5. Double-Transistor Circuits (continued)

5.4. *Differential amplifier*

Our aim is to find a new circuit that is able to amplify a difference between two voltage signals, each relative to the ground (see Fig. 1). (Note that the elementary single-transistor amplifiers have only one input and are unable to amplify the difference between two voltage signals, each relative to the ground). We will call such a circuit differential amplifier. We will define (see Fig. 1) the small-signal voltages at the amplifier inputs as v_a and v_b, the differential small-signal voltage as

$$v_\varepsilon = v_a - v_b \quad (1)$$

and the common-mode small-signal voltage as

$$v_{cm} = \frac{v_a + v_b}{2}. \quad (2)$$

An ideal differential amplifier should amplify only the differential signal v_ε. Practical amplifiers, however, have also an undesirable common-mode voltage gain. The ratio of the small-signal differential voltage gain, $A_{v\varepsilon}$, and the small-signal common-mode voltage gain, A_{vcm}, is used to describe the capability of a differential amplifier *not* to amplify common-mode signals. We will refer to this ratio, measured in dB, as the common-mode rejection ratio

$$CMRR = 20\log_{10}\left|\frac{A_{v\varepsilon}}{A_{vcm}}\right|. \quad (3)$$

It will also be important for us to improve the linearity of the mutual transconductance gain, g_m, of the new circuit compared to that of the elementary CE amplifier.

Since we need two high impedance inputs, and we use the double-transistor configuration shown in Fig. 2. We will see below, that the architecture of the differential amplifier is based on its symmetry; hence, we chose identical transistors and define their static state with the help of a current mirror.

Static-state

The static state of the differential amplifier in Fig. 2 is defined by the static current source I_N, R_N=r_{oS}, representing the slave transistor of a current mirror circuit. The static voltages of the transistor base terminals are zero, and their base-to emitter voltages approximately equal 0.7 V.

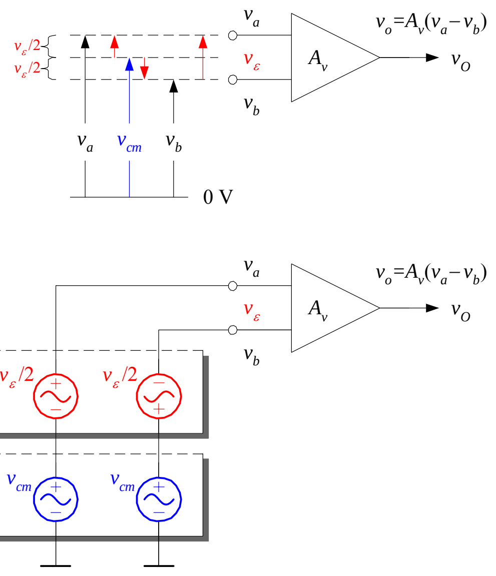

Fig. 1. Differential amplifier.

Therefore, the voltage drop on the static current source I_N, r_{oS} is $-V_{CC}$+0.7. Having this voltage and r_{oS}, one can easily find I_N that provides the needed currents of the transistor emitters. Amplified by α_F, the emitter currents give us the collector currents. Amplified by R_C, the collector currents give us the voltage drop on the load resistors. Subtracted from V_{CC}, the voltage drop on the load resistors gives us the collector potentials V_C. Adding 0.7 V, gives us the collector-to emitter voltages V_{CE}. Divided by β_F, for a given V_{CE}, the collector currents give us the base currents.

