7. Positive-Feedback Oscillators (continued)

7.3. *Oscillators for high frequencies: LC oscillators:*

Our aim is to develop oscillators with high frequency stability at high frequencies. We have to find a different approach because, as will be shown in the next lecture, the small-signal voltage gain decreases with frequency, and, therefore, the frequency stability of the Wien-bridge oscillator would be low at high frequencies.

Our approach will be based on employing parallel *RLC* resonant circuits as feedback networks. (At high frequencies inductances are small and inexpensive.) The phase response of a resonant circuit is real at the frequency of resonance, and the phase condition of the Barkhausen criterion will hold true; hence, we will set the oscillator frequency at the resonant frequency of its feedback network: $\omega_1=\omega_0$.

We saw in the previous lecture that to obtain high frequency stability the slope of the phase response of the feedback network, or its equivalent quality factor, should be high. By definition, equivalent quality factor, Q_{equiv}, of a parallel resonant circuit equals its intrinsic quality factor: $Q_{\text{equiv}}=Q_{RLC}=R/(\omega_0 L)$. Hence, to keep Q as high as possible for a given L and ω_0 we have to keep R as high as possible.

Before starting the development of an oscillator, let us first see what the physical meaning of R is in a parallel resonant circuit. Fig. 1 shows that for a given ω, for example, ω_0, R is inversely proportional to the resistance R_s of the inductance wire: the lower R_s: the higher R, the higher Q_{RLC}.

7.3.1. *Hartley and Colpitts oscillators*

According to the positive-feedback approach, we have to connect an amplifier to the *RLC* feedback network and to close the feedback loop to obtain an *LC* oscillator. In order not to reduce the quality factor of the *RLC* network, we have to choose only amplifiers with high output impedance. Otherwise, the output impedance of the amplifiers will shunt R in Fig. 1 and reduce Q_{RLC}. Hence, we choose either CE or CB amplifiers. (Note that it is also possible to connect an amplifier in series with a series resonant circuit. In this case, the output resistance of the amplifier should be much lower than the series resistance of the inductor to not to reduce the quality factor of the resonant circuit. A CC amplifier has the lowest output resistance, about 26 Ω at I_E=1 mA. However, compared to a lower than 1 Ω series resistance of the inductor, it is still too high. Of course, it will be low enough at I_E=100 mA, but it is too much of power consumption.)

In a general case, the small-signal voltage gains of these amplifiers are greater than one. Therefore the transmission of the *RLC* feedback network at the resonant frequency should be less than unity to satisfy the amplitude condition of the Barkhausen criterion. This transmission should also be
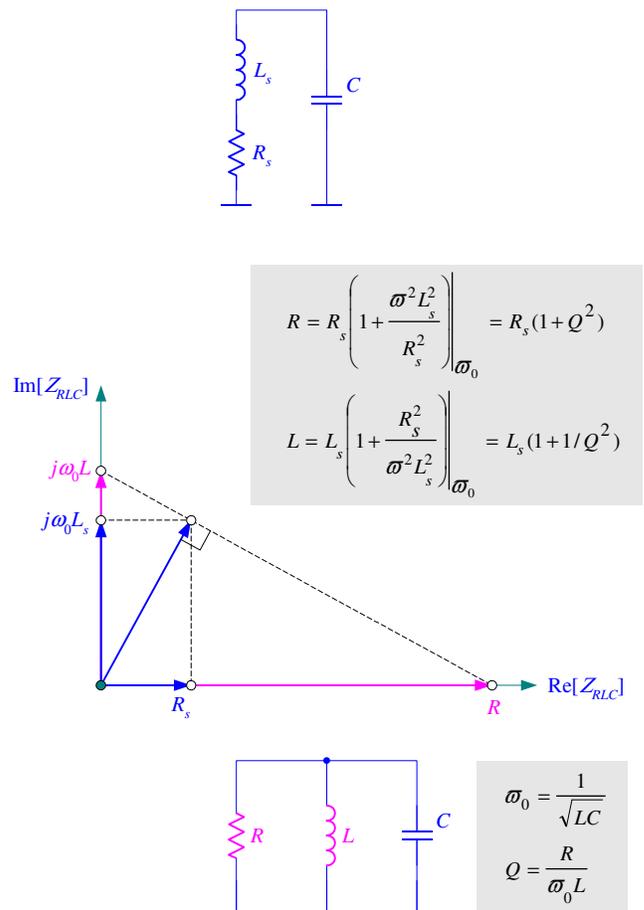

