant from the outermost turns from the point where the field is measured.

3. Optimal multilayer cylindrical coils

The found above optimum aspect ratio for a single-layer cylindrical coil, $(L/D)_{\text{opt}} = 0.86 : 1$, suggests a straightforward optimization rule for a multilayer cylindrical coil: the aspect ratio for each layer, L_j/D_j , should simply be equal to the $(L/D)_{\text{opt}}$. The induction coil that satisfies these

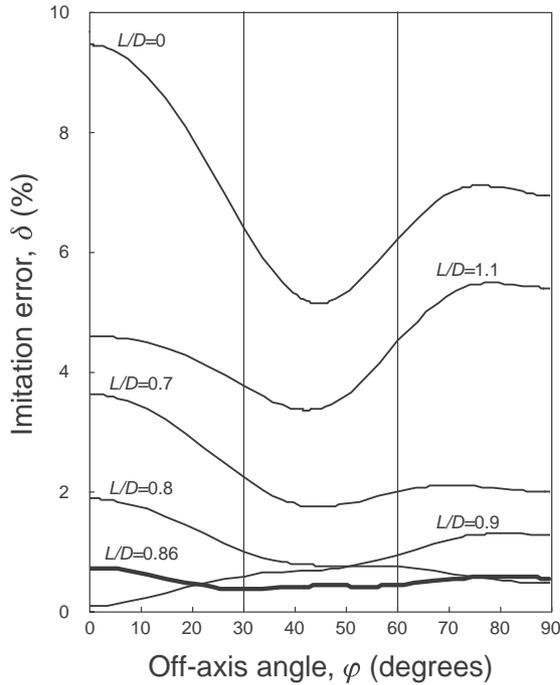


Fig. 3. Imitation error, δ , as a function of the off-axis angle, φ , and different values of the aspect ratio, L/D . The number of turns $N = 100$, and the distance from the coil's center $r = 4R$.

conditions has a trapezoidal cross section of the winding [see Fig. 4(a)].

More detailed analysis of the single-layer coil (see Fig. 5) suggests an alternative approach to optimizing a multilayer cylindrical coil. It is important to note in Fig. 5 that the imitation error components

$$\delta_y = \frac{H_{y \text{ coil}} - H_{y \text{ dipole}}}{H_{y \text{ dipole max}}} \times 100\% \quad (6)$$

and

$$\delta_z = \frac{H_{z \text{ coil}} - H_{z \text{ dipole}}}{H_{z \text{ dipole max}}} \times 100\% \quad (7)$$

change their signs while keeping approximately the same behavior for the aspect ratios that are somewhat below and above the $(L/D)_{\text{opt}} = 0.86 : 1$. As a result, a compensation of the imitation error

$$\delta = \frac{\sqrt{(\delta_y H_{y \text{ dipole max}})^2 + (\delta_z H_{z \text{ dipole max}})^2}}{H_{\text{coil}}} \times 100\% \quad (8)$$

appears possible for a couple of layers one of which has aspect ratio below and above the $(L/D)_{\text{opt}}$.

We have found that the above idea of compensation works quite well for a simple cylindrical coil—a coil with a rectangular cross section of the winding [see Fig. 4(b)]—provided that its L/D ratio is chosen properly (see Table 1).

It is interesting now to compare the optimal single- and multilayer coils. We have done it in Fig. 6 for the multilayer coils whose inner diameters are half of their outer diameters. As seen from Fig. 6, the multilayer cylindrical coils better

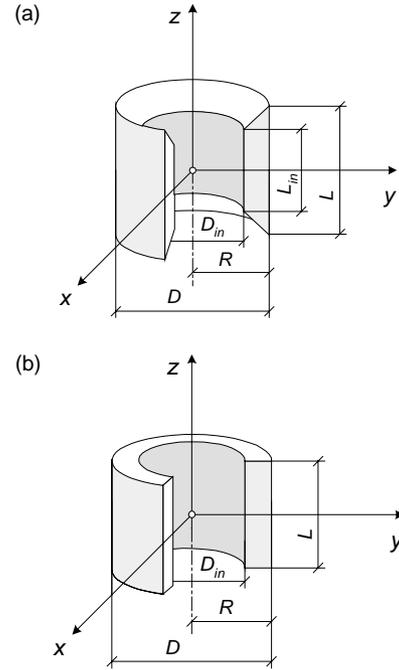


Fig. 4. Optimal multilayer cylindrical induction coils: (a) with variable layer lengths, L_j , and a constant, optimal aspect ratio, $L_j/D_j = (L/D)_{\text{opt}}$ (a coil with a trapezoidal cross section of the winding); and (b) with constant layer length, $L_j = L$, and a variable aspect ratio, L_j/D_j (a simple multilayer coil with a rectangular cross section of the winding).

