1. Elementary Electronic Circuits with a Diode

Nomenclature

	Mathematics	Electronic Circuits
Const.	X	V_D - *static* signal (dc)
Var.	x	v_D - *large* signal
Var. → 0	dx	v_d - *small* signal (dc or ac)

Note that $v_d \equiv \mathrm{d}v_D$.

Modeling Flow in the Electronic Circuits Analysis

Models: *physical*
analytical
graphical
electrical (equivalent circuit)
functional (block diagram)

Approaches to Solving Electronic Circuits

Approaches: *electrical* (KVL and KCL methods)
finding canonical subcircuits (voltage divider, current divider, Thévenin and Norton equivalents, Miller equivalents, etc.)
functional (solving block diagrams)

The Aim of the Course

Lumped linear time-invariant (LLTI) electric circuits do not provide any solution to the following five tasks (see Fig. 1), which are very important in Electrical Engineering:

1) It is impossible to control the circuit transfer function (*gain*) by an electrical signal, either voltage or current.
2) It is impossible to implement a circuit with a dc gain greater than one.
3) It is impossible to implement a circuit with a power gain greater than one.
4) It is impossible to implement a current source.
5) It is impossible to implement an oscillator (circuit generating a periodic signal), for example, a sine-wave oscillator.

The aim of the course is to solve all the above tasks by using electronic devices: diodes and transistors. To develop and study electronic circuits, we start from elementary circuits, analyze them, and then improve if there is a need.

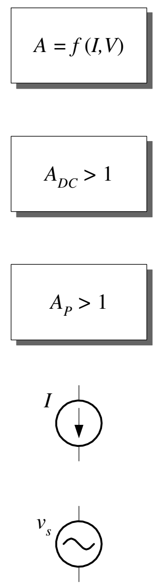

Fig. 1. Circuits that *cannot* be implemented by using LLTI components only: resistors, capacitors, and inductances.

1. Elementary Electronic Circuits with a Diode

Our main aim here is to build a circuit with a gain (not necessarily greater than one, note that the word "gain" in Electronic Circuits is used instead of "transfer function" and as a transfer function can have any value) that can be controlled by an electrical signal, either voltage or current. Namely, we would like to build a voltage-controlled voltage divider and a current-controlled current divider (homework).

To reach this goal, we develop physical, analytical, and graphical models of the diode. Based on the graphical model, we find equivalent *electric* circuits to replace a diode in an *electronic* circuit. This allows us to analyze single-diode electronic circuit by applying *electric* circuit theory. We also develop functional models of diode circuits to solve them for different *nonideal* input sources and different loading conditions.

1.1. *Diode: symbol, physical structure, analytical model and graphical characteristic*

The symbol of the diode and its physical structure are given in Fig. 2. To develop an analytical model of the diode we have to describe the dependence of the diode current, i_D, on the diode voltage, v_D.

Assuming that the n region is much more heavily doped that the p region, $n_{po} >> p_{no}$, we neglect the diode current due to the holes and consider only that due to the electrons. Neglecting the small effect of the weak electric field within the p region on the electrons, which are the minor charge carriers in this region, we conclude that the diode current is exclusively due to the diffusion current of electrons:

$$|i_D| = j_D A = D_n |q| \frac{\overbrace{n_p(0) - n_{po}}^{*}}{L_n} A$$

$$= D_n |q| \frac{\overbrace{n_{po} e^{v_D/V_T} - n_{po}}^{*}}{L_n} A \qquad , \qquad (1)$$

$$\underbrace{\frac{A\, D_n |q|\, n_{po}}{L_n}}_{\equiv I_{DS}} (e^{v_D/V_T} - 1) = I_{DS} (e^{v_D/V_T} - 1)$$

where j_D is the diode current density, A is the diode cross-section area, $n_p(0)$ is the concentration of the electrons in the p region at x=0 (see Fig. 2), n_{po} is the thermal-equilibrium concentration of the electrons in the p region (when the diode terminals are *open* circuited), D_n is the diffusion constant of the electrons, q is the charge of the electron, L_n is the diffusion length for the electrons, V_T is the thermal voltage, and I_{DS} is the saturation current of the diode.

