

BEN-GURION UNIVERSITY OF THE NEGEV
FACULTY OF ENGINEERING SCIENCES
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

FEEDBACK AS SUPERPOSITION

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE M.Sc DEGREE

By: Pavel Shanin

September 2013

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Supervised by: Prof. Eugene Paperno

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Supervisor:..... Date:.....

Chairman of Graduate Studies Committee:..... Date:.....

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Summary

This work is devoted to the analysis of linear feedback networks with multiple dependent sources. Feedback circuits are widely used in analog circuit design, mainly due to the fact that the circuit characteristics and the closed-loop gain value are highly independent of the operating conditions and the transistor parameters' variations. Modeling of such networks is very important for exploring the effect of feedback on the network transfer function. An analytical model enables us to apply control theory tools for analyzing the network behavior, as well as provides a better intuitive insight into its operation. Although any linear feedback network can be easily simulated, the obtained solution provides no insight into the network functional structure. These feedback circuits usually comprise many transistors; consequently, their small signal model would naturally include several dependent sources. Unfortunately, the existing theories do not provide analytical and functional models for feedback networks with multiple dependent sources.

The main goal of this work is to develop accurate analytical and functional models for the closed loop gain of linear feedback networks with multiple dependent sources. An additional goal is to study the effect of negative feedback on impedances seen from a pair of arbitrary terminals for such networks. This work also seeks to extend Middlebrook's formula for simulating return ratio to linear networks with multiple bilateral feedback loops.

The work develops an accurate analytical and functional model for the closed loop gain of linear feedback networks with multiple dependent sources and any number of feedback loops. The result is, a closed loop gain formula containing return

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ratios are obtained in a matrix form. This model describes linear feedback networks with multiple dependent sources analytically, in terms of return ratios. In the general case, in such networks, each dependent source effects, not only its own control signal but also the control signals of the other dependent sources. For reasons of consistency, this work refers to such cross contributions, normalized to the total control signal appearing in the dependent source, as cross return ratios. It is shown that return ratios and cross return ratios can be combined into a general return ratio. To the best of our knowledge, the present work shows for the first time, how to obtain for linear networks with multiple dependent sources a generalized return ratio as a function of network elements. This is achieved by using dissection and superposition in network analysis. Obtaining generalized return ratio, as a function of circuit components, is important for analyzing the network stability. This also turns a stability problem to a Nyquist plot calculation and allows applying other control theory tools.

In order to examine the effect of negative feedback on impedances seen from a pair of arbitrary terminals of linear feedback networks, we described them analytically as a function of return ratios. It has been shown that the Blackman's formula can be extended to the case of feedback networks with multiple dependent sources. To reach this aim, we revisit the proof of the Blackman's formula, proved it with a similar manner for feedback networks with two dependent sources and then extended to the case of networks with multiple dependent sources. We also showed that this formula can be written in a convenient matrix form.

As was already mentioned finding return ratio is very important for revealing the effect of feedback on the closed loop-gain function, impedances and stability. Conventionally, the return ratio is found by suppressing all the independent

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network's sources, assigning a fixed value to the dependent source, and calculating the signal that returns to its controlling terminals. However, this procedure is not suitable for simulating electronic circuits or for testing real electronic circuits. An alternative approach was suggested by Middlebrook and consists in connecting test sources to the accessible terminals of a transistor or operational amplifier. According to this method, two partial return ratios T_v and T_i are measured, one for a voltage injection and the other for a current injection. Then they are translated into the true loop gain T . This method is inaccurate at frequencies near loop gain crossover when $|T| < 1$. This defect is eliminated in an improved method suggested by Middlebrook. In the method a loop-gain measurement is done by two simultaneous voltage and current injection at a point of arbitrary impedance ratio. According to this method two partial return ratios are measured, one T_v^n is measured when injected signals are adjusted to null the current in a particular circuit point and the other T_i^n when the voltage on a dependent source equals zero. Then T_v^n and T_i^n are translated into the circuit's return ratio. In spite of the original proof of these methods is based on an ideal feedback model which does not account for nonzero reverse loop gains, our proof is based on a generic feedback model. As a result it is shown that two Middlebrook's formulas can be applied with no approximations to any linear feedback network with a single dependent source.

As a result of this work, the classical Bode and Blackman methods are extended to multiple transistor circuits, which are much wider class of circuits. Also the proposed theory could serve a useful reference to do quick calculations by hand, thus, to obtain a better intuitive insight into the effect of feedback on the closed loop network function and circuit impedances.

Keywords

Analytical model, cross return ratio, dependent sources, desensitivity factor, DSF, feedback as superposition, functional feedback model, general feedback theorem, Linear feedback networks, multiple dependent sources, multiple loop feedback amplifiers, multiple loop feedback networks, network theory, network transfer matrix, null return difference matrix, null return ratio matrix, return difference, return difference matrix, return ratio, return ratio matrix, successful current injection, successful voltage injection, superposition of dependent sources.

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List of abbreviations

adj - adjoint matrix.

CC - common collector.

det – determinant.

DSF - desensitivity factor.

GFT - general feedback theorem.

KCL - Kirchof's current law.

KVL – Kirchof's voltage law.

ppm – part per million.

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Chapter 1

Introduction

Studying of a feedback in electronic circuits is of great demand. Our literature survey shows that almost every analog design book contains chapter devoted to feedback [1]-[9].

The reason for such interest is that circuits with feedback are used widely in analog design. Also all operational amplifier applications rely on feedback concept. The use of feedback in analog circuits gives us the following advantageous:

- a. The open loop gain depends on a transistor small signal parameters that are very sensitive to the transistor technology. On the other hand feedback is made from the passive components that can be made arbitrary close to the desired values.
- b. Circuit characteristic can be made independent of operating conditions such as supply voltage and temperature.
- c. Frequency response and the gain-bandwidth trade-off can be controlled.

- d. Signal distortion, which is a result of the nonlinear nature of active devices, can be significantly reduced.

1.1 Linear feedback networks

1.1.1 Closed loop gain in feedback networks with a single dependent source

The ideal feedback system can be represented as a single-loop functional model [1] that can be seen on Figure 1.1. Where a is the forward gain and f represents a feedback ratio. It is worth to mention that the blocks a and f are unilateral.

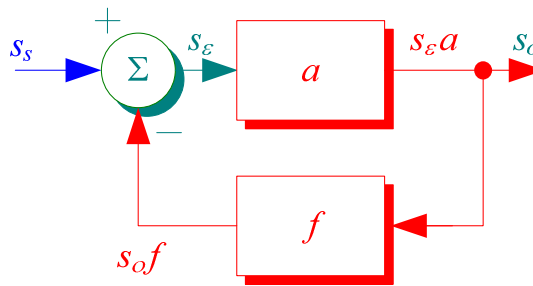


Figure 1.1: Ideal feedback functional model with unilateral blocks.

From the scheme, we obtain:

$$S_o = aS_\varepsilon = a(S_s - S_o f) \quad (1.1)$$

Then the ideal closed loop gain is:

$$A_{CL} = \frac{S_o}{S_s} = \frac{a}{1+af} = \frac{1}{f} \frac{af}{1+af} = \frac{1}{f} \frac{T}{1+T} \quad (1.2)$$

Where $T = af$ is the loop gain.

When the loop gain T becomes large compare to unity the closed loop gain approaches:

$$\lim_{T \rightarrow \infty} A_{CL} = \lim_{T \rightarrow \infty} \frac{S_0}{S_s} = \frac{1}{f} \quad (1.3)$$

We receive the closed loop gain depends only on the feedback network, which consists from passive elements that can be made arbitrary closed to desired value. As a result, the closed loop gain is independent of variations of the forward gain. The independence of closed-loop gain from the parameters of the active amplifier is the reason of the wide use of operational amplifier in analog circuits.

In contrast to ideal feedback model, most real analog circuits have not only bilateral feedback network but also bilateral forward gain. Therefore we introduce in chapter more deliberate feedback model, based on return ratio. The model describes networks with a single dependent source and was firstly introduced by Bode [10]. In the model, the closed-loop of feedback network is described through return ratio of a dependent source, in the small-signal model of a circuit. The return ratio analysis is often easier then two-port analysis and is topology independent [11]-[13]. It allows us to build functional model of a circuit, where each block can be described in terms of a network elements. As a result, we can derive from it, which elements of a circuit influence on the stability.

In order to introduce Bode feedback model let us consider a feedback amplifier that is shown in Figure 1.2 which consists of linear elements.

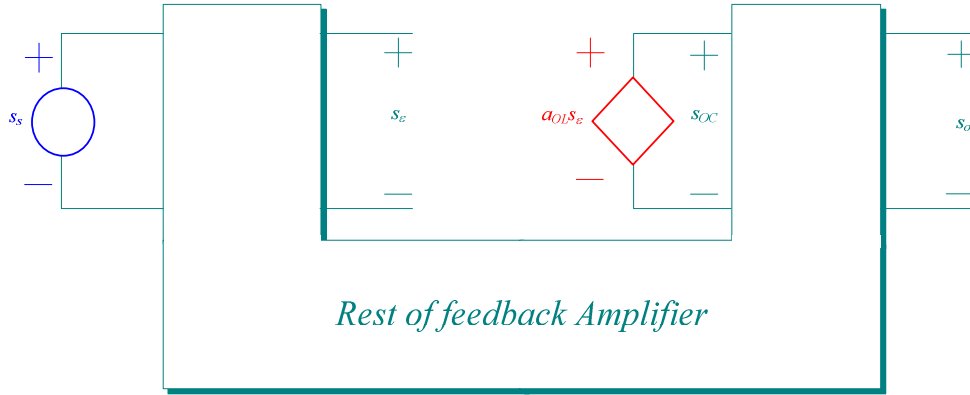


Figure 1.2: Feedback amplifier contains linear elements.

a_{ol} - is a controlled source value.

From the figure

$$S_{oc} = a_{ol}S_\epsilon \tag{1.4}$$

$$S_\epsilon = B_1S_s - HS_{oc} \tag{1.5}$$

$$S_o = dS_{in} + B_2S_{oc} \tag{1.6}$$

The terms B_1 , H , d and B_2 are defined by

$$\begin{aligned} G &= \left. \frac{S_\epsilon}{S_s} \right|_{S_{oc}=0} = \left. \frac{S_\epsilon}{S_s} \right|_{a_{ol}=0} \\ d &= \left. \frac{S_o}{S_s} \right|_{S_{oc}=0} = \left. \frac{S_o}{S_s} \right|_{a_{ol}=0} \\ B_2 &= \left. \frac{S_o}{S_{oc}} \right|_{S_s=0} \\ H &= - \left. \frac{S_\epsilon}{S_{oc}} \right|_{S_s=0} \end{aligned} \tag{1.7}$$

G is a transfer function from the input to the controlling signal and d is the transfer

function from the input to the output both evaluated with $a_{ol} = 0$. B_2 is a transfer function from the dependent source to the output and H is a transfer function from the output of the dependent source to the controlling signal times -1, both evaluated with the input source set to zero.

From equations (1.5), (1.6) and $S_{oc} = a_{ol}S_e$ we can find closed-loop gain

$$A_{CL} = \frac{S_o}{S_{in}} = G \frac{a_{ol}B_2}{1 + a_{ol}H} + d \quad (1.8)$$

According to Body [1] (see Chapter 8) and [10] the return ratio for a dependent source can be found by the following procedure:

1. Set all independent sources to zero.
2. Disconnect the dependent source from the rest of the circuit, which introduces a break in the feedback loop.
3. On the side of the break that is not connected to the dependent source, connect an independent test source S_t of the same sign and type as the dependent source.
4. Find the return signal S_r generated by the dependent source.

The return ratio T for the dependent source is $T = -\frac{S_r}{S_t}$.

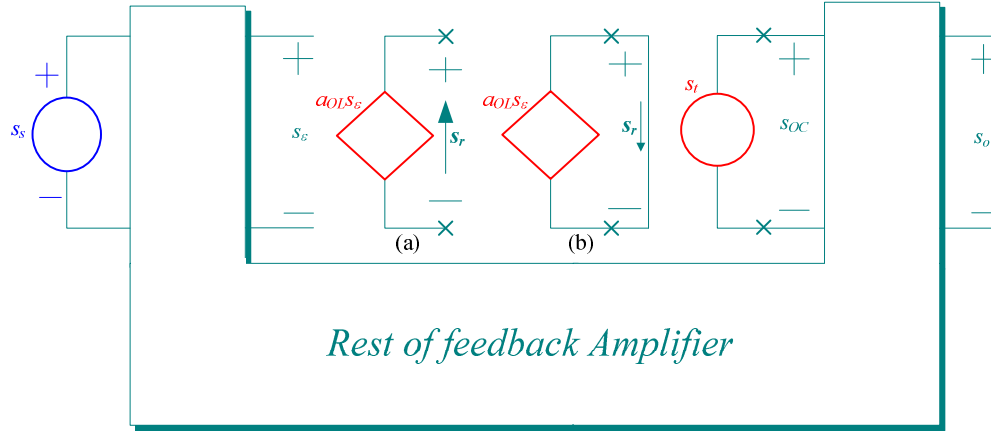


Figure 1.3: Finding of return ratio T in linear network with one dependent source, (a) for the voltage dependent source, (b) for the current dependent source.

The term $a_{ol}H$ in the denominator of (1.8) is equal to the return ratio. We can show it by setting $S_s = 0$, disconnecting the dependent source from the circuit. Connecting a test source S_t where the dependent source was connected. This is the procedure that was described earlier for finding the return ratio. After these changes, $s_{oc} = s_t$ and (1.5) becomes

$$S_\varepsilon = -HS_t \tag{1.9}$$

We became that the output of the dependent source is a return signal $S_r = a_{ol}S_\varepsilon = -a_{ol}HS_t$. The return ratio therefore

$$T = -\frac{S_r}{S_t} = a_{ol}H \tag{1.10}$$

The closed loop gain can be expressed in terms of it

$$A_{CL} = \frac{S_o}{S_{in}} = G \frac{a_{ol} B_2}{1+T} + d \quad (1.11)$$

or

$$A_{CL} = \frac{S_o}{S_{in}} = \frac{g}{1+T} + d \quad (1.12)$$

$$g = G \cdot a_{ol} B_2 \quad (1.13)$$

Definition $DSF=1+T$ is desensitivity factor or amount of feedback.

The feedback is negative if it decreases the closed loop gain, then $|DSF|>1$ and the feedback is positive if it increases the closed loop gain, then $|DSF|<1$. Obviously if the feedback is absent $|DSF|=1$.

Combining terms in (1.12) we become

$$A_{CL} = \frac{g}{1+T} + d = A_{\infty} \frac{T}{1+T} + \frac{d}{1+T} \quad (1.14)$$

With definition:

$$A_{\infty} = \frac{g}{T} + d \quad (1.15)$$

A functional model for the closed-loop gain formula can be seen on Figure 1.4.

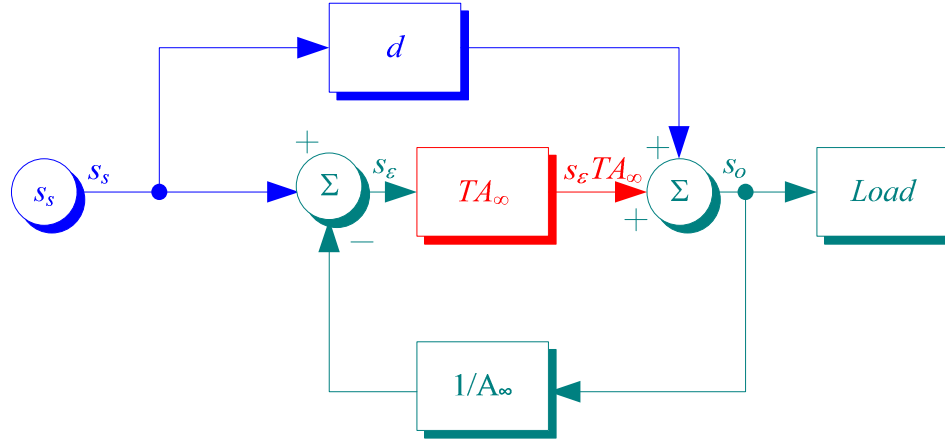


Figure 1.4: A functional model for the closed-loop gain formula.

