

Modeling of the magnetoelectric effect in finite-size three-layer laminates under closed-circuit conditions

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A theoretical model is presented for the low-frequency magnetoelectric (ME) effect in three-layered magnetostrictive-piezoelectric laminates. The model considers both the laminate finite size and the detection circuitry loading (closed-circuit conditions). The model development is based on a system of electroelasticity and magnetoelasticity equations and takes into account the boundary conditions at the inner and outer surfaces of the laminate. An averaging method is used to estimate the effective parameters of the laminate materials. The ME voltage coefficient for transverse fields is obtained theoretically. The obtained solution allows us to set up the laminate equivalent electrical circuits and to find their electrical parameters in terms of the physical properties of the laminate and its geometry. © 2010 American Institute of Physics. [doi:10.1063/1.3362925]

I. INTRODUCTION

Multiferroic materials have drawn increasing interest due to their potential for many modern devices, such as sensors, gyrators, energy harvesters, etc. The magnetoelectric (ME) effect in single-phase or composite multiferroics is defined as the induced polarization response to an applied magnetic field. The ME effect in multiferroic laminates is found to be several orders of magnitude stronger than that in single-phase multiferroics (see review¹ and references therein). In this work, we develop a theoretical model describing the ME interactions in three-layer magnetostrictive-piezoelectric laminates. In contrast to previously reported studies,¹⁻⁴ we consider the finite size of the laminates. We also assume that in practical cases the structure is under closed-circuit conditions.

We consider a three-layered disk of radius R and thickness t . The top and bottom surfaces of the inner piezoelectric layer of relative, compared with t , thickness t_p are assumed to be ideally bonded to the piezomagnetic layers of relative thickness $t_m/2$. Such a system can easily be synthesized by bonding the slices of piezoelectric ceramics [e.g., BaTiO₃ and lead zirconate titanate (PZT)] and Ni.⁵ For the sake of simplicity, we consider a poled state in both the piezoelectric and magnetostrictive phases with the poling directed along the out-of-sample plane orthogonal z axis. The applied ac magnetic field, $\delta\mathbf{H}$, is also assumed directed along z axis. As a result, the induced by $\delta\mathbf{H}$ polarization, $\delta\mathbf{P}$, also has the same direction, so-called longitudinal ME operation mode.¹

II. MAGNETOELECTRIC COUPLING IN UNCLAMPED THREE-LAYERS

In the low-frequency operation mode, the electric and magnetic fields, as well as the mechanical stresses and strains in the magnetostrictive-piezoelectric laminates have to be found by solving a joint set of magneto-electrostatic Maxwell equations (1) and the equations of elastic equilibrium with the compatibility conditions (2), simultaneously

$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \times \mathbf{E} = 0, \quad \nabla \times \mathbf{H} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad (1)$$

where \mathbf{D} is the electric displacement, ρ is the free electric charge density, \mathbf{E} is the electric field strength, \mathbf{B} is the magnetic induction, and \mathbf{H} is the magnetic field strength

$$\frac{\partial \sigma_{ik}^{p,m}}{\partial x_k} = 0, \quad \Delta u_{ik}^{p,m} + \frac{\partial^2 u_{ll}^{p,m}}{\partial x_i \partial x_k} - \frac{\partial^2 u_{il}^{p,m}}{\partial x_k \partial x_l} - \frac{\partial^2 u_{kl}^{p,m}}{\partial x_i \partial x_l} = 0, \quad (2)$$

where $\sigma_{ik}^{p,m}$ and $u_{ik}^{p,m}$ are the mechanical stress and strain tensor components in either piezoelectric or magnetostrictive phases, the indexes p and m stand for the piezoelectric and magnetostrictive materials, respectively, while i and k , equal to 1, 2, and 3, denote spatial coordinates x, y, z . The summation convention applies to the suffixes occurring twice in the vector and tensor expressions.

Assuming that both the piezoelectric and magnetostrictive layers are sufficiently thin, i.e., $t \ll R$, the deformation may be regarded as uniform over their thickness. The strain tensors are then the functions of r and ϑ (the $r-\vartheta$ plane being that of the laminate) and are independent of z . The boundary conditions on both the top and bottom surfaces of the laminate are $\sigma_{ik}^m n_k = 0$, or, since the normal vector is parallel (or anti-parallel) to z axis, $\sigma_{rz}^m = \sigma_{\vartheta z}^m = \sigma_{zz}^m = 0$. Due to the boundary conditions on both the surfaces between the mag-