Large-signal transconductance gain

To find the large-signal transconductance gain of the differential amplifier we solve the following system of equations.

$$\begin{cases} i_{EA} = I_{ES} e^{v_{BEA}/V_T} = I_{ES} e^{(v_A - v_E)/V_T} \\ i_{EB} = I_{ES} e^{v_{BEB}/V_T} = I_{ES} e^{(v_B - v_E)/V_T} \end{cases}. \quad (4)$$

$$\Rightarrow \frac{i_{EA}}{i_{EB}} = e^{(v_A - v_B)/V_T} \Rightarrow i_{EA} = i_{EB}\, e^{v_\varepsilon/V_T}$$

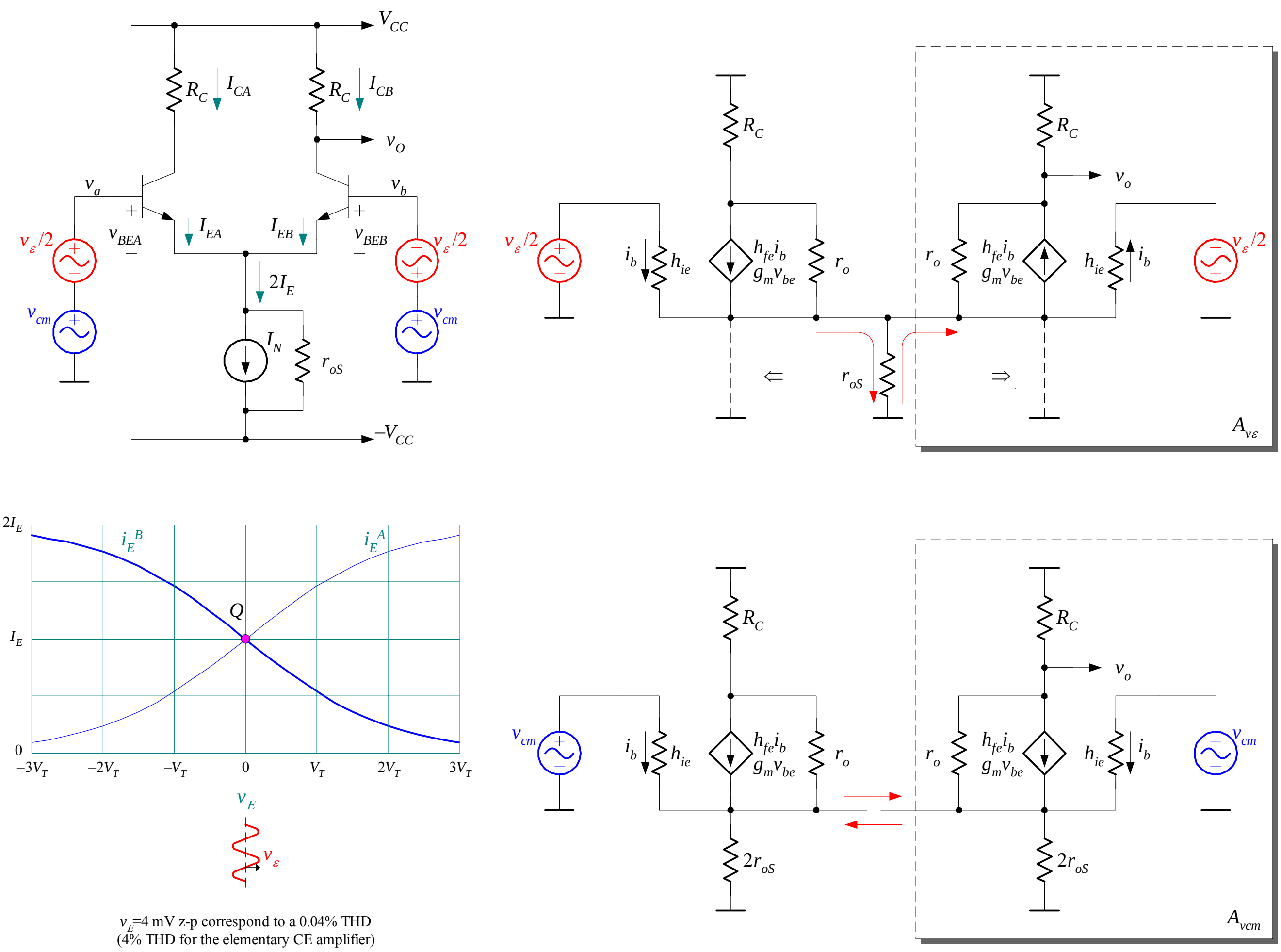

Fig. 2. The large-signal transconductance gain, the small-signal differential voltage gain, and the small-signal common-mode voltage gain of the differential amplifier.