Example

As an example to finding return ratio T we consider common collector amplifier that is shown in Figure 1.5.

$$i_b = -\frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}} i_t \quad (1.16)$$

$$i_r = h_{fe} i_b \quad (1.17)$$

$$T = -\frac{i_r}{i_t} = -\frac{h_{fe} i_b}{i_t} = h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}} \quad (1.18)$$

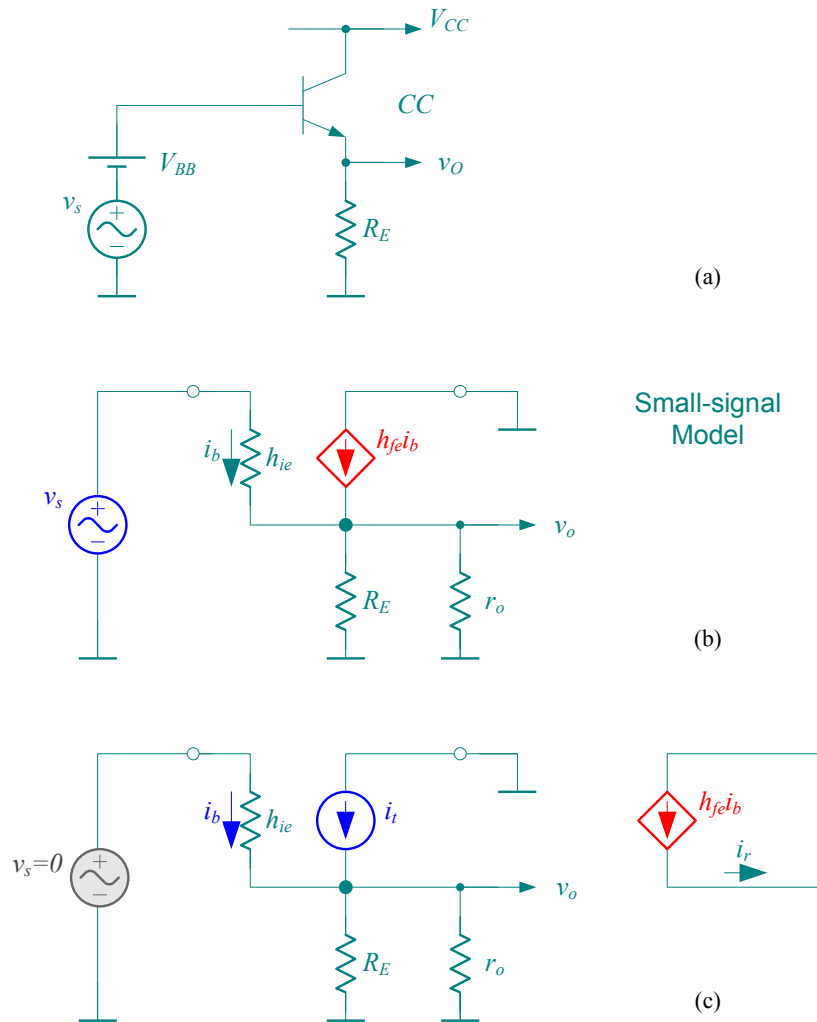


Figure 1.5: (a) Common collector amplifier, (b) Small signal model, (c) Circuit for finding of return ratio.

1.1.2 Impedances of feedback networks with a single dependent source

Feedback affects input and output impedances of a circuit [14]. An expression of finding the impedance at an arbitrary port x in a feedback network is called Blackman's formula (1.19). In the formula impedance is expressed in terms of the returned ratio. This formula can be used with arbitrary linear network with no more than one controlled source. The derivation of this formula can be found in [1].

$$R_X = R_X (a_{ol} = 0) \frac{1+T(\text{short circuit})}{1+T(\text{open circuit})} \quad (1.19)$$

Where $R_X (a_{ol} = 0)$ - is an impedance of the port x with suppressed dependent source, $T(\text{short circuit})$ - is a return ratio with the port x short circuit, $T(\text{open circuit})$ - is a return ratio with the port x open circuit. The return ratios are computed with respect to the same controlled source. An advantage of the formula is that we can use it with any type of feedback in linear circuit.

Example

As an example to finding impedance of input port we consider common collector amplifier see Figure 1.5.

$$R_X (a_{ol} = 0) = R_X (h_{fe} = 0) = h_{ie} + R_E \parallel r_o \quad (1.20)$$

When input port is shorted we have the same circuit like in the previous example Figure 1.5 (c):

$$T(\text{short circuit}) = -\frac{i_r}{i_t} = -\frac{h_{fe} i_b}{i_t} = h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}} \quad (1.21)$$

$$T(\text{opencircuit}) = 0 \quad (1.22)$$

$$R_X = h_{ie} + (1 + h_{fe})(R_E \parallel r_o) \quad (1.23)$$

1.1.3 Feedback networks with multiple dependent sources

1.1.1.1 The closed-loop transfer-function matrix of the multiple-loop feedback amplifier

In this section the analysis of feedback networks with multiple dependent sources reviewed, that was introduced by Chen in [9], (see Chapter 32). He introduces the concept of return difference matrix for a multiplicity of controlled sources. The model is based on representation of electrical network in a matrix equation form. From the solution of this matrix equation we can extract the closed loop gain.

In order to find transfer matrix of a network, we will study a general configuration of a multiple-input, multiple-output and multiple-loop feedback amplifier shown in Figure 1.6.

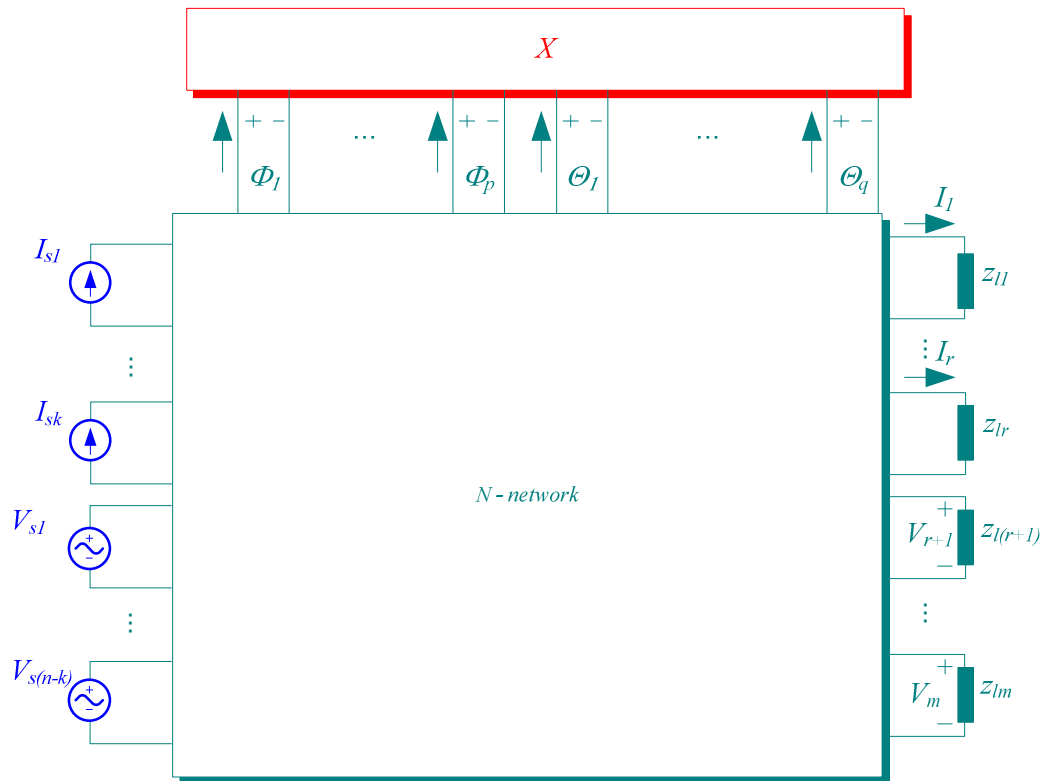


Figure 1.6: The general configuration of a multiple-input, multiple output and multiple-loop feedback amplifier.

In the scheme, the input, output and feedback variables may be either currents or voltages.

$u(s)$ - is n dimensional input vector and $y(s)$ - is m dimensional output vector.

$$u(s) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-2} \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} I_{s1} \\ \vdots \\ I_{sk} \\ V_{s1} \\ \vdots \\ V_{s(n-k)} \end{bmatrix}, \quad y(s) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{m-2} \\ y_{m-1} \\ y_m \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_r \\ V_{r+1} \\ \vdots \\ V_m \end{bmatrix}$$

X - is the matrix of order $p \times q$ that relates the controlled and controlling variables.

The matrix represents the dependent sources.

Φ - is a p dimensional controlling vector and Θ - is a q dimensional controlled vector of either currents or voltages.

The ϕ_k elements of Φ is a controlling variables while θ_k elements of Θ is a controlled variables of the dependent sources.

$$\Theta = X \cdot \Phi \tag{1.24}$$

Feedback amplifier can be represented in more convenient way see Figure 1.7.

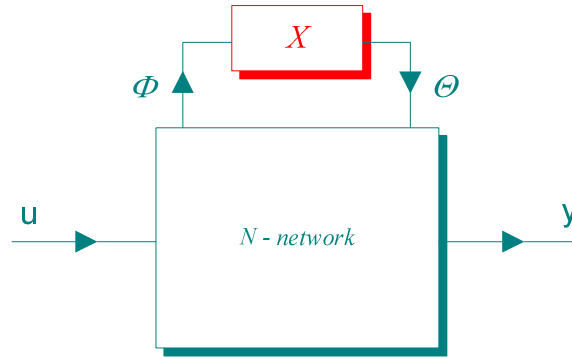


Figure 1.7: The block diagram of the general feedback configuration.

If X represents parameters of the dependent sources Θ and Φ are of the same dimension. In the block diagram of feedback model Figure 1.7 vectors u and θ are inputs and y and ϕ its outputs. Because the network is linear we can write matrix equations:

$$\Phi = A\Theta + Bu \quad (1.25)$$

$$y = C\theta + Du \quad (1.26)$$

A, B, C and D are transfer-function matrices of dimensions $p \times q$, $p \times n$, $m \times q$ and $m \times n$ respectively.

The closed-loop transfer-function matrix of the multiple-loop feedback amplifier of order $m \times n$ is defined by

$$y = W(X)u \quad (1.27)$$

In order to get the equation for $W(X)$ we combine the equations (1.24), (1.25) and (1.26).

$$\Phi = (I_p - AX)^{-1} Bu \quad (1.28)$$

$$y = CX(I_p - AX)^{-1} Bu + Du \quad (1.29)$$

Finally applying (1.27) we have:

$$W(X) = D + CX(I_p - AX)^{-1} B \quad (1.30)$$

I_p - is $p \times p$ identity matrix.

Alternatively we can write (1.30):

$$\begin{aligned} W(X) &= D + CX(I_p - AX)^{-1} X^{-1} XB = D + CX \left((X^{-1} X)^{-1} (I_p - AX) \right)^{-1} B \\ &= D + CX \left(X^{-1} X (I_p - AX) \right)^{-1} B = D + CX \left(X^{-1} X (I_p - AX) \right)^{-1} B \\ &= D + CX \left(X^{-1} (X - XAX) \right)^{-1} B = D + CX \left(X^{-1} (I_q - XA) X \right)^{-1} B \\ &= D + CXX^{-1} (I_q - XA)^{-1} XB = D + C(I_q - XA)^{-1} XB \end{aligned} \quad (1.31)$$

We can summarize:

$$W(X) = D + C(I_q - XA)^{-1} XB \quad (1.32)$$

Where we have use the matrix property $(AB)^{-1} = B^{-1}A^{-1}$ when A, B - are non-singular matrixes.

Obviously $W(0) = D$ and $y = Du$ means that closed loop is composed of a direct transmission only.

When X square and nonsingular, equation (1.32) can be written as:

$$W(X) = D + C(X(X^{-1} - A))^{-1}XB = D + C(X^{-1} - A)^{-1}X^{-1}XB \quad (1.33)$$

$$W(X) = D + C(X^{-1} - A)^{-1}B \quad (1.34)$$

Example

We will solve double-transistor amplifier circuit with this method.

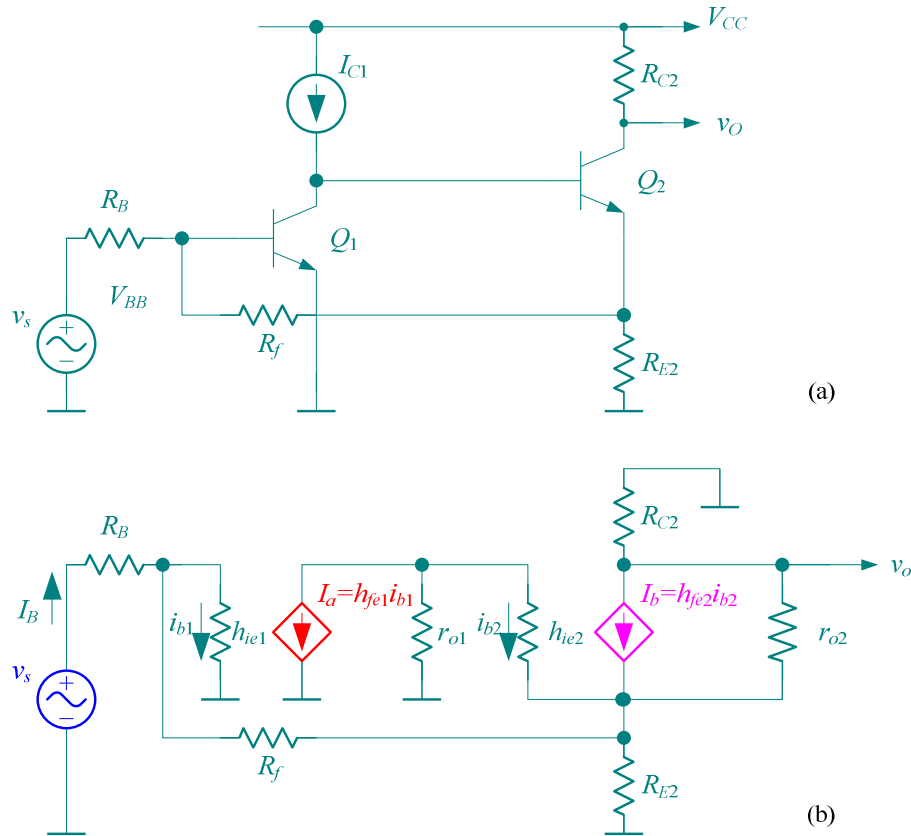


Figure 1.8: (a) Double Transistor Amplifier Circuit. (b) Small Signal Model of Double Transistor Amplifier Circuit with the values of dependent sources $I_a = h_{fe1}i_{b1}$ and $I_b = h_{fe2}i_{b2}$.

The following parameters values are used for this model.

$$R_C = 1.3\text{ k}\Omega, R_E = 1\text{ k}\Omega, R_f = 8\text{ k}\Omega, R_B = 2200\Omega$$

$$h_{ie1} = h_{ie2} = 23.5\text{ k}, h_{fe1} = h_{fe2} = 287, r_{o1} = r_{o2} = 25\text{ k}\Omega$$

$$X = \begin{bmatrix} h_{fe1} & 0 \\ 0 & h_{fe2} \end{bmatrix} = \begin{bmatrix} 287 & 0 \\ 0 & 287 \end{bmatrix} \quad (1.35)$$

Equation (1.24) can be written as:

$$\Theta = \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} 287 & 0 \\ 0 & 287 \end{bmatrix} \begin{bmatrix} i_{b1} \\ i_{b2} \end{bmatrix} = X\Phi \quad (1.36)$$

Assume that the output voltage v_o and input current I_B are the output variables. The seven port network N is defined by the variables $I_a, I_b, i_{b1}, i_{b2}, v_o, I_B$ and v_s . The matrix equations (1.25) and (1.26) of the network are:

$$\Phi = \begin{bmatrix} i_{b1} \\ i_{b2} \end{bmatrix} = A \begin{bmatrix} I_a \\ I_b \end{bmatrix} + B[v_s] \quad (1.37)$$

$$y = \begin{bmatrix} v_o \\ I_B \end{bmatrix} = C \begin{bmatrix} I_a \\ I_b \end{bmatrix} + D[v_s] \quad (1.38)$$

According to (1.27)

$$y = W(X)u$$

$$W(X) = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \quad A_{cl} = \frac{v_o}{v_s} = w_0 \quad Z_{in} = \frac{v_s}{I_B} = \frac{1}{w_1} \quad (1.39)$$

From solving the circuit in Figure 1.8 (b) where i_{b1} is a current on h_{ie1} and i_{b2} is a current on h_{ie2} we become (1.37) and (1.38) with the following values:

$$\begin{bmatrix} i_{b1} \\ i_{b2} \end{bmatrix} = \begin{pmatrix} -3.804 \cdot 10^{-3} & 7.016 \cdot 10^{-3} \\ -506.290 \cdot 10^{-3} & -16.918 \cdot 10^{-3} \end{pmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} + \begin{pmatrix} 0.031 \cdot 10^{-3} \\ 0.002 \cdot 10^{-3} \end{pmatrix} [v_s] \quad (1.40)$$

$$\begin{bmatrix} v_o \\ I_B \end{bmatrix} = \begin{pmatrix} -21.992801 & -1195.184378 \\ 0.040637 & -0.074939 \end{pmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} + \begin{pmatrix} 3.897 \cdot 10^{-3} \\ 0.115 \cdot 10^{-3} \end{pmatrix} [v_s] \quad (1.41)$$

Finally from the substitution of A, B, C and D matrix values from (1.40) and (1.41) into (1.34) we became:

$$W(X) = \begin{bmatrix} 5190.445 \cdot 10^{-3} \\ 0.246 \cdot 10^{-3} \end{bmatrix} \quad A_{cl} = \frac{v_o}{v_s} = 5.190445 \quad Z_{in} = \frac{v_s}{I_B} = \frac{1}{w_1} = 2232.578413 \Omega$$

1.1.1.2 The return difference and null return difference matrixes

In order to introduce the return difference matrix let us study matrix flow graph of a system of matrix equations (1.24), (1.25) and (1.26) from the previous section.