$$R = R_s \left(1 + \left. \frac{\varpi^2 L_s^2}{R_s^2} \right|_{\varpi_0} \right) = R_s(1+Q^2)$$

$$L = L_s \left(1 + \left. \frac{R_s^2}{\varpi^2 L_s^2} \right|_{\varpi_0} \right) = L_s(1+1/Q^2)$$

$$\varpi_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{R}{\varpi_0 L}$$

Fig. 1. A practical R_sL_sC circuit (above), a transformation of R_s and L_s into L and R (in the middle), and an electrical equivalent of the physical R_sL_sC circuit: a parallel *RLC* resonant circuit (below).

negative to satisfy the phase condition of the Barkhausen criterion for a CE amplifier and positive for a CB amplifier.

These requirements can be met if we divide the voltage at the output of the *RLC* network as shown in Fig. 2. Without the reactive voltage divider in Fig. 2(b), the feedback network transmission at $\omega_1=\omega_0$ would be equal to unity. This is so because the ideal resonant circuit (tank) has an infinite impedance at ω_0

$$Z_{LC} = \left. \frac{Z_L Z_C}{Z_L + Z_C} \right|_{\text{at } \varpi_0 \, Z_L=-Z_C} = \infty . \qquad (1)$$

Note also from Fig. 2 that the amplifier input impedance should be high in order not to shunt the feedback network.

Let us now find the Barkhausen conditions for the generic model [see Fig. 2(b)] for high-frequency oscillators.

$$A_{OL}\beta(j\varpi) \equiv +\frac{s''_\varepsilon}{s_\varepsilon} \equiv +\frac{v''_\varepsilon}{v_\varepsilon}\bigg|_{v_\varepsilon=1} = A_{OL}\frac{\dfrac{Z_1}{Z_1+Z_2+Z_3}Z_3}{R_o+\dfrac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}}$$

$$.\ (2)$$

$$= A_{OL}\frac{Z_1 Z_3}{R_o(Z_1+Z_2+Z_3)+Z_1(Z_2+Z_3)}$$

At $\omega_1=\omega_0$,

$$Z_1+Z_2+Z_3=0$$

for $Z_1=j\varpi_0 L_1$, $Z_2=\dfrac{1}{j\varpi_0 C_2}$, and $Z_3=\dfrac{1}{j\varpi_0 C_3}$. $\quad$ (3)

$$\varpi_0=\frac{1}{\sqrt{L_1\dfrac{C_2 C_3}{C_2+C_3}}}$$

the phase condition is met, and (2) sets the amplitude condition

$$A_{OL}\beta(j\varpi_1)=A_{OL}\frac{Z_1 Z_3}{Z_1(Z_2+Z_3)}=-A_{OL}\frac{Z_3}{Z_1}=1$$

$$.\ (4)$$

$$\Rightarrow A_{OL}=-\frac{Z_1}{Z_3}$$

One can see from (4) that the amplifier gain A_{OL} should be positive if the Z_1 and Z_3 impedances are of different types, and A_{OL} should be negative if the Z_1 and Z_3 impedances are of the same type.

To illustrate this intuitively recall that at ω_0 the tank has an infinite impedance, and therefore no current is flowing in R_o. Thus, the current flowing in the tank does not leave it: the same current flows through all the tank impedances. Note however that this current flows in the opposite directions through the Z_1 and Z_3 impedances. Therefore, if these impedances are of different types the voltage drops on them are of the same sign [see Fig. 2(b)] and the transmission of the tank is positive. Would Z_1 and Z_3 be of the same type, the voltage drops on them would be of different signs, and the transmission of the tank would be negative.

Note from (2) that for $R_o=0$, the amplitude condition can be satisfied for any frequency. Therefore, R_{oA} should not equal zero if we want—and we do want—our circuit to oscillate at a single frequency.