$$i_{EA} + i_{EB} = i_{EB}\, e^{v_\varepsilon / V_T} + i_{EB} = 2I_E \qquad (5)$$

$$\Rightarrow i_{EB} = \frac{2I_E}{1 + e^{v_\varepsilon / V_T}}$$

The obtained transfer characteristic i_{EB}–v_ε is shown in Fig. 2. Note that at the operating point, the second derivative of the transfer characteristic changes its sign. This provides the differential amplifier much better linearity than that of the elementary CE amplifier. For example, amplifying a v_ε=4 mV z-p (zero-to-peak) results only in a 0.04% THD (for THD see Appendix) in the output current. Distortions of the elementary CE amplifier are greater by two orders of magnitude.

Small-signal differential voltage gain $A_{v\varepsilon}$

To find the small-signal differential voltage gain, $A_{v\varepsilon}$, we note that the equivalent small-signal circuit of the differential amplifier is *symmetrical*, whereas its excitation by the $+v_\varepsilon/2$ and the $-v_\varepsilon/2$ small-signal sources is *asymmetrical*. (Note that we find the small-signal gains by applying superposition and, therefore, ground the v_{cm} source.) We can conclude, therefore, that no current is flowing in the r_{oS} resistor. This is so because the contributions (think superposition!) of the small-signal sources are equal in magnitudes but different in signs. No current through r_{oS} means no voltage drop across it, hence, we can ground the emitters of both the transistors without changing any small signal current or voltage in the circuit. This leaves us with two independent sub-circuits. The function of the left sub-circuit is to ground r_{oS} and to maximize in this way the differential voltage gain $A_{v\varepsilon}$. We will see below while finding the small-signal common-mode voltage gain A_{vcm}, that r_{oS} will not be affected by the left sub-circuit (remember the equivalent black box that grounds the static current source employed in the bias of integrated circuits?!).

The left sub-circuit is connected to the amplifier output, and it gain can easily be found

$$A_{v\varepsilon} \equiv \frac{v_o}{v_\varepsilon} = \frac{v_o}{2\frac{v_\varepsilon}{2}} = +\frac{1}{2} g_m (R_C \parallel r_o)\Big|_{R_C << r_o} = +\frac{1}{2}\frac{\alpha_f}{r_e} R_C \quad . \tag{6}$$

Note that this sub-circuit resembles the elementary CE amplifier. The only difference is that it sees the negative half of the input signal v_ε. (The other half is "invested" into the left sub-circuit to shortcircuit r_{oS}.)

Small-signal common-mode voltage gain A_{vcm}

To find the small-signal differential voltage gain A_{vcm}, we note that both the equivalent small-signal circuit of the differential amplifier and its excitation are symmetrical. We can conclude, therefore, that no current is flowing between the two identical sub-circuits. (Note that we have split out the r_{oS} resistor into an equivalent pair of resistors, each having a $2r_{oS}$ value.) Thus, the left sub-circuit do not affect the right one, including its emitter resistor $2r_{oS}$ (remember the black box?!).

To find the small-signal voltage gain of the right sub-circuit we neglect the effect of the dynamic impedance r_o of the transistor, assuming that $r_o >> R_C$, and $r_o \approx 2r_{oS}$ (prove the latter!). This lets us very easily find

$$A_{vcm} \equiv \frac{v_o}{v_{cm}}\Big|_{\substack{r_o \to \infty \\ i_b = 1}} = -\frac{h_{fe}}{h_{fe} + (1 + h_{fe})\, 2r_{oS}} R_C = -\frac{\alpha_f}{r_e + 2r_{oS}} R_C = -\frac{\alpha_f}{r_e + r_o} R_C\Big|_{r_e << r_o} = -\frac{\alpha_f}{r_o} R_C \tag{7}$$