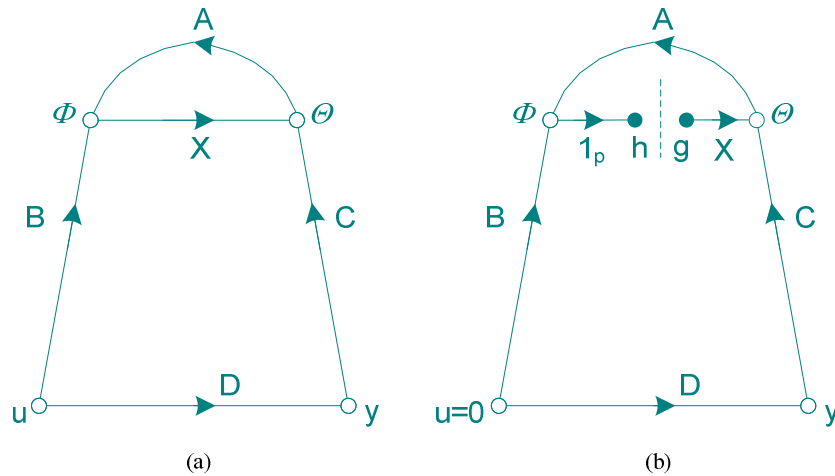


Figure 1.9: (a) Fundamental matrix flow graph, (b) Matrix flow graph with zero input vector and break in the branch with transmittance X .

In Figure 1.9 (b) we set the input vector u to zero, break the input of the branch with transmittance X and apply a signal p – vector g to the right of the breaking mark.

The returned signal p – vector h to the left of the breaking mark is

$$h = AXg \quad (1.42)$$

AX – is loop-transmission matrix.

The return ratio matrix is:

$$T(X) = -AX \quad (1.43)$$

The difference between the applied signal g and returned signal h is given by the following formula:

$$g - h = (I_p - AX)g \quad (1.44)$$

The return difference matrix with respect to X is:

$$F(X) = I_p - AX \quad (1.45)$$

In another form using (1.43) and (1.45) we have

$$F(X) = I_p + T(X) \quad (1.46)$$

For the circuit from previous example the return ratio matrix is:

$$T(X) = -AX = \begin{pmatrix} 1.091837 & -2.013463 \\ 145.305252 & 4.855334 \end{pmatrix} \quad (1.47)$$

Similar to the return difference matrix the null return difference matrix can be defined. In fundamental matrix flow graph on Figure 1.9 (b) input vector u is adjusted such that the output vector y is zero:

$$Du + CXg = 0 \quad (1.48)$$

$$u = -D^{-1}CXg \quad (1.49)$$

With such an inputs u and g the return signal is

$$h = Bu + AXg = (-BD^{-1}CX + AX)g \quad (1.50)$$

The null return difference matrix with respect to X is:

$$\hat{F}(X) = g - h = I_p + \hat{T}(X) = I_p - AX + BD^{-1}CX = I_p - \hat{A}X \quad (1.51)$$

The null return ratio matrix $\hat{T}(X)$ is a square matrix:

$$\hat{T}(X) = -AX + BD^{-1}CX = -\hat{A}X \quad (1.52)$$

$$\hat{A} = A - BD^{-1}C \quad (1.53)$$

Example

From the double-transistor amplifier circuit, we define vector y to be v_o is output voltage, and then instead (1.41) we have:

$$y = [v_o] = (-21.992801 \quad -1195.184378) \begin{bmatrix} I_a \\ I_b \end{bmatrix} + (3.897 \cdot 10^{-3}) [v_s] \quad (1.54)$$

From (1.54):

$$C = (-21.992801 \quad -1195.184378), \quad D = (3.897 \cdot 10^{-3})$$

Substituting these into (1.53) with A and B from (1.40) we have:

$$\hat{A} = A - BD^{-1}C = \begin{pmatrix} 175.477 \cdot 10^{-3} & 9749.945 \cdot 10^{-3} \\ -497.116 \cdot 10^{-3} & 481.629 \cdot 10^{-3} \end{pmatrix} \quad (1.55)$$

The null return difference matrix (1.51) with respect to X is:

$$\hat{F}(X) = I_2 - \hat{A}X = \begin{pmatrix} -49.361921 & -2798.234262 \\ 142.672360 & -137.227477 \end{pmatrix} \quad (1.56)$$

1.1.1.3 Useful impedance matrix relationship formulas

For network with single dependent source we have seen the Blackman formula (1.19) that can be rewritten according to [9] chapter 30 as:

$$z(x) = z(0) \frac{f(\text{input short circuit})}{f(\text{input open circuited})} \quad (1.57)$$

$z(x)$ - is an input impedance looking into a terminal pair and x - represents the controlling parameter of a controlled source in a single-loop feedback amplifier. When in (1.57) scalar $f(\cdot)$ denotes the return difference.

A useful relationship between determinants is:

$$\det W(X) = \det W(0) \frac{\det \hat{F}(X)}{\det F(X)} \quad (1.58)$$

A proof of it can be found in [9]. The expression (1.58) can be used to derive an expression of a general Blackman formula. If $w(X)$ denotes the impedance matrix of an n -port network. In this case similar to (1.57) $F(X) = F(\text{input open-circuited})$ and $\hat{F}(X)$ is the return difference matrix with respect to X for the input port-current vector I_S and the output port-voltage vector V under the condition that V is identically zero. Note: In this case output port-voltage vector V is the voltage on the impedance ports, where I_S is the current. We receive $\hat{F}(X) = F(\text{input short-circuited})$ and (1.58) can be written as:

$$\det Z(X) = \det Z(0) \frac{\det F(\text{input short-circuited})}{\det F(\text{input open-circuited})} \quad (1.59)$$

Example

For double-transistor amplifier circuit using (1.46) we have:

$$\det F(X) = \det [I_2 + T(X)] = 304.815200 \quad (1.60)$$

$$\det \hat{F}(X) = 406004.496874 \quad (1.61)$$

If v_s is chosen as an input and v_o as an output:

$$w(X) = \frac{v_o}{v_s} = w(0) \frac{\det \hat{F}(X)}{\det F(X)} = 5.190445 \quad (1.62)$$

Where $w(0) = d_o = 3.897 \cdot 10^{-3}$ is taken from (1.41).

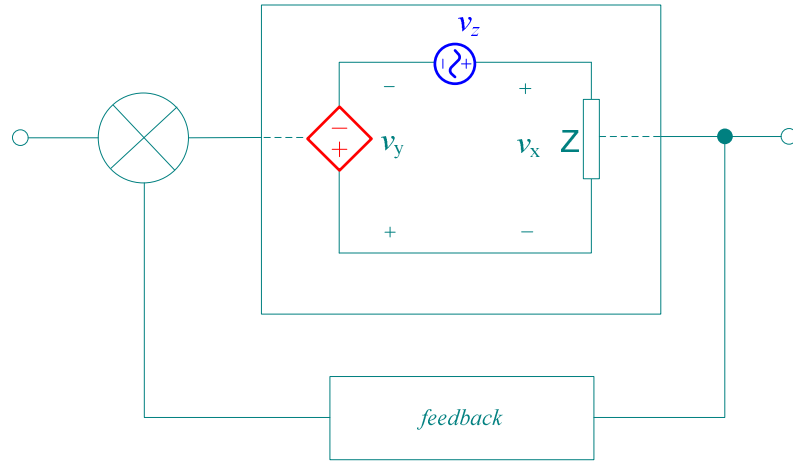
The result is closed loop gain. In a similar manner we can calculate the impedance of an arbitrary port of a network.

1.2 Measurements return ratio in SPICE simulations

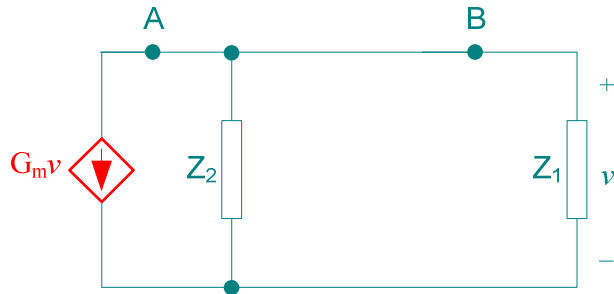
Practical method of measurement of the loop gain as a function of frequency are given by Middlebrook [15]. The method allows making experimental measurements of a loop gain without opening the loop in circuits with one dependent source.

The original circuit is shown in Figure 1.10 (a). The point of interest in the feedback loop at which the driving signal is represented neither by an ideal voltage source nor by an ideal current source is represented in Figure 1.10 (b). The driving signal is represented as Norton equivalent.

Expression for loop gain T can be simply obtained by current injection at point A, see Figure 1.10 (b). However, in the real network, the point A is not accessible. The accessible point B is with an arbitrary impedance ratio z_2 / z_1 . Voltage injection from a non-ideal voltage source with impedance z_v is shown in Figure 1.11 (a). The injection must be done in such a way that $v_z = v_x + v_y$. By measurement of the voltages v_y and v_x we become a ratio $T_v = v_y / v_x$.



(a)



(b)

Figure 1.10: (a) Feedback circuit with the focus on point of interest, in measurement of loop gain by single voltage injection. (b) Original equivalent feedback circuit that was used by Middlebrook in his proof of return ratio formula from successive voltage and current injection.

$$T_v = \left(G_m + \frac{1}{z_1} \right) z_2 = \left(1 + \frac{z_2}{z_1} \right) T + \frac{z_2}{z_1} \quad (1.63)$$

Then a current injection from a non-ideal current source of impedance z_i is performed, that shown on Figure 1.11 (b). The injection signal must satisfy $i_z = i_x + i_y$. By measurement of the currents i_x and i_y we become $T_i = i_y / i_x$.

$$T_i = \left(G_m + \frac{1}{z_2} \right) z_1 = \left(1 + \frac{z_1}{z_2} \right) T + \frac{z_1}{z_2} \quad (1.64)$$

True loop gain T of a circuit from Figure 1.10 (b) is given by:

$$T = G_m(Z_1 \parallel Z_2) \tag{1.65}$$

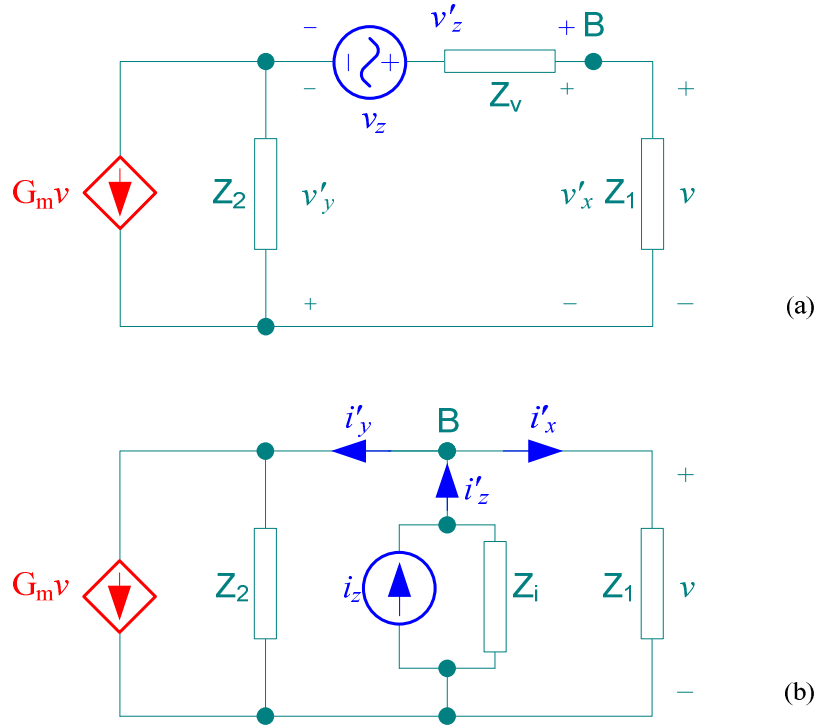


Figure 1.11: Successful (a) voltage and (b) current injection at the accessible point B of arbitrary impedance ratio z_2 / z_1 .

Combining (1.63) and (1.64) the loop gain is:

$$T = \frac{T_v T_i - 1}{2 + T_v + T_i} \tag{1.66}$$

The disadvantage of this method is that it can be used only for ideal feedback model. That means that only for the circuits with reverse one literal loop gain [16]. However, most of electrical circuits with one dependent source have several bilateral

feedback loops. Therefore it is important to widespread this method to such networks.

In his further work Middlebrook describes general feedback theorem for measurement of a closed loop gain and its components [16]. The GFT does not start from a functional model but is developed on a general property of a linear system. According to this property, called the dissection theorem, any “first level” transfer function of a linear system can be dissected into a combination of three “second level” transfer functions. The “second level” transfer functions are generally calculated with circuit simulator and in some cases can not be calculated in analytical way. Therefore this method is out of scope of our work.

1.3 Superposition with dependent sources

In this section we will reexamine the topic of superposition in networks with dependent sources that will be used in this work. The fact is that most widely used circuit books say that we can't deactivate a dependent source when making superposition in a circuit [17]-[21]. That is also learned in most introductory electrical engineering courses.

The proof that superposition of dependent sources can be done analyzing active circuits is published in [22] and [23].

Consider the dependent sources on Figure 1.12 (a) and (b).

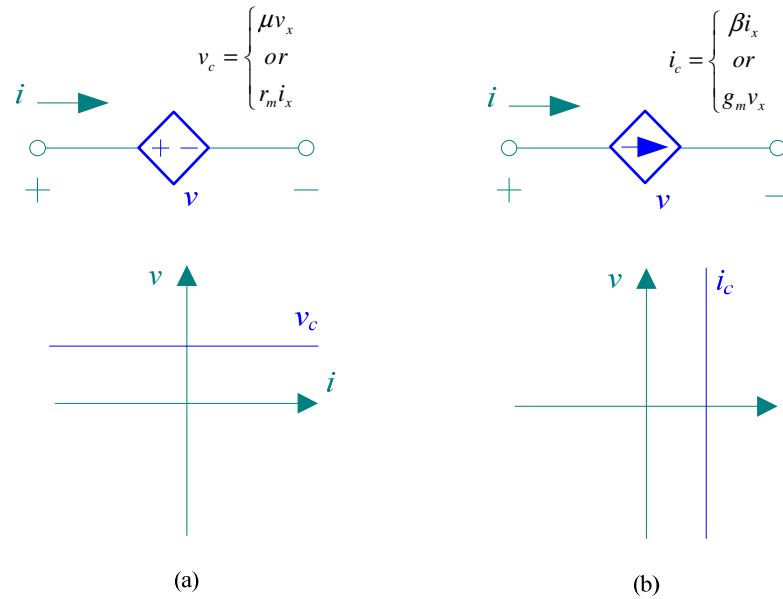


Figure 1.12: (a) Dependent voltage/ current - controlled voltage source $v-i$ characteristics, (b) dependent current/ voltage -controlled current source with their $v-i$ characteristics.

From circuit theory every element is defined completely by its $v-i$ characteristic. We can see from Figure 1.12 that $v-i$ characteristic of any type of dependent source is exactly the same as that of the corresponding type of independent one. Only the value of dependent source depends upon another circuit variable.

We can think that the dependency relation of a dependent source is labeled on it. Then we place a piece of masking tape with the symbol v_c for a dependent voltage source or i_c for a dependent current source over that label. This procedure is called “taping” and temporarily converts a dependent source into an independent one of the same type with unknown value v_c or i_c . Then the circuit is analyzed in a way one would analyze an equivalent circuit with all sources are of the independent type. After writing of circuit equations, we “untape” the dependent sources and expressing their values in terms of the unknowns.

As an example let us analyze the circuit on Figure 1.13 (a). The goal is to find i .