It is worth noting, that at room temperature

$$V_T = \left.\frac{kT}{q}\right|_{T=300^\circ K} \approx 26\,\text{mV} , \qquad (2)$$

where k is the Boltzmann constant, and T is the absolute temperature.

The typical value of I_{DS} is 10 fA. Due to the strong dependence of n_{po} on temperature, the I_{DS} current doubles its value per a 5° increase of the diode temperature.

Based on (1), we draw in Fig. 3 the diode i_D-v_D characteristic.

1.2. *Static and dynamic impedances*

Note (see Fig. 4) that the characteristic of the diode is *nonlinear* whereas that of a resistor is *linear*. As a result, a diode translates (amplifies) *differently* static, I_D and V_D, and incremental (dynamic), dv_D≡v_d and di_D≡i_d, signals:

$$G_D \equiv \frac{I_D}{V_D} \neq \text{(in general)}\ g_d \equiv \frac{i_d}{v_d} . \qquad (3)$$

It is obvious that for a resistor

$$G_R \equiv \frac{I_R}{V_R} = g_r \equiv \frac{i_r}{v_r} . \qquad (4)$$

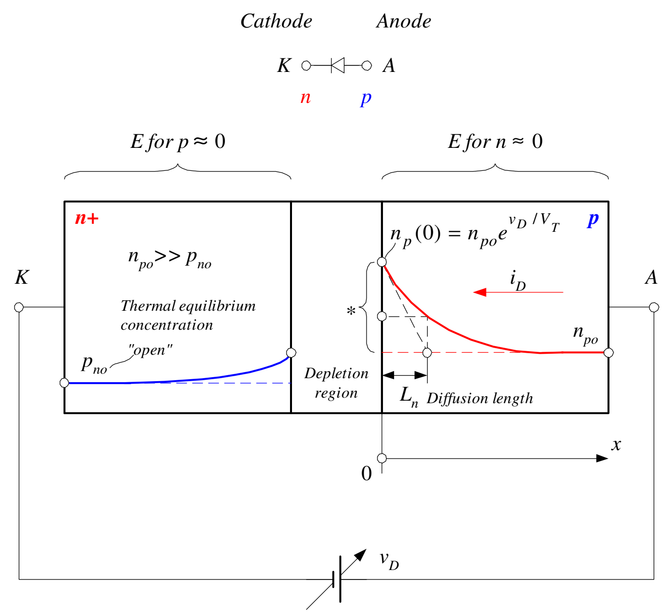

Fig. 2. Symbol of the diode and its physical structure. * see equation (1).