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netostrictive and piezoelectric layers $\sigma_{ik}^m n_k^m = \sigma_{ik}^p n_k^p$, the quantities σ_{iz}^p must also be small throughout the thickness of the piezoelectric layer, and we can therefore take them as approximately zero everywhere in the laminate (a state of plane stress). Furthermore, assuming the effective compliances for both the piezoelectric and piezomagnetic layers to be isotropic in the laminate plane, the strain tensor and the stress tensor components in both the piezoelectric and piezomagnetic layers are related to the electric and magnetic field strengths, E_z and H_z , in the corresponding layers via so-called coupling equations

$$\begin{aligned} u_{rr}^p &= \frac{1}{K_p}(\sigma_{rr}^p - \nu_p \sigma_{\vartheta\vartheta}^p) + d \delta E_z, \\ u_{rr}^m &= \frac{1}{K_m}(\sigma_{rr}^m - \nu_m \sigma_{\vartheta\vartheta}^m) + q \delta H_z, \\ u_{\vartheta\vartheta}^p &= \frac{1}{K_p}(\sigma_{\vartheta\vartheta}^p - \nu_p \sigma_{rr}^p) + d \delta E_z, \\ u_{\vartheta\vartheta}^m &= \frac{1}{K_m}(\sigma_{\vartheta\vartheta}^m - \nu_m \sigma_{rr}^m) + q \delta H_z. \end{aligned} \quad (3)$$

where, $K_{p,m}$ is Young's modulus, $\nu_{p,m}$ is Poisson's ratio, and d and q are the components of the converse piezoelectric and piezomagnetic coefficients tensor. Using a strict condition of $\sigma_{zz}=0$, we thus eliminate the displacement u_z ; one can then regard the laminate as a two-dimensional medium (an "elastic plane") and reduce the three-dimensional displacement vector \mathbf{u} to a two-dimensional vector with the only one, due to the axial symmetry of the problem, component u_r . For the conditions of a plane stress problem, a stress function χ satisfying the biharmonic equation can be introduced.⁶ To see it, one should notice that for the plane stress conditions, Eq. (2) can be written as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\vartheta\vartheta}}{r} = 0, \quad \frac{\partial u_{rr}}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_{\vartheta\vartheta}}{\partial r} \right) = 0. \quad (4)$$

Introducing the stress function defined by

$$\sigma_{rr} = \frac{1}{r} \frac{\partial \chi}{\partial r}, \quad \sigma_{\vartheta\vartheta} = \frac{\partial^2 \chi}{\partial r^2},$$

one finds that χ satisfies the biharmonic equation $\Delta \Delta \chi = 0$, $\chi = Cr^2$, and $\sigma_{rr} = \sigma_{\vartheta\vartheta} = 2C$. To satisfy the boundary condition of $\sigma_{rr} = 0$ at $r=R$, one should consider the laminate as effective medium with averaging stress tensor components,⁷ which yields

$$\sigma_{rr}^m t_m + \sigma_{rr}^p t_p = 0, \quad \sigma_{\vartheta\vartheta}^m t_m + \sigma_{\vartheta\vartheta}^p t_p = 0. \quad (5)$$

Furthermore, because of the boundary conditions at the interface surfaces between the magnetostrictive and piezoelectric layers the displacements in both the phases have to be equated (perfect bonding). This gives the two following equations:

$$\frac{1}{K_m}(\sigma_{rr}^m - \nu_m \sigma_{\vartheta\vartheta}^m) + q \delta H_z = \frac{1}{K_p}(\sigma_{rr}^p - \nu_p \sigma_{\vartheta\vartheta}^p) + d \delta E_z, \quad (6)$$

$$\frac{1}{K_m}(\sigma_{\vartheta\vartheta}^m - \nu_m \sigma_{rr}^m) + q \delta H_z = \frac{1}{K_p}(\sigma_{\vartheta\vartheta}^p - \nu_p \sigma_{rr}^p) + d \delta E_z.$$

Assuming that the charges present only at the metallic (Ni) piezomagnetic layer surfaces and applying standard procedure to the first equation in Eq. (1), we obtain

$$d(\sigma_{rr}^p + \sigma_{\vartheta\vartheta}^p) + k \delta E_z = \frac{Q}{\pi R^2}, \quad (7)$$

where Q is the total charge of the magnetostrictive layer (for open-circuit conditions $Q=0$) and k is the permittivity. To take into account the losses, we define the complex permittivity as $k = k' + j(k'' + \sigma/\omega)$, where k'' describes the losses associated with the dielectric polarization, and σ describes the losses associated with the current flow. Solving Eqs. (5)–(7) with respect to δE_z , and integrating the solution over z from the bottom to the top of the piezoelectric disk and then differentiating it in time, we obtain the Fourier transform of the ME voltage across the laminate

$$V_{\text{ME}} = \alpha_{\text{ME33}} \delta H_z - I_{\text{ME}} Z_{\text{eff}} = V_m - I_{\text{ME}} Z_{\text{eff}}, \quad (8)$$