Will classify the oscillators with two *inductances* and two *capacitors* in the tank as the Hartley and Colpitts oscillators, correspondingly.
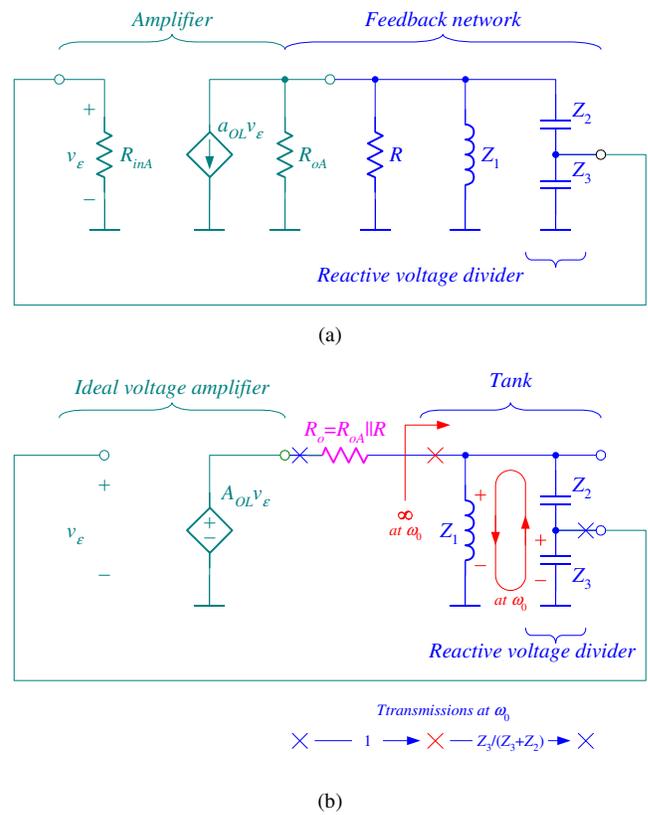

(a)

(b)

Fig. 2. Generic small-signal equivalent circuits for *LC* oscillators.

Example circuit

$$\varpi_1 = \varpi_0 = \sqrt{L_2 \frac{C_1 C_3}{C_1 + C_3}}$$

$$-g_m r_o = -\frac{\dfrac{1}{j\varpi_1 C_1}}{\dfrac{1}{j\varpi_1 C_3}} = -\frac{C_3}{C_1} \qquad \text{(E1)}$$

$$\Rightarrow \frac{C_3}{C_1} = \frac{I_C}{V_T} \frac{V_A + V_{CE}}{I_C} \underset{\substack{V_A = 100\,\text{V} \gg V_{CE} \\ V_T = 25\,\text{mV}}}{} = 4000$$
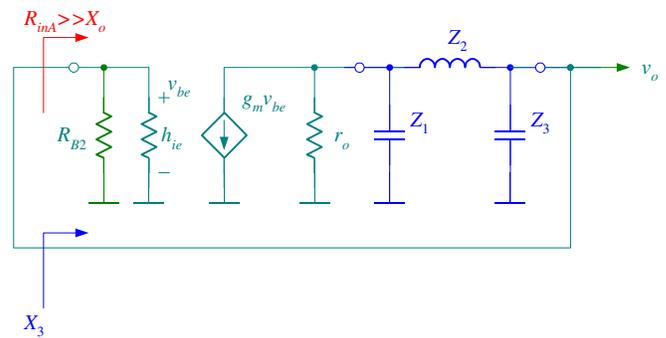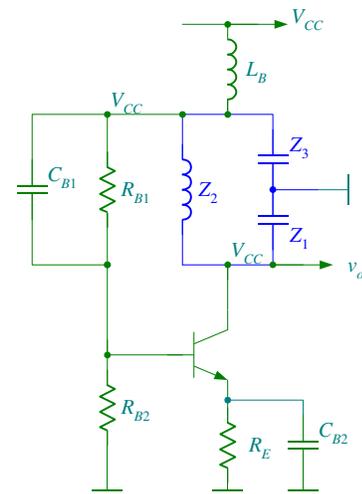
Fig. 3. Example circuit: a Colpitts oscillator.

7.3.2. *Quartz crystal oscillators*

Skin effect

The typical value of the quality factor of *RLC* resonant circuits at high frequencies is 100. It cannot be increased significantly either by increasing frequency or by increasing the diameter of the wire of the inductor because of the skin effect. At frequencies starting from about 100 kHz, the current distribution within the wire becomes inhomogeneous (see Fig. 4): the eddy currents decrease the current density at the wire center and increase it at the edges of the wire cross section. As a result, the *effective* cross-sectional area of the wire decreases with the square root of frequency and the wire resistance increases.

It can be shown that the quality factor of a parallel resonant circuit, $Q_{RLC}=R/(\omega_0 L)$, equals to that of the inductor $Q_L=\omega_0 L_s/R_s$ (see Fig. 1). Therefore, increasing ω_0 does not increase Q_L because R' increases with ω_0.