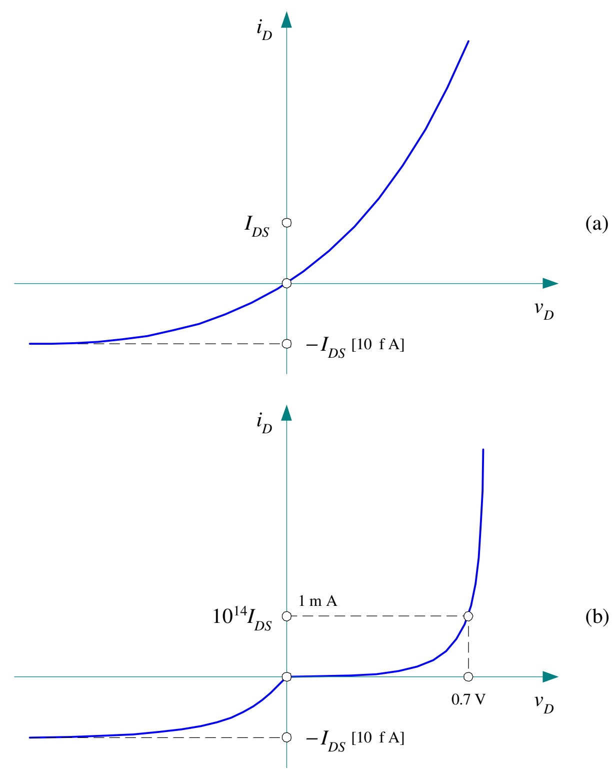

Fig. 3. The i_D-v_D characteristic of the diode: (a) the positive and negative parts of the i_D axis have the same scale, (b) the scales are different.

Let us denote the infinitely small incremental signals as small signals. In electronic circuits, static signals usually define *operating points* of electronic devices to provide a required translation (gain) for small signals. Static signals are defined by the designer.

The origin of small signals is usually external. They enter the circuit through either an antenna or sensor; they can also be generated by testing instruments (function generators, etc.), or by other electronic circuits. Thus, many electronic circuits are dedicated to the processing of small signals.

Note that static signals are always (at least in our course) dc and small signals can be either dc or ac (a dc signal can also have an infinitely small magnitude).

The small-signal conductance and resistance of a diode can be found as follows

$$g_d \equiv \frac{i_d}{v_d} \equiv \left.\frac{\mathrm{d}i_D}{\mathrm{d}v_D}\right|_Q$$

$$= \left.\frac{\mathrm{d}I_{DS}(e^{v_D/V_T}-1)}{\mathrm{d}v_D}\right|_Q = \left.\frac{I_{DS}e^{v_D/V_T}}{V_T}\right|_Q \quad . \tag{5}$$

$$= \left.\frac{I_{DS}e^{v_D/V_T} - I_{DS} + I_{DS}}{V_T}\right|_Q = \frac{I_D + I_{DS}}{V_T}$$

$$r_d \equiv \frac{1}{g_d} = \left.\frac{V_T}{I_D + I_{DS}}\right|_{I_D = 1\,\text{mA},\ 300^\circ\,\text{K}} \approx 26\,\Omega \,. \tag{6}$$

Note that the small-signal (or dynamic) conductance and resistance, r_d, are a function of the diode operating point, namely, a function of the static diode current, I_D.

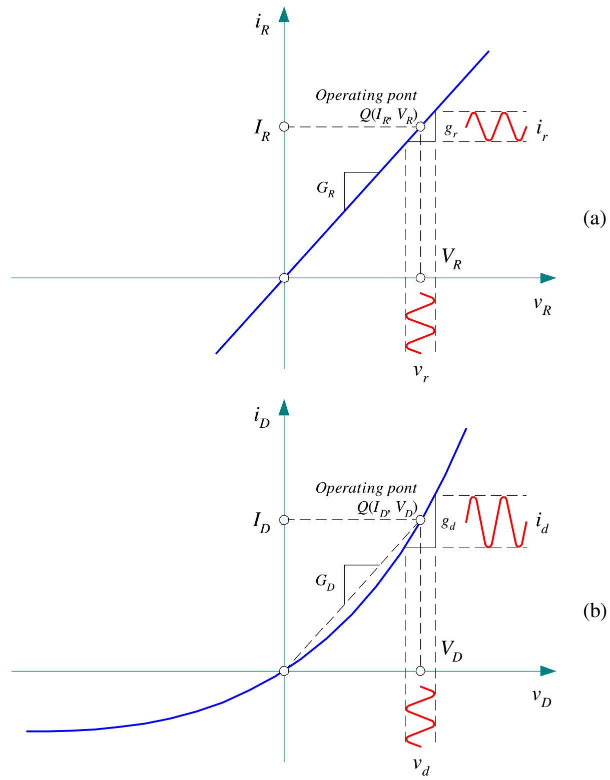

Fig. 4. Static and dynamic gains for (a) a resistor and (b) a diode. Note that in (a) *G*=*g*, whereas in *G*≠*g* in (b).