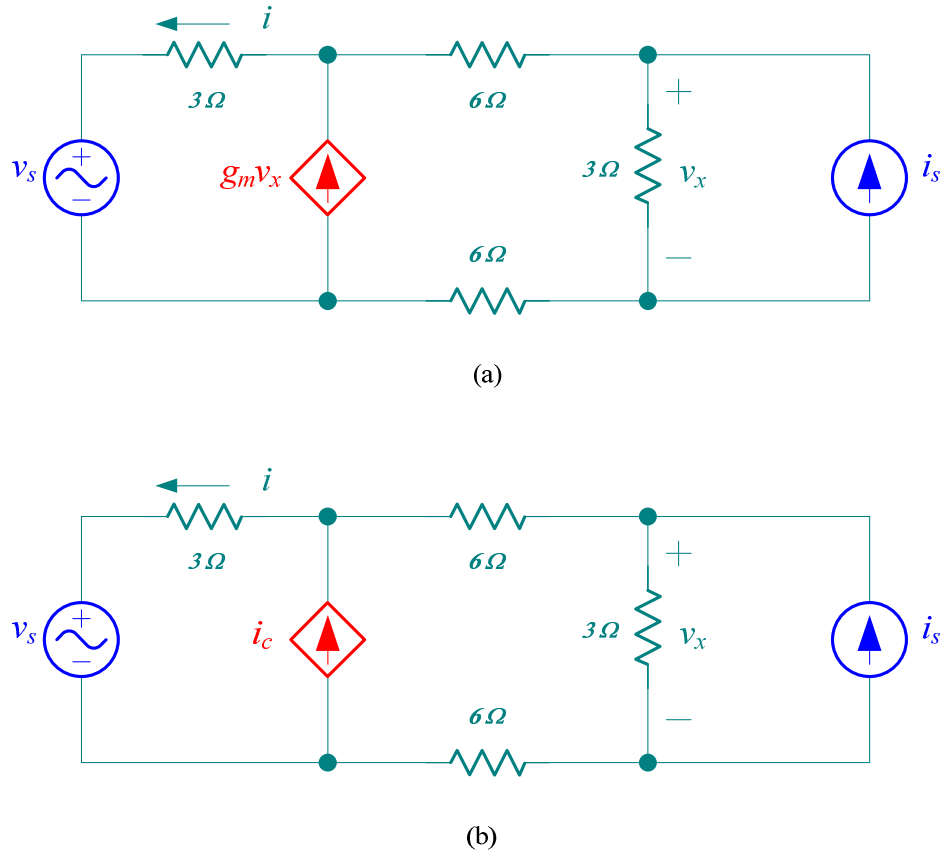


Figure 1.13: (a) active circuit example, (b) example circuit after taping.

Using superposition on all the sources on Figure 1.13 (b) we have

$$v_x = \frac{1}{6}v_s + \frac{5}{2}i_s + \frac{1}{2}i_c \quad (1.67)$$

$$i = -\frac{1}{18}v_s + \frac{1}{6}i_s + \frac{5}{6}i_c \quad (1.68)$$

If we “untape” the dependent source and use (1.67) we have

$$i_c = g_m v_x = \frac{g_m}{6}v_s + \frac{5g_m}{2}i_s + \frac{g_m}{2}i_c \quad (1.69)$$

$$i_c = \frac{g_m}{3(2-g_m)}v_s + \frac{5g_m}{2-g_m}i_s \text{ with } g_m \neq 2 \quad (1.70)$$

If $g_m = 2$ there are infinite number of solutions.

From (1.68) and (1.70)

$$i = \frac{3g_m - 1}{9(2-g_m)}v_s + \frac{12g_m + 1}{3(2-g_m)} \quad (1.71)$$

It can be verified that solving this circuit without superposition by KCL and KVL methods gives the same result.

Another argument for superposition of dependent sources is that when we write a matrix linear equation for the circuit $G\bar{v} = \bar{u}_s$, in the circuit without “taping” Figure 1.13 (a) a dependent source can be seen on the left hand side of matrix equation as a function of arguments of \bar{v} (1.72).

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{6} & 0 \\ -g_m - \frac{1}{6} & \frac{1}{2} & -\frac{1}{3} \\ g_m & -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_s \\ \frac{1}{3}v_s \\ -\frac{1}{3}v_s \end{bmatrix} \quad (1.72)$$

On the other hand, when we have circuit with “taping” Figure 13 (b), we considered the dependent source as an independent with the value i_c and obviously we can apply superposition for such circuits. Then equation $G\bar{v} = \bar{u}_s$ is of the form:

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{6} & 0 \\ -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_s \\ \frac{1}{3}v_s + i_c \\ -\frac{1}{3}v_s - i_c \end{bmatrix} \quad (1.73)$$

When we substitute to the last equation $i_c = g_m v_x = g_m v_1$ and after rearranging we became the same equation as in (1.72). As a conclusion we can make superposition of dependent sources in an active circuit.

1.4 Research Objectives

The aim of the present work is to develop exact analytical and functional models for the closed loop gain of linear feedback networks with multiple dependent sources. Additional goal is to examine the effect of negative feedback on impedances seen from a pair of arbitrary terminals for such networks. Another goal is to extend Middlebrook's formula for simulating return ratio to linear networks with multiple bilateral feedback loops. Such networks are very common among analog circuits. The method allows measurement of return-ratio without opening the loop and changing operation point of a circuit.

1.5 Method

In all proofs in our work we use generic linear circuits. Such circuits can have bilateral feedback loops and in this way making our proofs valid for general linear networks.

We use superposition of dependent sources, described in this chapter, and dissection throughout network analysis in this work. All those afford us to analyze

active circuits in a more efficient way and to receive general closed loop gain formula as a function of return ratios.

The classical Bode procedure of finding return ratio is modified and extended by introducing of cross return ratios to much wider class of circuits containing multiple transistors.

In order to check the above theory experimentally several circuits were simulated in Spice. Measurements of components of closed loop function of these circuits were performed. The circuits with bilateral feedback loops were also simulated in Spice in order to illustrate the accuracy of finding a return ratio by applying Middlebrook's method.

Chapter 2

Modeling Linear Feedback Networks with Multiple Dependent Sources

2.1 Introduction

In this section we suggest a new approach to the description of feedback in linear networks with multiple dependent sources, which is intuitively simple and comprehensible. It involves no approximations and is applicable to linear feedback networks with any number of dependent sources and any number of feedback loops. Any linear feedback network can easily be simulated however the obtained solution provides no insight into the network functional structure. The suggested approach gives parametric equation for closed loop gain function that provides better intuitive insight into feedback network operation, allows building of functional model of a system and applying control theory tools to analyze the stability of it. Because the closed loop function is expressed through the circuit parameters we can see from it what elements of a network influence the stability of it.

Unfortunately, existing literature [24]-[33] suggests exact analytical or functional modeling only for linear feedback networks with a single dependent source. In some cases, feedback networks with two or more dependent sources can be approximated

by models based on a single dependent source. It is not always clear, however, how accurate this approximation is.

The proposed approach is based on the 'return ratio' concept [1] and superposition of depended sources that was proved in the previous section.

With the help of superposition we can easily find all the partial open-loop gains and to translate them into the closed loop gain.

Return ratios, T_i describe the contribution of dependent sources to their own control terminals. To consider contributions of each dependent source to the control terminals of other dependent sources in a network, we introduce cross return ratios $T_{ij}, i \neq j$. This allows us to extend the canonical Body feedback model [10] to linear networks with multiple dependent sources.

Moreover, we show that the return ratios T_i and cross return ratios T_{ij} , which can be written as a return ratio matrix and can be combined into a generalized return ratio T_Σ . With the help of generalized return ratio we can analyze the stability of the network.

Firstly we revisit the modeling of linear feedback networks with a single dependent source, and then develop models for linear feedback networks with two and three dependent sources. Following the proposed approach, we extend the developed models to feedback networks with multiple dependent sources.

2.2 Feedback networks with a single dependent source

Let us consider a generic linear feedback network with a single dependent source, as shown in Figure 2.1 (a).

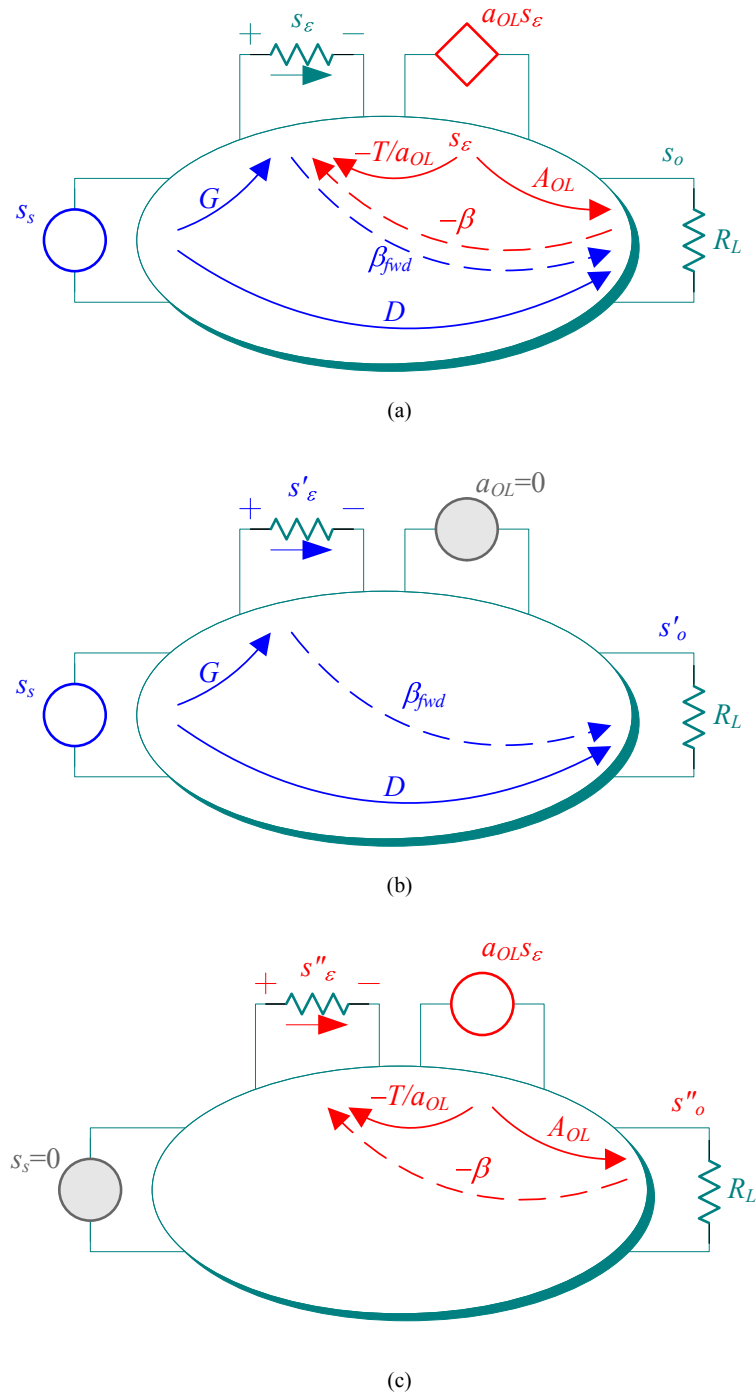


Figure 2.1: Finding the partial open-loop gains of a generic single-transistor circuit by applying superposition. (a) Original network. (b) The network, where the independent source is the only active one. (c) The network, where the equivalent independent source $a_{OL} s_{\epsilon}$, replacing the dependent source, is the only active one.

To define the partial open-loop gains we use the superposition of dependent sources, the method that was proved in the previous chapter. We first find the contributions of the independent source s_s , to the control terminal of the dependent source s_ε and output s_o signals. To do this, we suppress the dependent source as shown in Figure 2.1 (b). The contributions of the independent source to the control and output signals are s'_ε and s'_o respectively.

To find the contributions of the dependent source, $a_{OL}s_\varepsilon$ we suppress the independent source and force the dependent source to have its value from original circuit Figure 2.1 (a) the resulting circuit is on Figure 2.1 (c). The contributions of the dependent source to the control and output signals are $-s''_\varepsilon$ and s''_o respectively.

We note, that the signals in Figure 2.1 with the prime symbol correspond to the case, where the independent source is the only active one, and the signals with the double prime symbol correspond to the case, where the dependent source is the only active one.

The return ratio can be found more easily, then in the procedure that was described in chapter 1.1.1 and in [10]. The original procedure of finding return ratio was described in introduction chapter and is depicted on Figure 1.3. The return ratio is:

$$T = -\frac{s_r}{s_t} \quad (2.1)$$

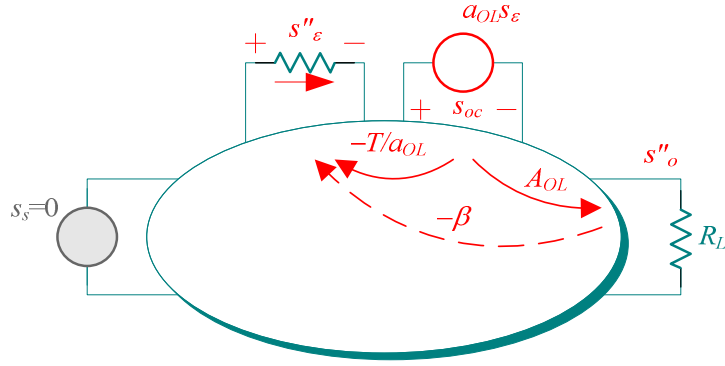


Figure 2.2: Generic single-transistor circuit for finding of return ratio.

Let us investigate the circuit on Figure 2.2, with the independent source set to zero, and the dependent source is substituted with the independent one with the value of dependent one in the original circuit. We define a transfer function H_o to

be $H_o = -\frac{s''_{\epsilon}}{s_{oc}}$. On the other hand from the Figure 1.3: $H_o = -\frac{s_{\epsilon}}{s_t}$

And from Figure 1.3 (a):

$$s_r = a_{OL}s_{\epsilon} \quad \text{and} \quad s_{\epsilon} = -H_o s_t, \quad s_r = -a_{OL}H_o s_t \tag{2.2}$$

On the other side from Figure 2.2:

$$s''_{\epsilon} = -H_o a_{OL} s_{\epsilon} \tag{2.3}$$

Combining (2.2) and (2.3) we became that

$$T = -\frac{s_r}{s_t} = -\frac{s''_{\epsilon}}{s_{\epsilon}} \tag{2.4}$$

We now can define the following open-loop partial gains:

$$\text{input transition:} \quad G \equiv \frac{s'_\varepsilon}{s_s} \quad (2.5)$$

$$\text{feed forward transmission:} \quad \beta_{fwd} \equiv \frac{s'_o}{s'_\varepsilon} \quad (2.6)$$

$$\text{direct transmission:} \quad D \equiv G\beta_{fwd} \equiv \frac{s'_o}{s_s} \quad (2.7)$$

$$\text{open-loop gain:} \quad A_{OL} \equiv \frac{s''_o}{s_\varepsilon} \quad (2.8)$$

$$\text{feedback transition:} \quad \beta \equiv -\frac{s''_\varepsilon}{s''_o} \quad (2.9)$$

$$\text{return ratio:} \quad T \equiv A_{OL}\beta \equiv -\frac{s''_\varepsilon}{s_\varepsilon} \quad (2.10)$$

To find the closed-loop gain, with the help of superposition of sources, we combine all the contributions to obtain the original control and output signals:

$$s_\varepsilon = s'_\varepsilon + s''_\varepsilon = s_s G - s_\varepsilon T \quad (2.11)$$

$$s_o = s'_o + s''_o = s_s G \beta_{fwd} + s_\varepsilon A_{OL} \quad (2.12)$$

From (2.11) and (2.12) we can find the closed loop:

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{s_\varepsilon A_{OL} + s_s G \beta_{fwd}}{s_s} = \frac{s_\varepsilon A_{OL}}{s_s} + G \beta_{fwd} = \frac{s_\varepsilon A_{OL}}{s_\varepsilon (1+T)} + D \quad (2.13)$$

$$A_{CL} = G \frac{A_{OL}}{1+T} + D \quad (2.14)$$

Based on (2.14) and Figure 2.1, a functional model of the feedback can be developed (see Figure 2.3). It is important to note that the functional block \mathcal{B} , combining the

feedback and feed forward transmissions, represents a bidirectional feedback network.