and

$$CMRR = 20\log_{10}\frac{\frac{1}{2}\frac{\alpha_f}{r_e}R_C}{\frac{\alpha_f}{r_o}R_C} = 20\log_{10}\frac{1}{2}\frac{r_o}{r_e}\Big|_{V_A >> V_{CE}} = 20\log_{10}\frac{1}{2}\frac{I_E}{V_T}\frac{V_A}{I_C} = 20\log_{10}\frac{1}{2}\frac{1}{\alpha_f}\frac{V_A}{V_T} \approx 20\log_{10}2000 = 20\log_{10}\sqrt{2}\times\sqrt{2}\times 1000 \approx 3\,\text{dB} + 3\,\text{dB} + 60\text{dB} = 66\,\text{dB} \quad . \tag{8}$$

To find the small-signal input impedances for the differential and common-mode signals we refer to Fig. 3

$$R_{in\varepsilon} = 2h_{ie}$$

$$R_{incm}\big|_{r_o \to \infty} = \frac{1}{2}(1 + h_{fe})(r_e + 2r_{oS}) = \frac{1}{2}(1 + h_{fe})(r_e + r_o) \; . \tag{9}$$

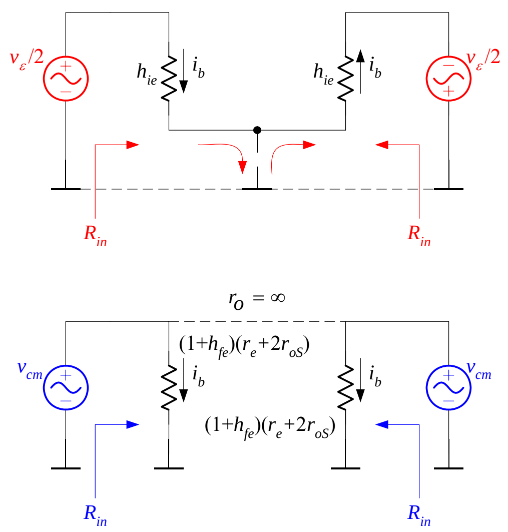

Fig. 3. Finding input impedances for the differential and common-mode signals.

5.5. *Darlington configurations*

The main aim of Darlington configurations is to increase the input impedance of a transistor. There are two different types of this configuration (see Fig. 4): the basic Darlington configuration and the complementary Darlington configuration. In both the cases, the double-transistor properties are defined by the master transistor. Due to this, the *npn* slave in the complementary Darlington configuration behaves like a *pnp* transistor. Thus the complementary Darlington configuration can also be used to replace a *pnp* transistor with a better *npn* one.

To find the increase of the small-signal input impedance due to the Darlington configurations, we first note that the operating points of the master and the slave transistors are very different (see Fig. 4): $I_{BM}=I_{BS}/(1+\beta_M)$ and $I_{BM}=I_{BS}/\beta_M$ for the basic and complementary configurations, respectively. As a result, $h_{ieM}=(1+h_{feM})h_{ieS}$ and $h_{ieM}=\beta_M\, h_{ieS}$ (see Fig. 5).

Note in Fig. 5, that the Darlington configurations increase the input impedance of the elementary CE amplifier by two orders of magnitude: by a factor of $2(1+h_{fe})$ for the basic configuration and by a factor of β_M for the complementary configuration.