$$\begin{cases} v_O = v_D, \;\; i_O = i_D \\ i_D = I_{DS}(e^{v_D/V_T} - 1) \\ i_D = i_R = \dfrac{(V_{AA} + v_s) - v_D}{R_A} \\ \qquad = -\dfrac{1}{R_A}[v_D - (V_{AA} + v_s)] \end{cases} \quad . \tag{7}$$

1.3. *Voltage-controlled voltage divider*

To build a voltage-controlled voltage divider that should attenuate small-signals according to a static voltage, we utilize the above dependence of r_d on I_D and connect a diode and resistor R_A [see Fig. 5(a)] similarly to a resistive voltage divider, where one of the resistors is replaced by a diode. We then connect in series to the resistor and diode a static voltage source, V_{AA}, to define (control) – together with R_A – the diode operating point, and a small-signal voltage source v_s.

To find the circuit voltage gain $A_v \equiv v_o/v_s$ as a function of V_{AA}, we use the following system of equations:

The system of equations (7) is a nonlinear one and does not necessarily have an analytical solution. However we can easily find its graphical solution [see Fig. 5(b)], which illustrates very well the signal translation by the circuit.

Note in Fig. 5(b) that the small signal is translated only by the infinitely small part of the diode characteristic about the operating point. It is also very important to note that the tangent line to the diode characteristic, drawn through the operating point, translates the small signal in the *exactly* same way. Therefore, we can substitute the diode in Fig. 5(a) with an equivalent circuit [see Fig. 5(c)] having the characteristic that is identical to the tangent line in Fig. 5(b).

As a result, the circuits in Figs. 5(b) and (d) are equal in terms of the small-signal gain. We will call the circuit in Fig. 5(d) "large" signal equivalent circuit. The quotes mean that such a circuit translates exactly only static and dynamic signals. (A real large-signal equivalent circuit should exactly translate any signals, for example, non-infinitely-small signals.)

An important advantage of the circuit in Fig. 5(d) is that it is a *linear* one and, therefore, can be solved by applying superposition. Since we are interested in finding small-signal gains only, we suppress all the static signal sources [see the dashed lines in Fig. 5(d) that short-circuit the static signal sources]. This gives us equivalent *small-signal* circuit in Fig. 5(e). This circuit can easily be solved by applying elementary electric circuit theory:

$$A_v \equiv \frac{v_O}{v_s} = \frac{r_d}{r_d + R_A} = \begin{cases} 1, & V_{AA} \to -\infty \\ 0, & V_{AA} \to +\infty \end{cases}. \quad (8)$$

$$A_i \equiv \frac{i_O}{i_s} = 1. \quad (9)$$

$$A_p = \frac{v_O}{v_s}\frac{i_O}{i_s} = A_v A_i < 1. \quad (10)$$

where A_i is the small-signal current gain, and A_p is the small-signal power gain.