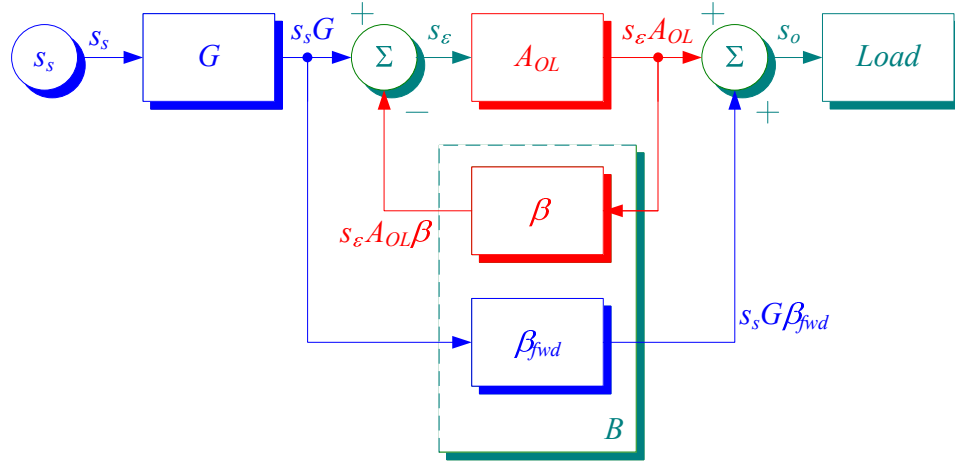


Figure 2.3: Functional model of the generic single-transistor circuit of Figure 2.1. The block B represents a bidirectional network.

2.3 Feedback networks with multiple dependent sources

1.1.4 Feedback networks with two dependent sources

A generic double-transistor electronic circuit is shown in Figure 2.4. The contributions of all its sources to the original control and output signals can be found by applying superposition:

$$s_{\epsilon 1} = s_s G_1 - s_{\epsilon 1} T_1 - s_{\epsilon 2} T_{21} \quad (2.15)$$

$$s_{\epsilon 2} = s_s G_2 - s_{\epsilon 2} T_2 - s_{\epsilon 1} T_{12} \quad (2.16)$$

$$s_o = s_{\epsilon 1} A_{OL1} + s_{\epsilon 2} A_{OL2} + s_s D \quad (2.17)$$

Where

$$A_{OL1} \equiv \frac{s''_o}{s_{\epsilon 1}}, \quad A_{OL2} \equiv \frac{s'''_o}{s_{\epsilon 2}}, \quad G_1 \equiv \frac{s'_{\epsilon 1}}{s_s}, \quad G_2 \equiv \frac{s'_{\epsilon 2}}{s_s}, \quad T_1 \equiv -\frac{s''_{\epsilon 1}}{s_{\epsilon 1}}, \quad T_2 \equiv -\frac{s'''_{\epsilon 2}}{s_{\epsilon 2}},$$

$$T_{12} \equiv -\frac{s''_{\epsilon 2}}{s_{\epsilon 1}}, \quad T_{21} \equiv -\frac{s''_{\epsilon 1}}{s_{\epsilon 2}}.$$

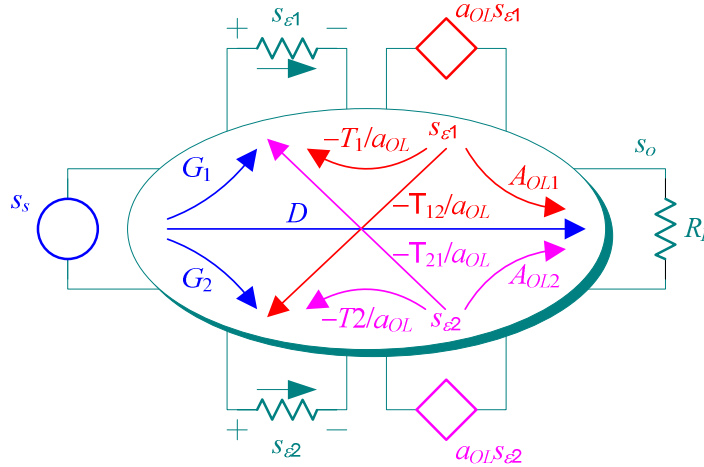


Figure 2.4: Defining the partial open-loop gains for a generic feedback network with two dependent sources.

Note that the signals with the double and triple prime symbol correspond to the case, where the only active sources are $a_{OL1}s_{\epsilon 1}$ and $a_{OL2}s_{\epsilon 2}$ ones, respectively.

Equations (2.15) and (2.16) can be solved for the independent source value:

$$s_s = \frac{1 + T_1 + T_2 + T_1 T_2 - T_{12} T_{21}}{G_1 + G_1 T_2 - G_2 T_{21}} s_{\epsilon 1} \quad (2.18)$$

$$s_s = \frac{1 + T_1 + T_2 + T_1 T_2 - T_{12} T_{21}}{G_2 + G_2 T_1 - G_1 T_{12}} s_{\epsilon 2} \quad (2.19)$$

Considering (2.17) - (2.19), the closed loop gain can be found as follows:

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{A_{OL1}l_1}{1+T_\Sigma} + \frac{A_{OL2}l_2}{1+T_\Sigma} + D \quad (2.20)$$

where

$$l_1 = G_1 + G_1T_2 - G_2T_{21} \quad (2.21)$$

$$l_2 = G_2 + G_2T_1 - G_1T_{12} \quad (2.22)$$

$$T_\Sigma = T_1 + T_2 + T_1T_2 - T_{12}T_{21} \quad (2.23)$$

Equations (2.20) - (2.23) can be represented as the feedback functional model shown in Figure 2.5.

1.1.5 Feedback networks with three dependent sources

The above analysis of linear feedback networks with two dependent sources can easily be extended to feedback networks with multiple dependent sources. For example, for a generic feedback network with three dependent sources, the control and output signals can be found as follows:

$$s_{\varepsilon 1} = s_s G_1 - s_{\varepsilon 1} T_1 - s_{\varepsilon 2} T_{21} - s_{\varepsilon 3} T_{31} \quad (2.24)$$

$$s_{\varepsilon 2} = s_s G_2 - s_{\varepsilon 2} T_2 - s_{\varepsilon 1} T_{12} - s_{\varepsilon 3} T_{32} \quad (2.25)$$

$$s_{\varepsilon 3} = s_s G_3 - s_{\varepsilon 3} T_3 - s_{\varepsilon 2} T_{23} - s_{\varepsilon 1} T_{13} \quad (2.26)$$

$$s_o = s_s D + s_{\varepsilon 1} A_{OL1} + s_{\varepsilon 2} A_{OL2} + s_{\varepsilon 3} A_{OL3} \quad (2.27)$$

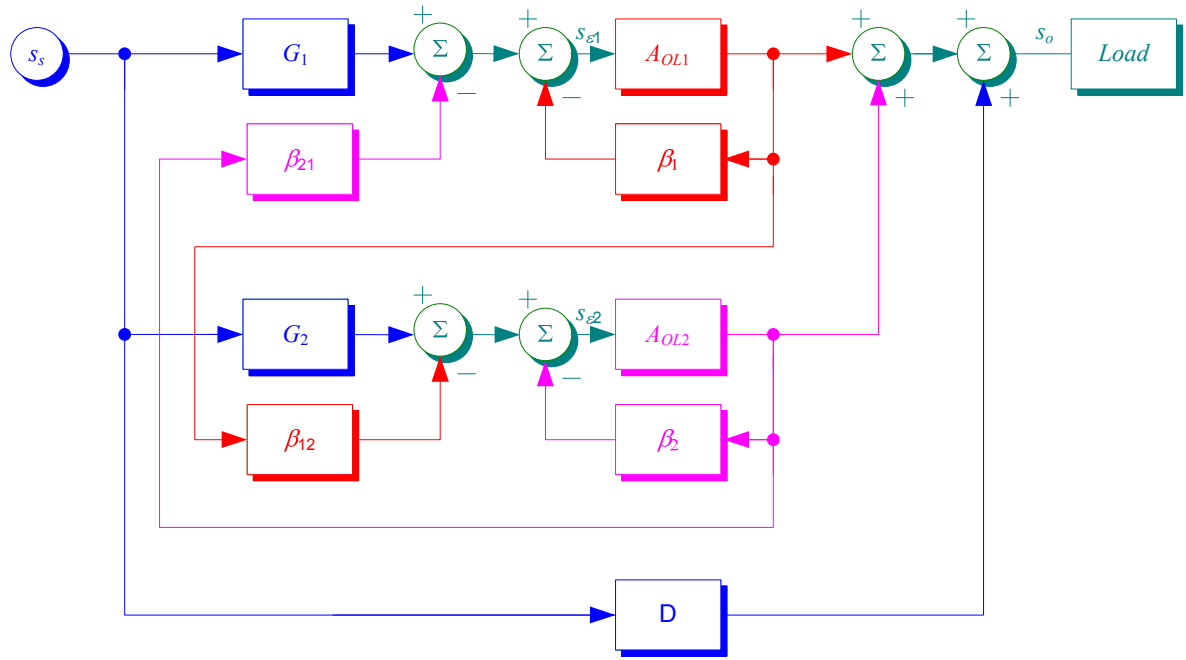


Figure 2.5: Functional model of a feedback network with two dependent sources. Note that $A_{OL1}\beta_{12} = T_{12}$ and $A_{OL2}\beta_{21} = T_{21}$.

From (2.24) - (2.27), the closed loop gain can be found as follows:

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{A_{OL1}I_1}{1+T_\Sigma} + \frac{A_{OL2}I_2}{1+T_\Sigma} + \frac{A_{OL3}I_3}{1+T_\Sigma} + D \quad (2.28)$$

Where

$$\begin{aligned} I_1 = & G_1 [1 + T_2(1 + T_3) + T_3 - T_{23}T_{32}] \\ & + G_2 [T_{23}T_{31} - T_{21}(1 + T_3)] \\ & + G_3 [T_{21}T_{32} - T_{31}(1 + T_2)] \end{aligned} \quad (2.29)$$

$$\begin{aligned} I_2 = & G_2 [1 + T_1(1 + T_3) + T_3 - T_{13}T_{31}] \\ & + G_1 [T_{13}T_{32} - T_{12}(1 + T_3)] \\ & + G_3 [T_{12}T_{31} - T_{32}(1 + T_1)] \end{aligned} \quad (2.30)$$

$$\begin{aligned} I_3 = & G_3 [1 + T_1(1 + T_2) + T_2 - T_{12}T_{21}] \\ & + G_1 [T_{12}T_{23} - T_{13}(1 + T_2)] \\ & + G_2 [T_{13}T_{21} - T_{23}(1 + T_1)] \end{aligned} \quad (2.31)$$

$$\begin{aligned} T_\Sigma = & T_1(1 - T_{23}T_{32}) + T_2(1 - T_{13}T_{31}) + T_3(1 - T_{12}T_{21}) \\ & + T_1T_2 + T_1T_3 + T_2T_3 \\ & - T_{12}T_{21} - T_{13}T_{31} - T_{23}T_{32} \\ & + T_1T_2T_3 + T_{12}T_{23}T_{31} + T_{13}T_{21}T_{32} \end{aligned} \quad (2.32)$$

Equations (2.28) - (2.32) can be represented as a feedback functional model shown in Figure 2.6.

1.1.6 General formula for closed-loop gain in feedback networks with multiple dependent sources

We can derive general formula for a closed-loop gain for networks with any number of dependent sources. A generic multiple-dependent sources network is shown in Figure 2.7. For simplicity the return-ratios are shown only for the second dependent source.

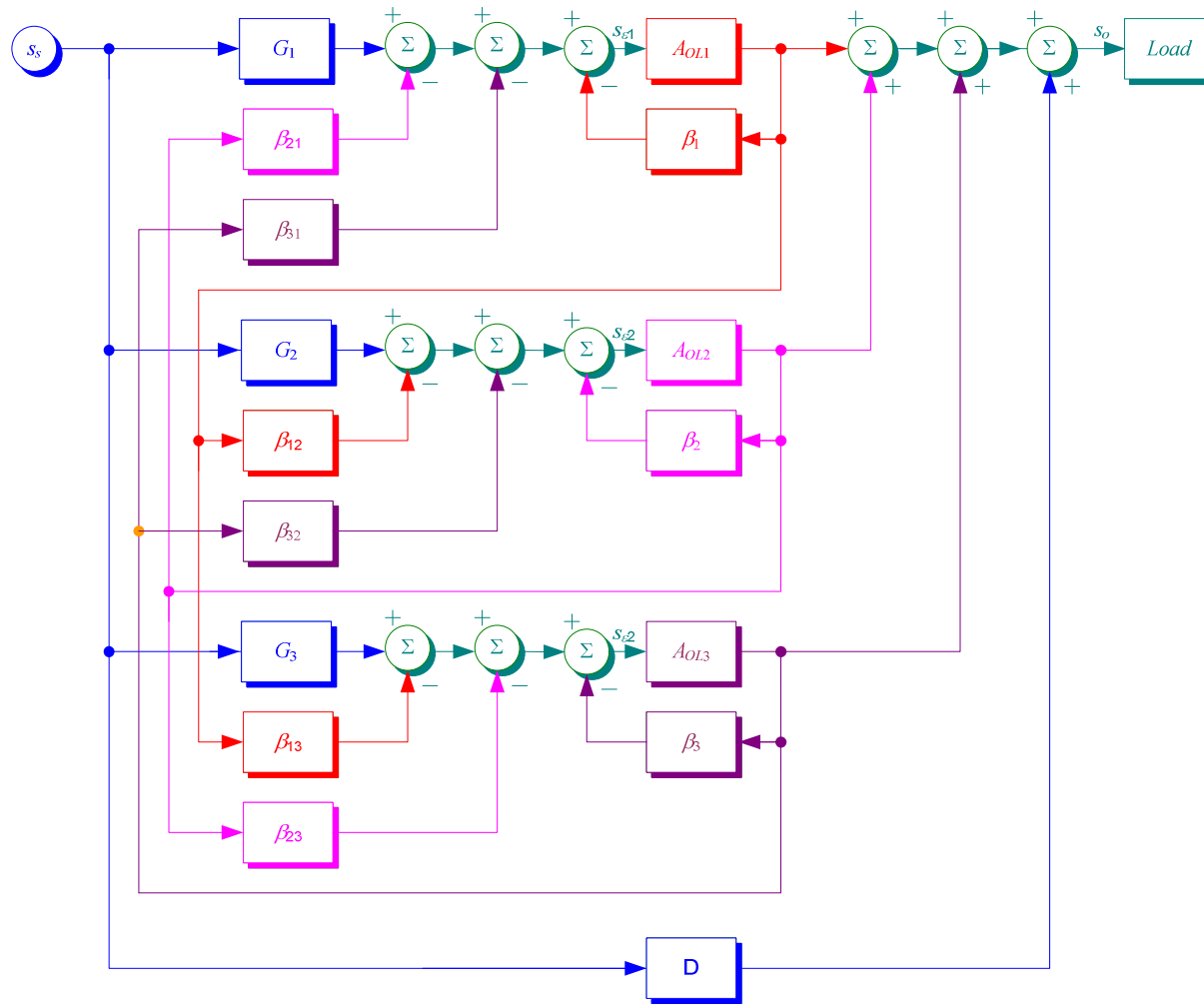


Figure 2.6: Functional model of a feedback network with three dependent sources.

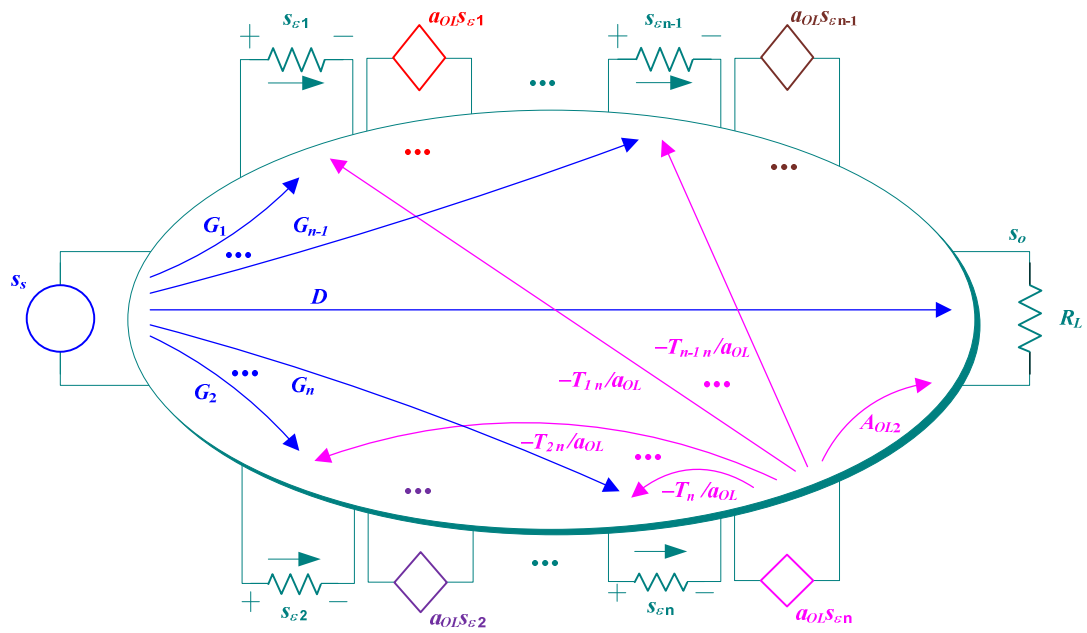


Figure 2.7: Defining the partial open-loop gains for a generic feedback network with multiple dependent sources.