where α_{ME33} is the longitudinal ME voltage coefficient,¹ I_{ME} is the electric current flowing through the laminate, V_m is the voltage induced by the external magnetic field, and Z_{eff} is the effective impedance of the laminate. The longitudinal ME voltage coefficient and the effective impedance can be given as

$$\begin{aligned} \alpha_{\text{ME33}} &= \frac{2dq t_p (1-t_p) t}{\left[\frac{1-\nu_p}{K_p} (1-t_p) + \frac{1-\nu_m}{K_m} t_p \right] k - 2d^2 (1-t_p)}, \quad (9) \\ Z_{\text{eff}} &= \frac{t_p}{j\omega k \pi R^2} \left[1 - \frac{1}{k} \frac{2d^2 (1-t_p)}{\frac{1-\nu_m}{K_m} t_p + \frac{1-\nu_p}{K_p} (1-t_p)} \right]^{-1} \\ &= R_{\text{eff}} + \frac{1}{j\omega C_{\text{eff}}}, \quad (10) \end{aligned}$$

where R_{eff} represents the active part of impedance Z_{eff} , and C_{eff} represents its reactive part. The very structure of Eq. (8) reflects the fact that, in the ME laminate, the dielectric polarization of the piezoelectric phase is induced by both the induced stresses and electric field. Considering Eq. (8), a Thévenin equivalent circuit of the laminate loaded by the input impedance of a preamplifier, Z_{in} , can be drawn (see Fig. 1). I_{ME} in this circuit should be equal to $V_m / (Z_{\text{eff}} + Z_{\text{in}})$. As a result, Eq. (8) can be rewritten as follows:

$$V_{\text{ME}} = \alpha_{\text{ME33}} \delta H_z \left(1 - \frac{Z_{\text{eff}}}{Z_{\text{in}} + Z_{\text{eff}}} \right), \quad (11)$$

which is a theoretical model of the voltage across the laminate as a function of the applied magnetic field, physical properties and dimensions of the laminate, and the input impedance of the preamplifier.

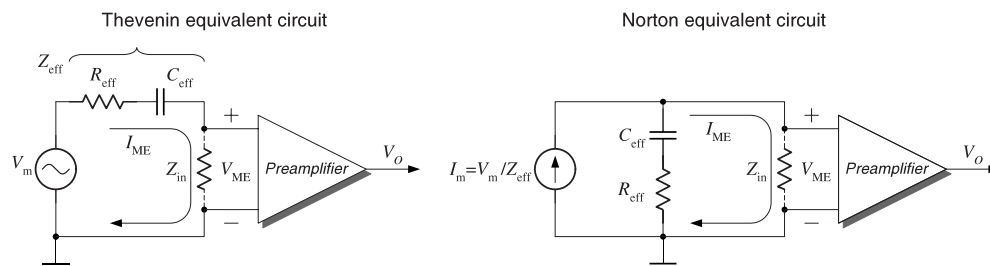


FIG. 1. Equivalent electrical circuits of the magnetostrictive-piezoelectric laminate loaded by the input impedance, Z_{in} , of the preamplifier. (V_O is the output voltage of the preamplifier.)

The field, δH_z , inside the piezomagnetic layers can be related to the external magnetic field by using existing approximations for thin ferromagnetic disks or oblate spheroid.⁸ It should be noted that, in contrast to Ref. 9, we obtain the laminate impedance Z_{eff} directly in terms of physical properties, $K_{m,p}$, $\nu_{m,p}$, d , q , k , and losses ($\tan \delta$) of the magnetostrictive and piezoelectric layers and their geometry.

III. CONCLUSIONS

Simplified theoretical models for magnetostrictive-piezoelectric laminates have been obtained. This models considers the finite size of the laminates and the closed-circuit conditions. The models include both the analytical description of the laminates and the equivalent electrical circuits. The equivalent circuits and their electrical parameters have been derived from Eq. (8), which is obtained by solving Eqs. (1) and (2), based on the first-principles theory, under appropriate boundary conditions. To gain an insight into the ME effect in a finite size laminate, we have considered a thin disk thus adopting the plane stress state approximation,⁶ where the stresses depend only on r . To further simplify the task, we have assumed that the stresses, strains, and electric field are uniform. After that, the above discussed strict conditions at $r=R$ could hardly be fulfilled, so we have to average the stresses, see Eq. (5). Though this approximation is rough, the

solution, Eq. (8), has clear physical sense. Moreover, Z_{eff} given by Eq. (10) allows one to compare the theoretical predictions against measurements.

It is important to note that the suggested approach allows one to further correct Eqs. (9) and (10), by considering different laminate shapes, and properly including elastic loading conditions (unclamped, clamped samples) as well as to explore the dependence of the ME voltage on the interface bonding imperfections. The suggested theoretical models can be very helpful and instructive for designing various types of ME devices.

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