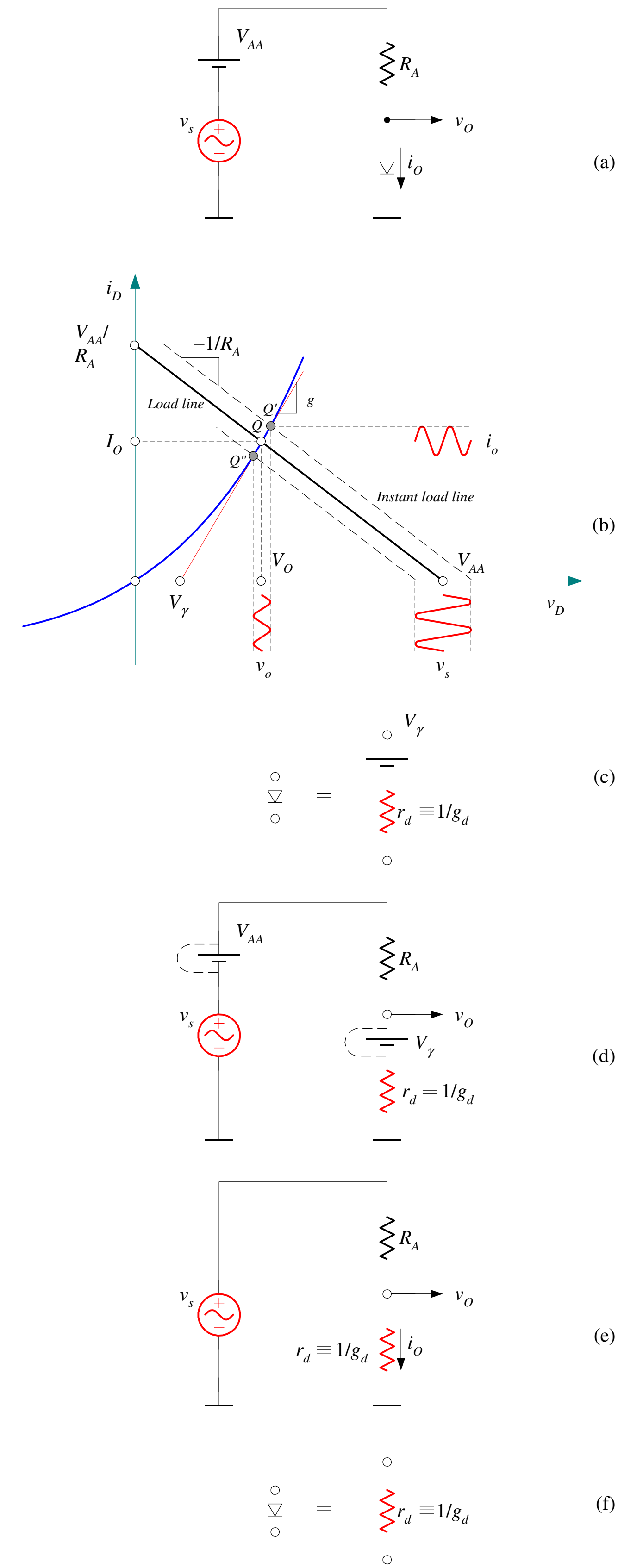

Fig. 5. (a) Voltage-controlled voltage divider, (b) graphical solution, (c) "large" signal equivalent circuit (model) for the diode, (d) "large" signal equivalent circuit, (e) small-signal equivalent circuit, (f) small-signal equivalent circuit (model) of the diode.

Small-signal input and output impedances

Functional models (block diagrams) simplifies circuit analysis: a circuit is divided in blocks, and then the blocks' gains are found and combined into the gain of the whole circuit.

Let us assume, for example, that a nonideal input source, v_s and its internal resistance r_s, is connected to the voltage-controlled voltage divider [see Fig. 6(a)] analyzed in the previous section for an ideal input source.

Since the new circuit in Fig. 6(a) differs from the previous one in Fig. 5(e) due to adding the input source resistance, r_s, its gain will also have a different value. Let us denote the gain of the previous circuit as A_v' and the gain of the new circuit as A_v.

To find A_v, we can solve the new circuit once again, but it is time consuming to solve a new circuit each time a component is added or removed. Instead, it is worth using the previous solution as a part of the new one. Note in Fig. 6(a) that $A_v=GA_v'$, where G is the input gain from the input source v_s to the input of the previous circuit. Hence, we can use already found A_v' and have to only find G to find A_v.

Finding G alone is much easier than solving the whole new circuit. It is so because the circuit connected to the nonideal input source can be replaced with its equivalent input impedance R_{in} [see Fig. 6(b)], provided, of course, that this circuit comprises no independent sources, then G can very easily be found [see Fig. 6(b)].