Having a limited Q_{RLC}, we have limited frequency stability. If we want to improve it, we have to do something with the inductor. The effective series resistance (ESR) of capacitors is relatively small and can be neglected.

Piezoelectric effect

We will replace the inductor with a grain of sand, or—which is the same—with a quartz crystal. When a quartz crystal is deformed by an external mechanical stress (see Fig. 5), electric charges appear on the crystal surface (think of a quartz gas lighter!). This is called the *direct* piezoelectric effect, and the materials that exhibit it are called piezoelectric materials. Conversely, an external electric field causes the strain, which changes the crystal dimensions. This is called the *reverse* piezoelectric effect.

Suppose now that we connect for a short time a dc voltage source to a quartz crystal having electrodes on its opposite faces (see Fig. 6). The applied voltage will stress the crystal, and after removing it, the crystal—like a tuning fork after striking it—will generate mechanical oscillations at a self-resonant frequency. Due to the alternating mechanical stress within the crystal we will see sinusoidal voltage between the crystal electrodes. This voltage will decay due to mechanical losses in the crystal. It can be shown that the time $t_{0.9\text{-}0.1}$ that it takes to the envelope of the crystal voltage to decrease from the level of 90% to 10% of its maximum value is proportional, for a given resonant frequency, to the crystal quality factor (see the equation in Fig. 6). Thinking of a 1-kHz tuning fork, $t_{0.9\text{-}0.1}$ can easily be seconds, which corresponds to a quality factor of thousands. The typical quality factor of quartz crystals is even much higher; it can approach 10^5.
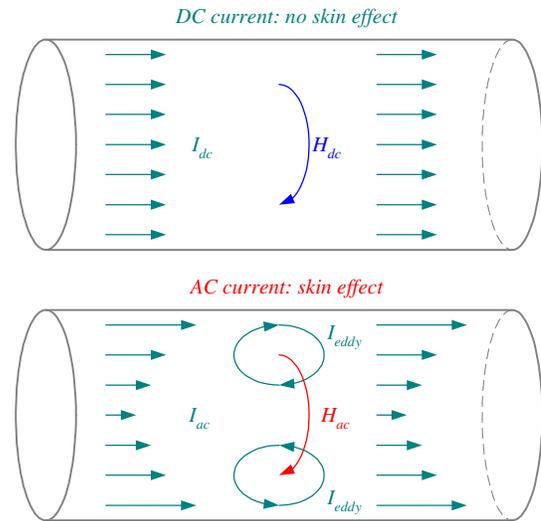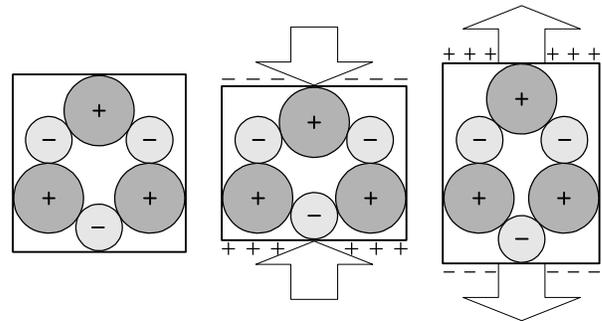

Fig. 4. Skin effect.

Fig. 5. Piezoelectric effect.

To understand how an inductor with a low Q_L can be replaced by a quartz crystal with a high Q_{XTAL}, let us develop its equivalent electric circuit. Since a quartz crystal is a passive system with no internal energy sources, except those that are related to the initial conditions (initial stress), we have to replace it with a passive electric circuit. This circuit should be able to produce decaying oscillations in response to initial conditions and should also have, like a quartz crystal, a very high dc resistance.

A *series* resonant circuit (see Fig. 6) perfectly suits these conditions. This circuit includes the series resonance capacitance C_s, the inductance L, and the resistance R' that represent such mechanical properties of the crystal as the elasticity, mass, and energy losses, respectively. The parallel resonance capacitance C_p represents the crystal electric capacitance (measured between the crystal electrodes). We chose such, C_s, L, R_s, and C_p that provide us with the same natural response of the equivalent circuit as the specific quartz crystal have.