In this section we define cross return ratios as:

$$T_{km} \equiv -\frac{S_{\varepsilon k} \Big|_{s_s=0, a_1=0 \dots a_m \neq 0 \dots a_n=0}}{S_m}; \quad 1 \leq k, m \leq n \quad (2.33)$$

Matrix equations, similar to (1.24) - (1.26) that we used in chapter 1, for the generic network in Figure 2.7 are:

$$\Theta = X\Phi \quad (2.34)$$

$$\Phi = A\Theta + Bu \quad (2.35)$$

$$y = C\Theta + Du \quad (2.36)$$

Θ is a controlled vector, consists from the values of the dependent sources.

Φ is a controlling vector, consists from the values of the controlling terminals of the dependent sources.

We will use the technique of superposition of the dependent sources, that was described in introduction, in order to find the elements of the matrices A , B , C and D .

$$\Theta = \begin{bmatrix} a_{OL1} S_{\varepsilon 1} \\ \vdots \\ a_{OLn} S_{\varepsilon n} \end{bmatrix} = \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_n \end{bmatrix} \quad (2.37)$$

θ_1, θ_n - is a "taping" of dependent sources.

$$\Phi = \begin{bmatrix} s_{\varepsilon 1} \\ \vdots \\ s_{\varepsilon n} \end{bmatrix} \quad (2.38)$$

$$y = [v_o] \quad (2.39)$$

$$u = [v_s] \quad (2.40)$$

A , B , C and D are transfer-function matrices of dimensions $n \times n$, $n \times 1$, $1 \times n$ and 1×1 respectively. Because of the fact that we are interested in transfer function for simplicity we can set all the values of the gain of the dependent sources $a_{OL} = 1$. We have n dependent sources, so we need to examine $n+1$ circuits. We denote with the prime symbol signals in a circuit with s_s active and all the dependent sources are set to zero. For integer k such that $1 \leq k \leq n$ we denote with ' k ' symbol signals in a circuit with only one dependent source $\theta_k = a_{OLk} s_{\varepsilon k}$ active and all the other sources are set to zero. Then we have:

- Prime-circuit: s_s - is active, all the dependent sources are set to zero.

$$B = \begin{bmatrix} \frac{s'_{\varepsilon 1}}{s_s} \\ s_s \\ \vdots \\ \frac{s'_{\varepsilon n}}{s_s} \\ s_s \end{bmatrix} = \begin{bmatrix} G_1 \\ \vdots \\ G_n \end{bmatrix} \quad (2.41)$$

$$D = \frac{s'_o}{s_s} = [D] \quad (2.42)$$

- '1' - circuit: θ_1 - is active, all other sources are set to zero.

We obtain the 1-st column of A matrix and the 1-st element of C matrix.

$$\begin{aligned}
 a_{11} &= \frac{s_{\varepsilon 1}^{\prime 1}}{\theta_1} = \frac{s_{\varepsilon 1}^{\prime 1}}{s_{\varepsilon 1}} = -T_1 \\
 a_{21} &= \frac{s_{\varepsilon 2}^{\prime 1}}{\theta_1} = \frac{s_{\varepsilon 2}^{\prime 1}}{s_{\varepsilon 1}} = -T_{21} \\
 &\quad \vdots \\
 a_{n1} &= \frac{s_{\varepsilon n}^{\prime 1}}{\theta_1} = \frac{s_{\varepsilon n}^{\prime 1}}{s_{\varepsilon 1}} = -T_{n1} \\
 \\
 c_1 &= \frac{s_o^{\prime 1}}{\theta_1} = \frac{s_o^{\prime 1}}{s_{\varepsilon 1}} = A_{OL1}
 \end{aligned}$$

- '2' - circuit: θ_2 - is active, all other sources are set to zero.

We obtain the 2-nd column of A matrix and the 2-nd element of C . matrix.

$$\begin{aligned}
 a_{12} &= \frac{s_{\varepsilon 1}^{\prime 2}}{\theta_2} = \frac{s_{\varepsilon 1}^{\prime 2}}{s_{\varepsilon 2}} = -T_{12} \\
 a_{22} &= \frac{s_{\varepsilon 2}^{\prime 2}}{\theta_2} = \frac{s_{\varepsilon 2}^{\prime 2}}{s_{\varepsilon 2}} = -T_2 \\
 &\quad \vdots \\
 a_{n2} &= \frac{s_{\varepsilon n}^{\prime 2}}{\theta_2} = \frac{s_{\varepsilon n}^{\prime 2}}{s_{\varepsilon 2}} = -T_{n2} \\
 \\
 c_2 &= \frac{s_o^{\prime 2}}{\theta_2} = \frac{s_o^{\prime 2}}{s_{\varepsilon 2}} = A_{OL2}
 \end{aligned}$$

- By analogy we can find other columns of matrices A and C from circuits with only one source of $\theta_3, \dots, \theta_{n-1}$ active.

- 'n' - circuit: θ_n - is active, all other sources are set to zero.

We obtain the n column of A matrix and the n element of C matrix.

$$\begin{aligned} a_{1n} &= \frac{s_{\epsilon 1}^{\prime n}}{\theta_n} = \frac{s_{\epsilon 1}^{\prime n}}{s_{\epsilon n}} = -T_{1n} \\ a_{2n} &= \frac{s_{\epsilon 2}^{\prime n}}{\theta_n} = \frac{s_{\epsilon 2}^{\prime n}}{s_{\epsilon n}} = -T_{2n} \\ &\vdots \\ a_{nn} &= \frac{s_{\epsilon n}^{\prime n}}{\theta_n} = \frac{s_{\epsilon n}^{\prime n}}{s_{\epsilon n}} = -T_n \end{aligned}$$

$$c_n = \frac{s_o^{\prime n}}{\theta_n} = \frac{s_o^{\prime n}}{s_{\epsilon n}} = A_{OLn}$$

We can summarize the following result in the matrix form:

$$A = \begin{bmatrix} -T_1 & -T_{12} & \cdots & -T_{1n} \\ -T_{21} & -T_2 & \cdots & -T_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -T_{n1} & -T_{n2} & \cdots & -T_n \end{bmatrix} \quad (2.43)$$

$$C = [A_{OL1} \quad \cdots \quad A_{OLn}] \quad (2.44)$$

$X = I_n$ - is an identity matrix

From (1.43) and (2.43) the return ratio matrix is:

$$T(X) = -AX = \begin{bmatrix} T_1 & T_{12} & \cdots & T_{1n} \\ T_{21} & T_2 & \cdots & T_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n1} & T_{n2} & \cdots & T_n \end{bmatrix} \quad (2.45)$$

We have seen in chapter 1, that the solution of matrix equations (2.34) - (2.36) is $W(X) = D + C(X^{-1} - A)^{-1}B$ and because of (2.39) it is equal to the closed loop gain:

$$\begin{aligned}
 A_{cl} &= \frac{v_o}{v_s} \\
 &= D + [A_{OL1} \quad \dots \quad A_{OLn}] \cdot \begin{bmatrix} 1+T_1 & T_{12} & \dots & T_{1n} \\ T_{21} & 1+T_2 & & T_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n1} & T_{n2} & \dots & 1+T_n \end{bmatrix}^{-1} \cdot \begin{bmatrix} G_1 \\ \vdots \\ G_n \end{bmatrix} \\
 &= D + [A_{OL1} \quad \dots \quad A_{OLn}] \cdot F(I_n)^{-1} \cdot \begin{bmatrix} G_1 \\ \vdots \\ G_n \end{bmatrix}
 \end{aligned} \tag{2.46}$$

$$A_{cl} = D + [A_{OL1} \quad \dots \quad A_{OLn}] \cdot \frac{adjF(I_n)}{detF(I_n)} \cdot \begin{bmatrix} G_1 \\ \vdots \\ G_n \end{bmatrix} \tag{2.47}$$

$F(I_n)$ - is a return difference matrix.

The last equation (2.47) is a general formula for a closed-loop gain in a network with n dependent sources. We have used an identity that for a nonsingular square matrix A , we have $A^{-1} = \frac{1}{detA} adj(A)$. From this equation we come to an important conclusion, in order to analyze the stability of a network we need to calculate the determinant of return difference matrix, which is composed from return-ratio factors.

Example A. A Single-Transistor Example Circuit

Let us solve an elementary common-collector (CC) amplifier (see Figure 2.8) for the closed-loop gain. From Figure 2.8 (c), we obtain:

$$G \equiv \frac{s'_\varepsilon}{s_s} = \frac{i'_b}{v_s} = \frac{1}{h_{ie} + R_E \parallel r_o} \quad (2.48)$$

$$\beta_{fwd} \equiv \frac{s'_o}{s'_\varepsilon} = \frac{v'_o}{i'_b} = R_E \parallel r_o \quad (2.49)$$

From Figure 2.8 (d),

$$A_{OL} \equiv \frac{s''_o}{s_\varepsilon} = \frac{v''_o}{i_b} = h_{fe} (h_{ie} \parallel R_E \parallel r_o) \quad (2.50)$$

$$T \equiv A_{OL}\beta \equiv -\frac{s''_\varepsilon}{s_\varepsilon} = -\frac{i''_b}{i_b} = h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}} \quad (2.51)$$

Considering (2.48) - (2.51) and (2.14), we finally obtain

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{v_o}{v_s} = \frac{1}{h_{ie} + R_E \parallel r_o} \frac{h_{fe} (h_{ie} \parallel R_E \parallel r_o)}{1 + h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}}} + \frac{1}{h_{ie} + R_E \parallel r_o} (R_E \parallel r_o) \quad (2.52)$$

The obtained closed-loop gain can be reduced to the well-known value for the elementary CC amplifier:

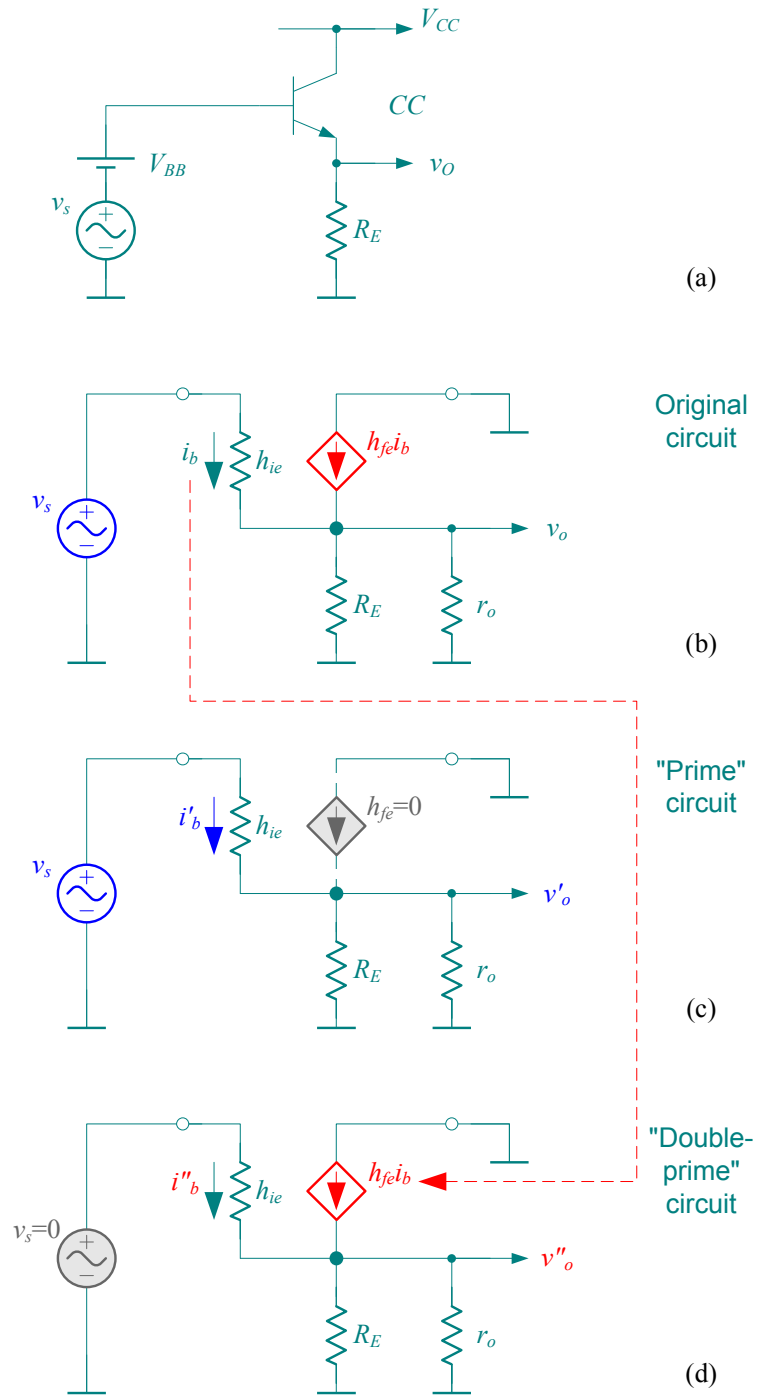


Figure 2.8: Finding the partial open-loop gains of a single-transistor circuit. (a) Example circuit. (b) Original equivalent small-signal circuit. (c) The “prime” circuit, where the independent source is the only active one. (d) The “double-prime” circuit, where the dependent source is the only active one. Note that the dependent source in (d) is controlled by the \mathbf{s}_ε signal of the original circuit (b).

$$\begin{aligned}
A_{CL} &= \frac{1}{h_{ie} + R_E \parallel r_o} \frac{h_{fe} \frac{h_{ie} (R_E \parallel r_o)}{h_{ie} + (R_E \parallel r_o)}}{1 + h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + h_{ie}}} + \frac{R_E \parallel r_o}{h_{ie} + R_E \parallel r_o} \\
&= \frac{R_E \parallel r_o}{h_{ie} + R_E \parallel r_o} \times \left[\frac{h_{fe} h_{ie}}{h_{ie} + R_E \parallel r_o + h_{fe} (R_E \parallel r_o)} + 1 \right] \\
&= \frac{R_E \parallel r_o}{h_{ie} + R_E \parallel r_o} \times \left[\frac{h_{fe} (1 + h_{fe}) r_e}{(1 + h_{fe}) r_e + (1 + h_{fe}) (R_E \parallel r_o)} + 1 \right] \\
&= \frac{R_E \parallel r_o}{h_{ie} + R_E \parallel r_o} \frac{h_{fe} r_e + r_e + (R_E \parallel r_o)}{(1 + h_{fe}) r_e + (1 + h_{fe}) (R_E \parallel r_o)} \\
&= \frac{R_E \parallel r_o}{r_e + R_E \parallel r_o}
\end{aligned} \tag{2.53}$$

Example B. A Double-Transistor Example Circuit

Let us solve a double-transistor amplifier (see Figure 2.9) for the closed-loop gain. From Figure 2.10 (b), we obtain:

$$\begin{aligned}
G_1 &\equiv \frac{s'_e1}{s_s} = \frac{i'_b1}{v_s} \\
&= \frac{1}{R_B + h_{ie1} \parallel \left[R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1}) \right]} \\
&\quad \times \frac{R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1})}{R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1}) + h_{ie1}}
\end{aligned} \tag{2.54}$$

$$\begin{aligned}
G_2 &\equiv \frac{s'_e2}{s_s} = \frac{i'_b2}{v_s} \\
&= \frac{1}{R_B + h_{ie1} \parallel \left[R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1}) \right]} \\
&\quad \times \frac{h_{ie1}}{R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1}) + h_{ie1}} \\
&\quad \times \frac{R_{E2} \parallel (r_{o2} + R_{C2})}{R_{E2} \parallel (r_{o2} + R_{C2}) + h_{ie2} + r_{o1}}
\end{aligned} \tag{2.55}$$

$$\begin{aligned}
D &\equiv \frac{s'_o}{s_s} = \frac{v'_o}{v_s} \\
&= \frac{1}{R_B + h_{ie1} \parallel [R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1})]} \\
&\quad \times \frac{h_{ie1}}{R_f + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (h_{ie2} + r_{o1}) + h_{ie1}} \\
&\quad \times \frac{R_{E2} \parallel (h_{ie2} + r_{o1})}{R_{E2} \parallel (h_{ie2} + r_{o1}) + r_{o2} + R_{C2}} R_{C2}
\end{aligned} \tag{2.56}$$