Another example, where the previous solution can be used as a part of the new one is given in Fig. 6(c). In this figure the voltage-controlled voltage divider [see Fig. 6(a)] is loaded by an additional resistor R_L. The linear circuit seen by the load R_L can be replaced with its Thévenin equivalent: v_sA_v' and R_o [see Fig. 6(d)]. The gain of the loaded circuit in Fig. 6(d) can be found as $A_v'=A_v'Q$, where the output gain Q can very easily be found [see Fig. 6(d)] as the gain from the Thévenin equivalent source v_sA_v' to the output v_o.

Note that knowing A_v', R_{in}, and R_o allows us to find A_v for any nonideal input source or any load, provided we know r_s and R_L. Or in other words, once A_v', R_{in}, and R_o are given, we do not need to know the circuit topology to find A_v. This is why manufacturers list in their data sheets A_v', R_{in}, and R_o.

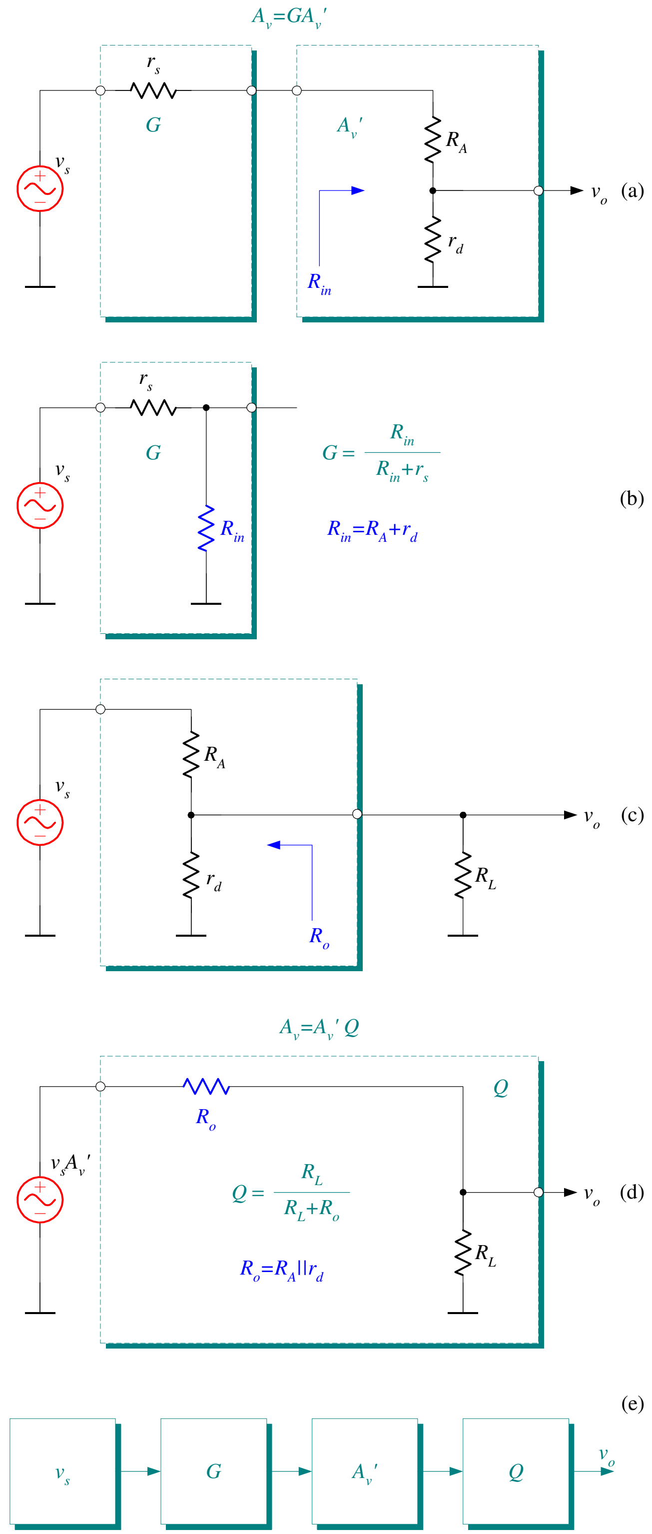

Fig. 6. Small-signal input and output impedances: (a) connecting a nonideal input source to the circuit given in Fig. 5(e), (b) finding the input impedance R_{in} and the input gain G, (c) loading the circuit given in Fig. 5(e), (d) finding the output impedance R_o and the output gain Q, (e) functional model.