To find qualitatively the frequency dependence of the quartz crystal impedance, we find for its equivalent circuit the frequencies of series (zero impedance; prove this!) and parallel (infinite impedance) resonance, neglecting R_s (for a very high $Q_{XTAL}=1/j\omega_0 C_s R_s$, R_s and C_s are very small, such that $R_s \to 0$, $C_s \ll C_p$):

$$\varpi_s\big|_{R_s=0} = \frac{1}{\sqrt{LC_s}}$$

$$\varpi_p\big|_{R_s=0} = \frac{1}{\sqrt{L\dfrac{C_s C_p}{C_s + C_p}}} = \frac{1}{\sqrt{LC_s}}\sqrt{\frac{C_s + C_p}{C_p}} \, . \qquad (5)$$

$$= \varpi_s \sqrt{1 + \frac{C_s}{C_p}}\Bigg|_{C_s \ll C_p} \to \varpi_s$$

Fig. 7 shows the quartz crystal impedance as a function of frequency. One can see that for $\omega_s \ll \omega_1 \ll \omega_p$, it is inductive. So we have an equivalent of inductance but with much less losses and a much higher quality factor.

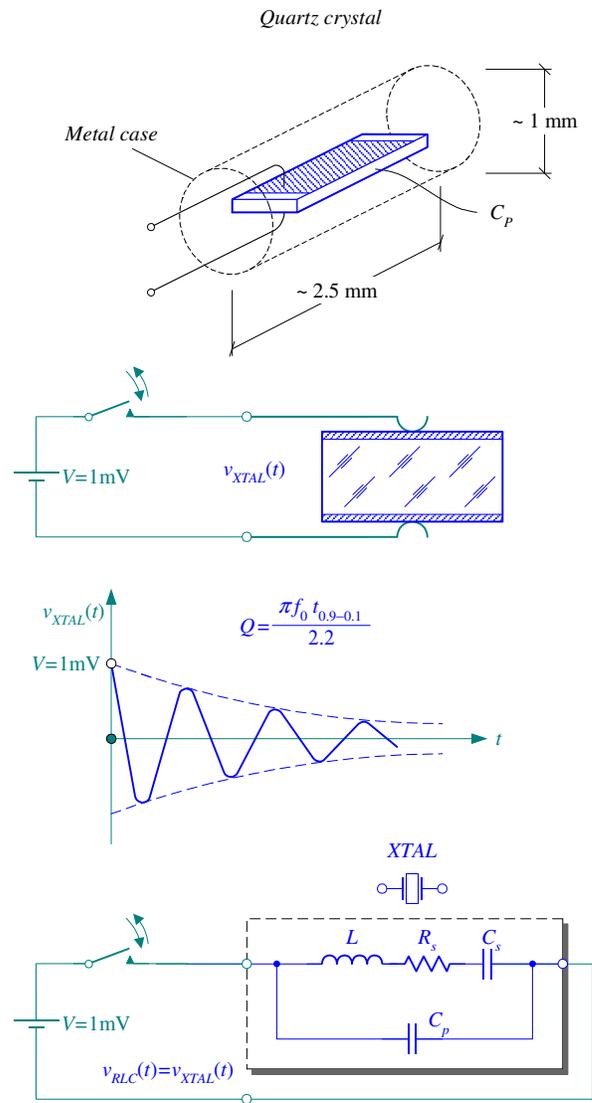It is interesting that trying to build a precise oscillator (clock), we use a tiny part of the ancient sand clock.

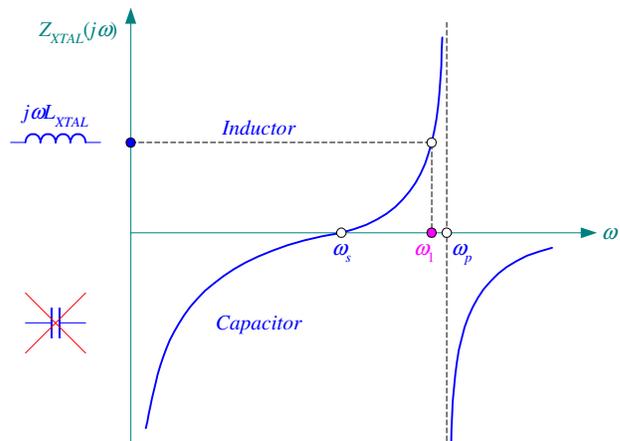Fig. 6. Quartz crystal, its natural response, and equivalent electric circuit.

Fig. 7. Impedance of the quartz crystal as a function of frequency.