From Figure 2.10 (d),

$$\begin{aligned}
A_{OL1} &\equiv \frac{s''_o}{s_{\varepsilon 1}} = \frac{v''_o}{i_{b1}} \\
&= -\frac{h_{fe1} r_{o1}}{r_{o1} + h_{ie2} + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (R_f + R_B \parallel h_{ie1})} \\
&\quad \times \frac{R_{E2} \parallel (R_f + R_B \parallel h_{ie1})}{R_{E2} \parallel (R_f + R_B \parallel h_{ie1}) + r_{o2} + R_{C2}} R_{C2}
\end{aligned} \tag{2.57}$$

$$\begin{aligned}
T_1 &\equiv -\frac{s''_{\varepsilon 1}}{s_{\varepsilon 1}} = -\frac{i''_{b1}}{i_{b1}} \\
&= \frac{h_{fe1} r_{o1}}{r_{o1} + h_{ie2} + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (R_f + R_B \parallel h_{ie1})} \\
&\quad \times \frac{R_{E2} \parallel (r_{o2} + R_{C2})}{R_{E2} \parallel (r_{o2} + R_{C2}) + R_f + R_B \parallel h_{ie1}} \frac{R_B}{R_B + h_{ie1}}
\end{aligned} \tag{2.58}$$

$$\begin{aligned}
T_{21} &\equiv -\frac{s''_{\varepsilon 2}}{s_{\varepsilon 1}} = -\frac{i''_{b2}}{i_{b1}} \\
&= \frac{h_{fe1} r_{o1}}{r_{o1} + h_{ie2} + R_{E2} \parallel (r_{o2} + R_{C2}) \parallel (R_f + R_B \parallel h_{ie1})}
\end{aligned} \tag{2.59}$$

From Figure 2.10 (c),

$$\begin{aligned}
A_{OL2} &\equiv \frac{s'''_o}{s_{\varepsilon 2}} = \frac{v'''_o}{i_{b2}} \\
&= -\frac{h_{fe2} r_{o2} R_{C2}}{r_{o2} + R_{C2} + R_{E2} \parallel (R_f + R_B \parallel h_{ie1}) \parallel (h_{ie2} + r_{o1})}
\end{aligned} \tag{2.60}$$

$$\begin{aligned}
 T_2 &\equiv -\frac{s'''\epsilon_2}{s_{\epsilon 2}} = -\frac{i'''\epsilon_2}{i_{b2}} \\
 &= -\frac{h_{fe2}r_{o2}}{r_{o2} + R_{C2} + R_{E2} \parallel (R_f + R_B \parallel h_{ie1}) \parallel (h_{ie2} + r_{o1})} \\
 &\quad \times \frac{R_{E2} \parallel (R_f + R_B \parallel h_{ie1})}{R_{E2} \parallel (R_f + R_B \parallel h_{ie1}) + h_{ie2} + r_{o1}}
 \end{aligned} \tag{2.61}$$

$$\begin{aligned}
 T_{12} &\equiv -\frac{s'''\epsilon_1}{s_{\epsilon 2}} = -\frac{i'''\epsilon_1}{i_{b2}} \\
 &= -\frac{h_{fe2}r_{o2}}{r_{o2} + R_{C2} + R_{E2} \parallel (R_f + R_B \parallel h_{ie1}) \parallel (h_{ie2} + r_{o1})} \\
 &\quad \times \frac{R_{E2} \parallel (h_{ie2} + r_{o1})}{R_{E2} \parallel (h_{ie2} + r_{o1}) + R_f + R_B \parallel h_{ie1}} \frac{R_B}{R_B + h_{ie1}}
 \end{aligned} \tag{2.62}$$

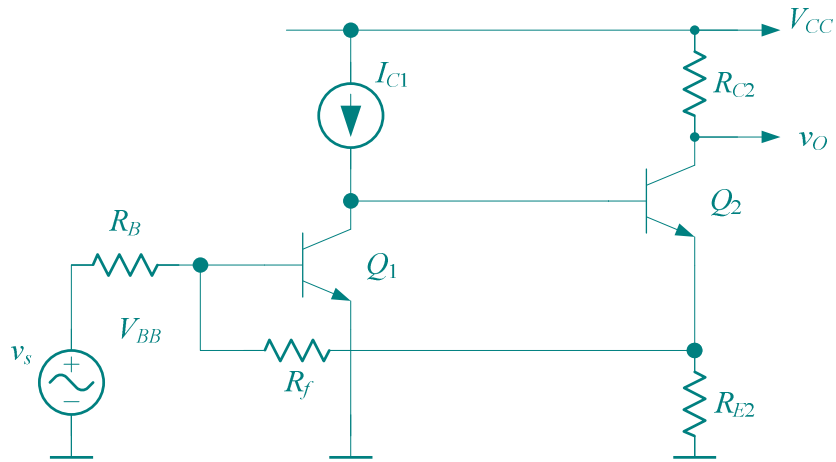


Figure 2.9: Double-transistor amplifier circuit.

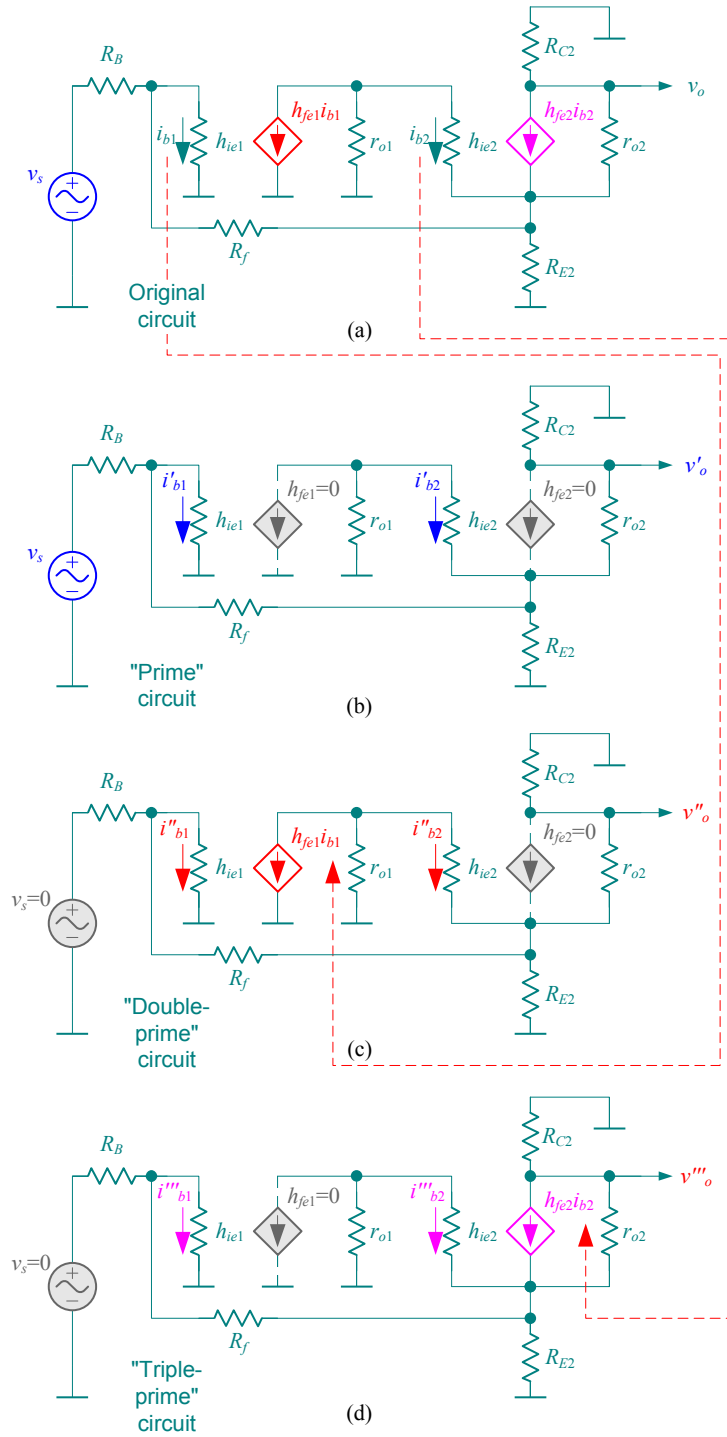


Figure 2.10: Finding the partial open-loop gains of a double-transistor circuit. (a) Original equivalent small-signal circuit. (b) The “prime” circuit, where the independent source is the only active one. (c) The “double-prime” circuit, where $h_{fe1}i_{b1}$ the only active source is. (d) The “triple-prime” circuit, where $h_{fe2}i_{b2}$ the only active source is. Note that the dependent sources in (c) and (d) are controlled by the corresponding signals of the original circuit (a).

The following parameters values are used for this model.

$$R_C = 1.3k\Omega, R_E = 1k\Omega, R_f = 8k\Omega, R_B = 2200\Omega$$

$$h_{ie1} = h_{ie2} = 23.5k, h_{fe1} = h_{fe2} = 287, r_{o1} = r_{o2} = 25k\Omega$$

After substitution of them we become:

$$G_1 = 0.032 \cdot 10^{-3} \quad G_2 = 0.002 \cdot 10^{-3}$$

$$D = 3.897 \cdot 10^{-3}$$

$$A_{OL1} = -6311.933941 \quad A_{OL2} = -343017.916496$$

$$T_1 = 1.091837 \quad T_{12} = -2.013463 \quad T_{21} = 145.305252 \quad T_2 = 4.855334$$

Finally form (2.47) and (2.45) we became the closed-loop gain and the return ratio:

$$A_{cl} = 5.190445$$

$$F(I_2) = I_2 + T(I_2) = \begin{bmatrix} 2.091837 & -2.013463 \\ 145.305252 & 5.855334 \end{bmatrix}$$

$$T_\Sigma = \det F(I_2) - 1 = 303.81500$$

2.4 Conclusion

Analytical and functional models for the closed loop gain of linear feedback networks with multiple dependent sources are developed. The proposed approach includes no approximations and can be applicable to linear feedback networks with any number of dependent sources and any number of feedback loops.

The model allows finding analytical closed loop gain formula for such networks that provides better intuitive insight into network operation. This formula can be written in a matrix form. The components of it are found by dissection and superposition and in that way making calculations much easier. It also allows us to represent a feedback network as functional diagram.

A generalized return ratio and a return ratio matrix which are a combination of return ratios and cross return ratios are obtained. As we seen, generalized return ratio can also be calculated by finding the determinant of return-difference matrix. Due to generalized return-ratio a stability analysis of the network can be done with the tools of control theory.

Another advantage of the model is that it allows us to obtain generalized return ratio and a closed loop gain function, as a function of circuit components. This allows us to modify the stability of the network by proper choice of network components.

Also the network analysis methods that were used in this work can be extended to the analysis of linear systems with feedback.

Chapter 3

Extension of Blackman's Formula to Feedback Networks with Multiple Dependent Sources

3.1 Introduction

To examine the effect of negative feedback [1]-[2], [6]-[8], [10]-[14] and [24]-[33] on impedances seen from a pair of arbitrary terminals of a linear feedback network, it is important to describe them analytically as a function of feedback partial gains, such as return ratios.

We have seen in introduction chapter, that the impedance seen from a pair of arbitrary terminals of a linear feedback network with a single dependent source can be described by the Blackman's formula [14]

$$Z_t = Z_t|_{a=0} \frac{1+T_{sc}}{1+T_{oc}} \quad (3.1)$$

where Z_t is the closed-loop impedance seen by a test source v_t [see Figure 3.1 (a)], a is the dependent source gain, $Z_t|_{a=0}$ is the open-loop impedance, T_{sc} is the return

ratio for the short-circuited terminals, and T_{oc} is the return ratio for the open-circuited terminals

$$T_{sc} \equiv -\frac{s_{\mathcal{E}}|_{v_i=0}}{s_{\mathcal{E}}} \quad (3.2)$$

$$T_{oc} \equiv -\frac{s_{\mathcal{E}}|_{i_i=0}}{s_{\mathcal{E}}} \quad (3.3)$$

As seen from the previous chapter and (3.2) and (3.3), the return ratio is defined as the contribution of a dependent source to its control signal, normalized to the total control signal $s_{\mathcal{E}}$ appearing in the dependent source value $as_{\mathcal{E}}$ (see Figure 3.1).

Since the network independent source are suppressed while finding return ratios, it is necessary to substitute a dependent source with an equivalent independent one having the same value $as_{\mathcal{E}}$ to find the contributions $s_{\mathcal{E}}|_{v_i=0}$ and $s_{\mathcal{E}}|_{i_i=0}$ of the dependent source to its control signal $s_{\mathcal{E}}$.

The aim of this chapter is to extend the Blackman's formula to the case of feedback networks with multiple dependent sources. To reach this aim, we revisit the proof of the Blackman's formula for feedback networks with a single dependent source, proof it in a similar way for feedback networks with two dependent sources, and then show how it can be extended to the case of feedback networks with multiple dependent sources. As a result we received a formula for impedances seen from a pair of arbitrary terminals for feedback networks with multiple dependent sources as a function of return-ratio matrices.

As we already seen in the previous chapter, in feedback networks with two or

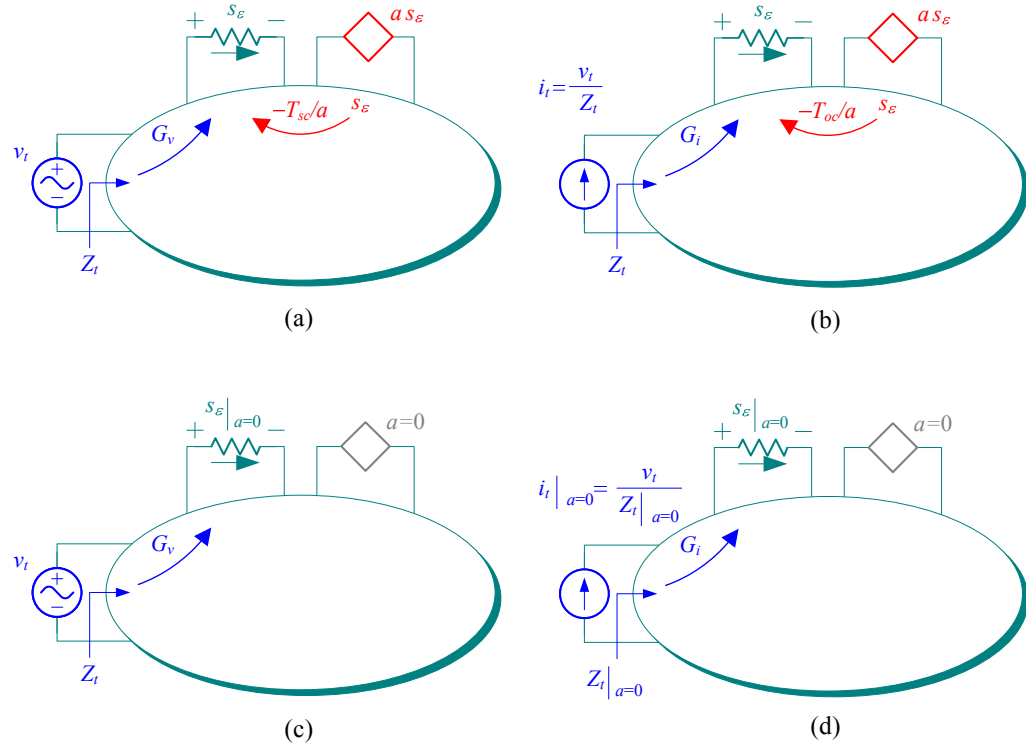


Figure 3.1: Finding the impedance seen by a test source connected to arbitrary terminals of a generic feedback network with a single dependent source. (a) Connecting a voltage test source and finding the control signal s_ϵ . (b) Replacing the source v_t with i_t such that $i_t = v_t/Z_t$ and finding a control signal s_ϵ . (c) Finding of G_v for an arbitrary v_t . (d) Finding of G_i for $i_t|_{a=0} = v_t/Z_t|_{a=0}$.

more dependent sources, each dependent source contributes in a general case not only to its own control signal terminal but also to the control signal terminals of other dependent sources. We defined such cross contributions, normalized to the dependent source value, as cross return ratios. These return ratios can be written in return-ratio matrix.