APPENDIX

Note (see Fig. A1) that the operating point in a graphical solution for the small-signal equivalent circuit of Fig. 5(e) is shifted to the origin of the coordinate system. This is because we suppress in the small-signal equivalent circuit all the sources responsible for the static conditions, namely, V_{AA} and V_γ.

HOME EXERCISE

Find R_{in}, R_o, G, and Q for the circuit in Fig. 5(e) that is connected to a nonideal input source with output resistance r_s and also loaded with a load R_L.

Find operating points, with the help of a graphical solution, for the two circuits in Fig. H1. Note that, $I_{DS1} \neq I_{DS2}$.

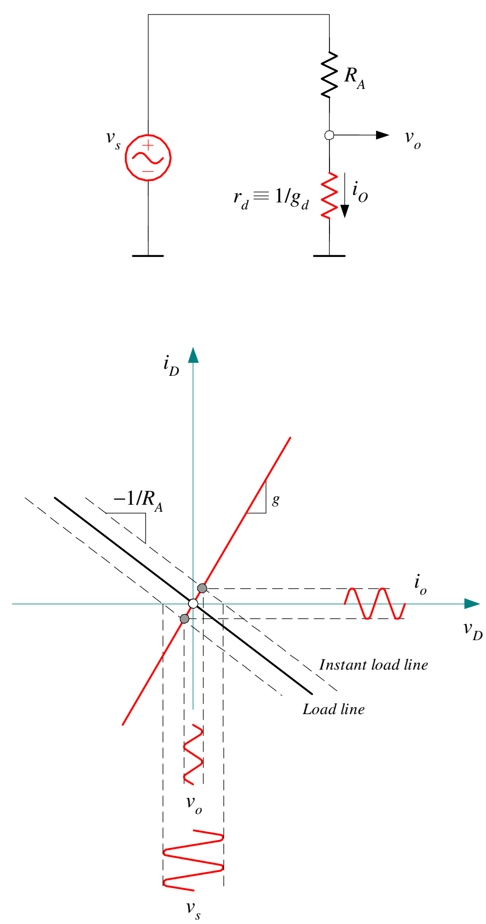

Fig. A1. The small-signal equivalent circuit of Fig. 5(e) and its graphical solution.

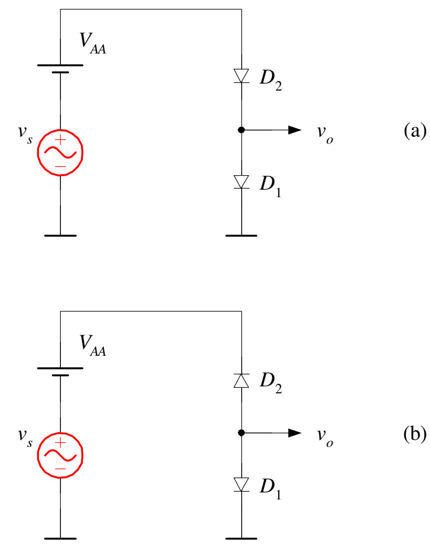

Fig. H1. Circuits for the home exercise.

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[2] A. S. Sedra and K. C.Smith, Microelectronic circuits.