Pierce oscillator

Let us now build a quartz crystal oscillator. We will use a generic approach shown in Fig. 2, choosing the Colpitts configuration because it comprises only a single inductor that we have to replace with the quartz crystal. We will also use a CMOS NOT logic gate instead of an analog amplifier. (We are actually building a clock circuit for an electronic watch, communication device, computer, etc. Such devices are usually built by CMOS technology.)

You will study logic gates in a special course, meanwhile it is enough for us to define the transfer characteristic of a NOT gate (see Fig. 8) Fig. 8 shows that the NOT gate simply translates the *low* and *high* voltages (binary zero and binary one) at its input into the *high* and *low* voltages (binary one and binary zero), respectively, at its output. Note also that the input impedance of a NOT gate is very high and its output impedance is very low.

In general, its transfer characteristic is nonlinear; however, there is a linear transition region. We will use this region with a high negative slope to amplify (translate) relatively small analog signals. To do this we have to first define the operating point in the middle part of the transition region. A resistor R_f connected between the input and output of the NOT gate helps us to do this easily: no current flows through this resistor in the static state, hence, no voltage drops across the resistor and $V_{IN}=V_O$. The small-signal voltage gain A_{OL} of the NOT gate equals the slope of its transfer characteristic at the operating point Q.

We now can connect to the NOT gate the Colpitts tank with the quartz crystal instead of the inductor, not forgetting to connect between the gate output and the tank the R_o resistor in order not to let its low output impedance to shunt the tank and reduce the tank quality factor.

Note as well that R_f should also have a high enough value in order not to shunt the tank. We divide R_f in Fig. 8 by $1+A_{OL}$ to find the impedance seen by Z_3 relative to the ground, because the voltage drop across R_f is $(1+A_{OL})\cdot 1V$ for a 1V unit voltage drop across Z_3. Reflecting R_f to the ground reduces the voltage drop across it to 1V. Therefore, we have to decrease the R_f value to compensate for the voltage decrease across it and keep its current the same.

The obtained oscillator is called after George Washington Pierce who was the first to use a quartz crystal to stabilize more precisely the frequency of oscillators (patented in 1923, based on lamps, not on NOT gates!).

Self resonant frequency of quartz crystals is high, above 10 kHz, this is why Pierce left a niche for Hewlett.
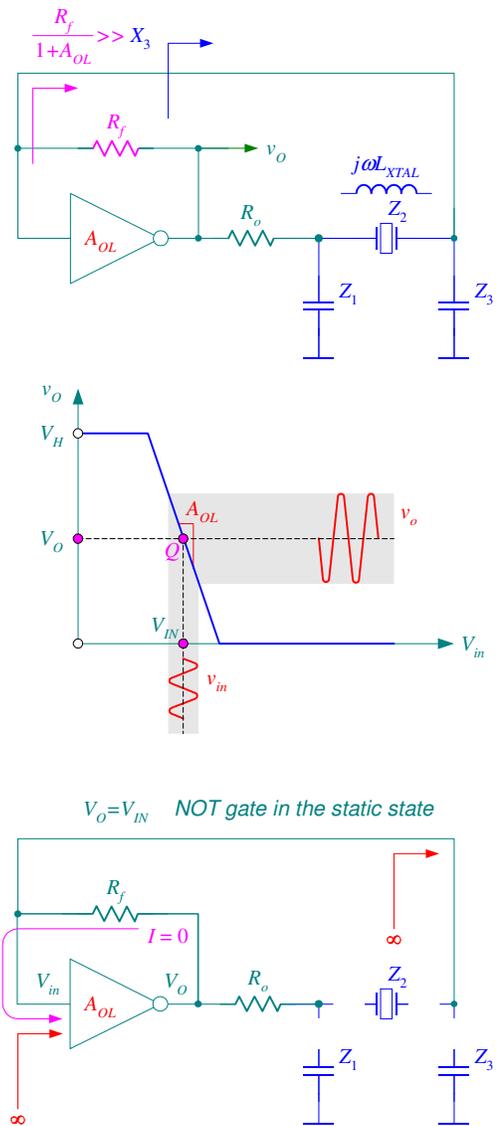

Fig. 8. Pierce oscillator based on a CMOS NOT logic gate.

HOME WORK

Prove that $Q_{RLC}= R/(\omega_0 L)=Q_L=\omega_0 L_s/R_s$.

REFERENCES

[1]   S. Sedra, K. C. Smith, Microelectronic Circuits, 4th ed. New York: Oxford University Press, 1998.