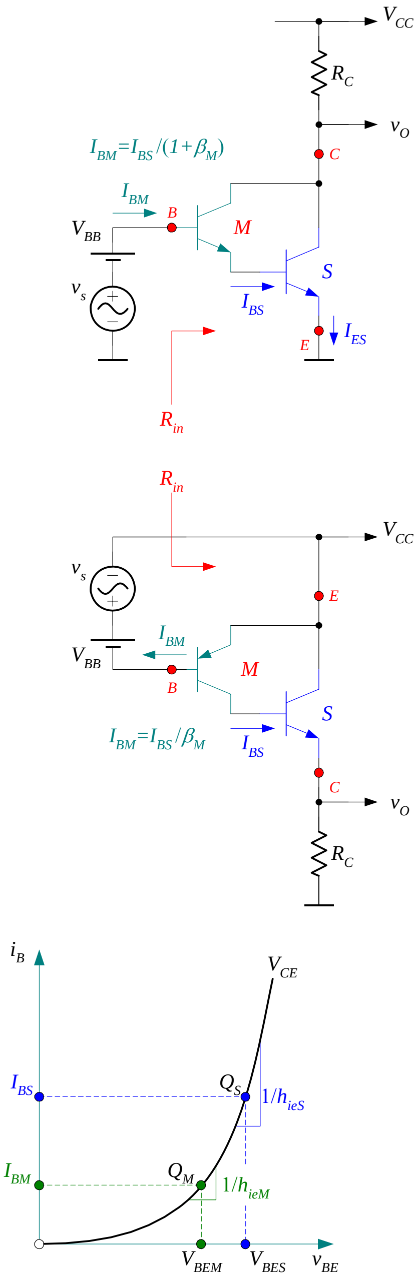

Fig. 4. Increasing the input impedance of the elementary CE amplifier with: (a) Darlington configuration and (b) complementary Darlington configuration.

APPENDIX

Recall that THD (total harmonic distortions) equals to the square root of the ratio of the total average power of all the harmonics divided by that of the fundamental, times 100%. Recall also that the average power of a signal is $1/T$ times the integral, from zero to T, of the squared value of the signal, where T is time interval. The average signal power is used to compare between two signals and is measured either in V^2 or I^2, whereas the electrical power is measured in W. Note that without defining the average signal power we cannot compare, for example a sine and cosine signals. This is so because at some times the sine signal is greater, and at the other times, the cosine signal is greater. (The average *signal power* can be converted into the *electrical power* by assuming that the electrical power is dissipated at a 1 Ω resistor.)

REFERENCES

[1] A. S. Sedra and K. C.Smith, *Microelectronic circuits*.

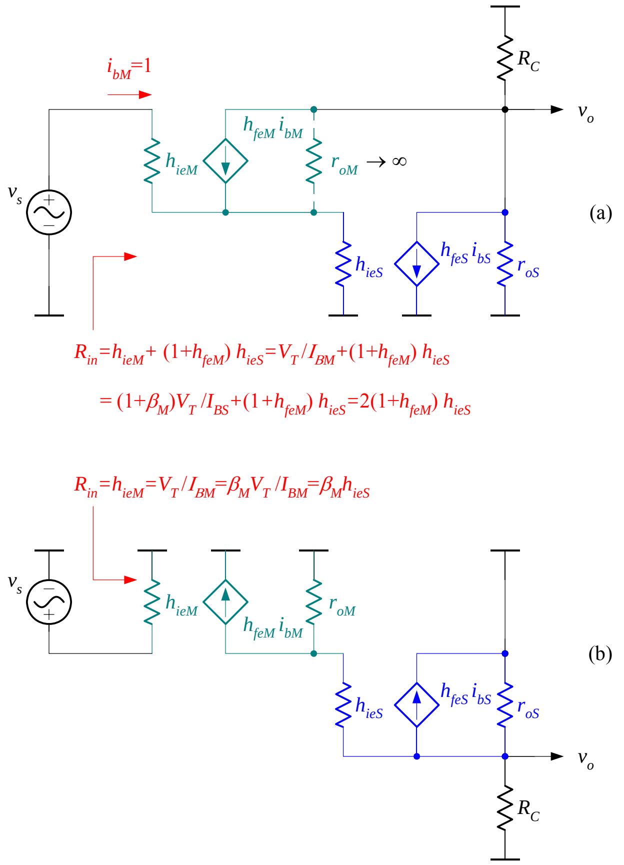

Fig. 5. Small-signal input impedances of the CE amplifier with: (a) Darlington configuration and (b) complementary Darlington configuration.