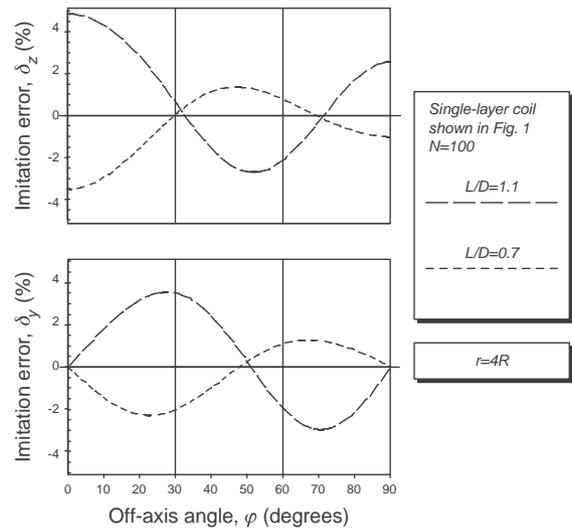


Fig. 5. Imitation error components as a function of the off-axis angle, φ , and different values of the aspect ratio, L/D .

Table 1
Optimized multilayer cylindrical coil shown in Fig. 4(b)

D_{in}/D	$(L/D)_{\text{opt}}$	$\delta_{\text{max}} (\%)^a$
0.50	0.72	0.28
0.60	0.74	0.31
0.70	0.76	0.40
0.80	0.79	0.48
0.90	0.82	0.59

^a $r = 4D$.

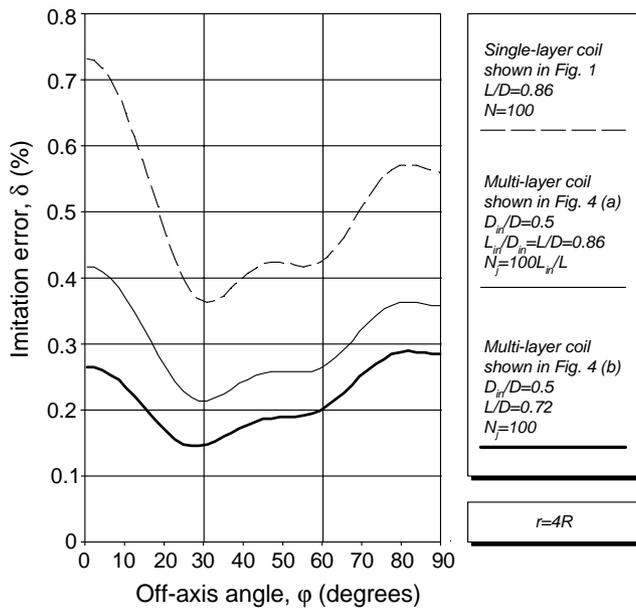


Fig. 6. Comparison between the optimized single-layer induction coil and two different multilayer cylindrical coils shown in Fig. 4(a) and (b).

imitate the ideal dipole compared to a single-layer coil of the same outer diameter. The maximum imitation error is reduced from 0.73 down to 0.42% for the coil type shown in Fig. 4(a) and down to 0.28% for the coil type shown in Fig. 4(b).

Finally, we investigate in Fig. 7 the behavior of the imitation error with increasing the distance from the multilayer coil of Fig. 4(b). This figure shows that the imitation error is decreasing by an order of magnitude when the distance from the coil is increasing by a factor of two: from $r = 4R$

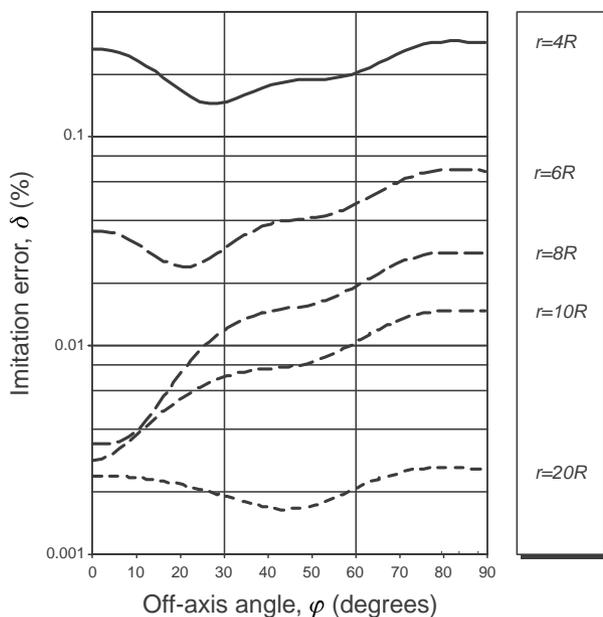


Fig. 7. Imitation error as a function of the distance from the multilayer coil with $D_{in}/D = 0.5$ shown in Fig. 4(b).

to $8R$. (It is interesting to note from Fig. 7 that for small values of the off-axis angle, $\varphi < 30^\circ$, the imitation error decrease is even more significant). At distances $r = 10R$, the imitation error is below 0.015% for any φ and at distances $r = 20R$ the imitation error is below 0.0027%.

4. Conclusion

Our conclusion is that a better imitation accuracy and simpler design make a multilayer cylindrical coil with a rectangular cross section of the winding (see Fig. 4(b)) the best choice to imitate the ideal dipole field. The optimum coil dimensions can be found in Table 1. The employment of such optimized coil allows one to reduce the imitation error relative to a single-turn coil by a factor of ~ 30 for a four radii distance from the coil's center. The remaining error (0.28%) can be neglected in many applications.

Acknowledgements

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Biographies

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