3.2 Feedback networks with a single dependent source

Let us consider a linear feedback network with a single dependent source (see Figure 3.1). To find the impedance seen from a pair of arbitrary terminals, we connect to

them a voltage test source in Figure 3.1 (a) and find the control signal by applying superposition

$$s_{\mathcal{E}} = v_t G_v - s_{\mathcal{E}} T_{sc} \quad (3.4)$$

where the input transmission for the voltage test source

$$G_v \equiv \frac{s_{\mathcal{E}} \Big|_{a=0}}{v_t} \quad (3.5)$$

From (3.4), v_t can be obtained as

$$v_t = \frac{s_{\mathcal{E}}(1 + T_{sc})}{G_v} \quad (3.6)$$

Let us now replace in Figure 3.1 (b) source v_t with i_t , such that $i_t = v_t / Z_t$ to keep the same conditions of the test source branch. Keeping the same branch voltage and current leaves the signal $s_{\mathcal{E}}$ unchanged. As a result

$$s_{\mathcal{E}} = i_t G_i - s_{\mathcal{E}} T_{oc} \quad (3.7)$$

where the input transmission for current test source

$$G_i \equiv \frac{s_{\mathcal{E}} \Big|_{a=0}}{i_t} \quad (3.8)$$

From (3.7), i_t can be obtained as

$$i_t = \frac{s_\varepsilon(1+T_{oc})}{G_i} \quad (3.9)$$

Hence

$$Z_t = \frac{v_t}{i_t} = \frac{G_i}{G_v} \frac{1+T_{sc}}{1+T_{oc}} \quad (3.10)$$

G_v and G_i in (3.10) can be found from Figure 3.1 (c) and (d)

$$G_v = \frac{s_\varepsilon|_{a=0}}{v_t} \text{ for an arbitrary } v_t \quad (3.11)$$

$$G_i = \frac{s_\varepsilon|_{a=0}}{i_t|_{a=0}} \text{ for } i_t|_{a=0} = \frac{v_t}{Z_t|_{a=0}} \quad (3.12)$$

Note that $i_t|_{a=0} = v_t / Z_t|_{a=0}$ in (3.12) keeps the same conditions of the test source branch for both v_t and $i_t|_{a=0}$ sources and, as a result, $s_\varepsilon|_{a=0}$ in (3.11) and (3.12) has the same value.

According to (3.11) and (3.12), $G_i / G_v = Z_t|_{a=0}$. Thus, considering (3.10), the Blackman's formula (3.1) can be obtained for feedback networks with a single dependent source.

3.3 Feedback networks with two dependent sources

Following the approach introduced in the previous section, we extend in this section the Blackman's formula to feedback networks with two dependent sources (see

Figure 3.2).

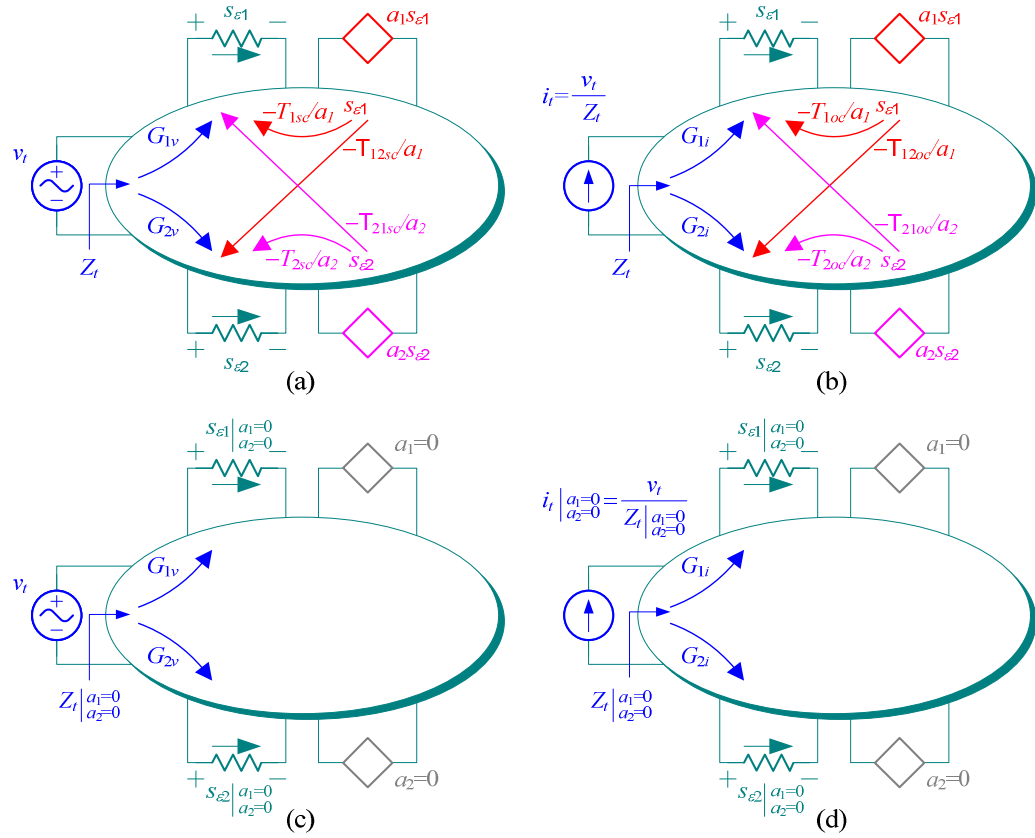


Figure 3.2: Finding the impedance seen by a test source connected to arbitrary terminals of a generic feedback network with two dependent sources.

Figure 3.2 (a) and (b) suggest the following equations for the control signals of the dependent sources

$$\begin{cases} s_{\epsilon 1} = v_t G_{1v} - s_{\epsilon 1} T_{1sc} - s_{\epsilon 2} T_{21sc} \\ s_{\epsilon 2} = v_t G_{2v} - s_{\epsilon 2} T_{2sc} - s_{\epsilon 1} T_{12sc} \end{cases} \quad (3.13)$$

$$\begin{cases} s_{\epsilon 1} = i_t G_{1i} - s_{\epsilon 1} T_{1oc} - s_{\epsilon 2} T_{21oc} \\ s_{\epsilon 2} = i_t G_{2i} - s_{\epsilon 2} T_{2oc} - s_{\epsilon 1} T_{12oc} \end{cases} \quad (3.14)$$

Where

$$T_{1sc} \equiv -\frac{s_{\epsilon 1} \Big|_{v_i=0}}{a_2=0} \quad T_{1oc} \equiv -\frac{s_{\epsilon 1} \Big|_{i_r=0}}{a_2=0} \quad (3.15)$$

$$T_{12sc} \equiv -\frac{s_{\epsilon 2} \Big|_{v_i=0}}{s_{\epsilon 1}} \quad T_{12oc} \equiv -\frac{s_{\epsilon 2} \Big|_{i_r=0}}{s_{\epsilon 1}} \quad (3.16)$$

$$T_{2sc} \equiv -\frac{s_{\epsilon 2} \Big|_{v_i=0}}{a_1=0} \quad T_{2oc} \equiv -\frac{s_{\epsilon 2} \Big|_{i_r=0}}{a_1=0} \quad (3.17)$$

$$T_{21sc} \equiv -\frac{s_{\epsilon 1} \Big|_{v_i=0}}{a_1=0} \quad T_{21oc} \equiv -\frac{s_{\epsilon 1} \Big|_{i_r=0}}{a_1=0} \quad (3.18)$$

Equations (3.13) and (3.14) can be solved for v_i and i_r :

$$v_i = \frac{1 + T_{1sc} + T_{2sc} + T_{1sc}T_{2sc} - T_{12sc}T_{21sc}}{G_{2v} + G_{2v}T_{1sc} - G_{1v}T_{12sc}} \quad (3.19)$$

$$i_r = \frac{1 + T_{1oc} + T_{2oc} + T_{1oc}T_{2oc} - T_{12oc}T_{21oc}}{G_{2i} + G_{2i}T_{1oc} - G_{1i}T_{12oc}} \quad (3.20)$$

Dividing (3.19) by (3.20) gives

$$Z_t = \frac{G_{2i}}{G_{2v}} \frac{1 + T_{1oc} - \frac{G_{1i}}{G_{2i}} T_{12oc}}{1 + T_{1sc} - \frac{G_{1v}}{G_{2v}} T_{12sc}} \frac{1 + T_{\Sigma_{sc}}}{1 + T_{\Sigma_{oc}}} \quad (3.21)$$

Where

$$T_{\Sigma_{sc}} = T_{1sc} + T_{2sc} + T_{1sc}T_{2sc} - T_{12sc}T_{21sc} \quad (3.22)$$

$$T_{\Sigma_{oc}} = T_{1oc} + T_{2oc} + T_{1oc}T_{2oc} - T_{12oc}T_{21oc}$$

The first term in (3.21) represents the open-loop impedance seen by v_i

$$\frac{G_{2i}}{G_{2v}} \equiv \frac{s_{\mathcal{E}2} \Big|_{\substack{a_1=0 \\ a_2=0}}}{i_t \Big|_{\substack{a_1=0 \\ a_2=0}}} \cdot \frac{v_t}{s_{\mathcal{E}2} \Big|_{\substack{a_1=0 \\ a_2=0}}} = \frac{v_t}{i_t \Big|_{\substack{a_1=0 \\ a_2=0}}} = Z_t \Big|_{\substack{a_1=0 \\ a_2=0}} \text{ for } i_t \Big|_{\substack{a_1=0 \\ a_2=0}} = \frac{v_t}{Z_t \Big|_{\substack{a_1=0 \\ a_2=0}}} \quad (3.23)$$

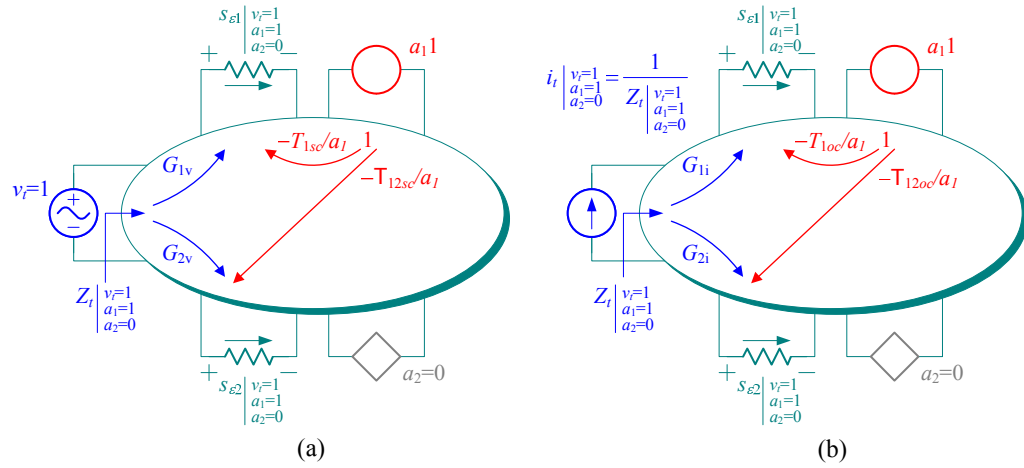


Figure 3.3: Generic feedback network with two dependent sources, where $v_t=1$, $s_{\mathcal{E}1}=1$ and $s_{\mathcal{E}2}=0$. (a) Connecting a voltage test source. (b) Replacing the voltage test source with the current test source with keeping the same conditions of the test source branch.

To find the second term in (3.21), let us consider Figure 3.3, where $v_t=1$, $s_{\mathcal{E}1}=1$ and $s_{\mathcal{E}2}=0$, and hence

$$\begin{cases} s_{\mathcal{E}1} \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} = G_{1v} - T_{1sc} \\ s_{\mathcal{E}1} \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} = i_t \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} G_{1i} - T_{1oc} \end{cases} \quad (3.24)$$

$$\begin{cases} s_{\mathcal{E}2} \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} = G_{2v} - T_{12sc} \\ s_{\mathcal{E}2} \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} = i_t \Big|_{\substack{v_t=1 \\ a_1=1 \\ a_2=0}} G_{2i} - T_{12oc} \end{cases} \quad (3.25)$$

From (3.24) and (3.25)

$$T_{1oc} - \frac{G_{1i}}{G_{2i}} T_{12oc} - G_{2v} \left(\frac{G_{1v}}{G_{2v}} - \frac{G_{1i}}{G_{2i}} \right) = T_{1sc} - \frac{G_{1i}}{G_{2i}} T_{12sc} \quad (3.26)$$

Considering that, in (3.26), in accordance with (3.23)

$$\frac{G_{1v}}{G_{1i}} = \frac{G_{2v}}{G_{2i}} = Z_t \Big|_{\substack{a_1=0 \\ a_2=0}} \Rightarrow \frac{G_{1v}}{G_{2v}} = \frac{G_{1i}}{G_{2i}} \quad (3.27)$$

(3.26) can be written as follows:

$$T_{1oc} - \frac{G_{1i}}{G_{2i}} T_{12oc} = T_{1sc} - \frac{G_{1v}}{G_{2v}} T_{12sc} \quad (3.28)$$

Considering (3.21), (3.23) and (3.28) Z_t can eventually be obtained as

$$Z_t = Z_t \Big|_{\substack{a_1=0 \\ a_2=0}} \frac{1 + T_{\Sigma_{sc}}}{1 + T_{\Sigma_{oc}}} \quad (3.29)$$

3.4 Feedback networks with multiple dependent sources

Following the approach given in the previous sections, the impedance seen from a pair of arbitrary terminals of feedback networks with multiple dependent sources can be obtained similar to (3.29).

For example, for a feedback network with three dependent sources, the control signals of the dependent sources

$$\begin{cases} s_{\varepsilon 1} = v_t G_{1v} - s_{\varepsilon 1} T_{1cs} - s_{\varepsilon 2} T_{21cs} - s_{\varepsilon 3} T_{31cs} \\ s_{\varepsilon 2} = v_t G_{2v} - s_{\varepsilon 2} T_{2cs} - s_{\varepsilon 1} T_{12cs} - s_{\varepsilon 3} T_{32cs} \\ s_{\varepsilon 3} = v_t G_{3v} - s_{\varepsilon 3} T_{3cs} - s_{\varepsilon 1} T_{13cs} - s_{\varepsilon 2} T_{23cs} \end{cases} \quad (3.30)$$

$$\begin{cases} s_{\varepsilon 1} = i_t G_{1i} - s_{\varepsilon 1} T_{1oc} - s_{\varepsilon 2} T_{21oc} - s_{\varepsilon 3} T_{31oc} \\ s_{\varepsilon 2} = i_t G_{2i} - s_{\varepsilon 2} T_{2oc} - s_{\varepsilon 1} T_{12oc} - s_{\varepsilon 3} T_{32oc} \\ s_{\varepsilon 3} = i_t G_{3i} - s_{\varepsilon 3} T_{3oc} - s_{\varepsilon 1} T_{13oc} - s_{\varepsilon 2} T_{23oc} \end{cases} \quad (3.31)$$

Equations (3.30) and (3.31) can be solved for $Z_t = v_t / i_t$

$$Z_t = \frac{G_{3i} B_{oc3} (1 + T_{\Sigma_{sc}})}{G_{3v} B_{sc3} (1 + T_{\Sigma_{oc}}} \quad (3.32)$$

where

$$\begin{aligned} B_{oc3} = & 1 + T_{1oc} + T_{2oc} + T_{1oc} T_{2oc} - T_{12oc} T_{21oc} \\ & + \frac{G_{1i}}{G_{2i}} (T_{12oc} T_{23oc} - T_{13oc} - T_{2oc} T_{13oc}) \\ & + \frac{G_{2i}}{G_{3i}} (T_{13oc} T_{21oc} - T_{23oc} - T_{1oc} T_{23oc}) \end{aligned} \quad (3.33)$$

$$\begin{aligned} B_{sc3} = & 1 + T_{1sc} + T_{2sc} + T_{1sc} T_{2sc} - T_{12sc} T_{21sc} \\ & + \frac{G_{1v}}{G_{2v}} (T_{12sc} T_{23sc} - T_{13sc} - T_{2sc} T_{13sc}) \\ & + \frac{G_{2v}}{G_{3v}} (T_{13sc} T_{21sc} - T_{23sc} - T_{1sc} T_{23sc}) \end{aligned} \quad (3.34)$$

$$\begin{aligned} T_{\Sigma_{sc}} = & T_{1sc} - T_{1sc} T_{23sc} T_{32sc} + T_{2sc} - T_{2sc} T_{13sc} T_{31sc} \\ & + T_{3sc} - T_{3sc} T_{12sc} T_{21sc} + T_{1sc} T_{2sc} + T_{1sc} T_{3sc} \\ & + T_{2sc} T_{3sc} - T_{12sc} T_{21sc} - T_{13sc} T_{31sc} - T_{23sc} T_{32sc} \\ & + T_{1sc} T_{2sc} T_{3sc} + T_{12sc} T_{23sc} T_{31sc} \\ & + T_{13sc} T_{21sc} T_{32sc} \end{aligned} \quad (3.35)$$

$$\begin{aligned}
T_{\Sigma_{oc}} = & T_{1oc} - T_{1oc}T_{23oc}T_{32oc} + T_{2oc} - T_{2oc}T_{13oc}T_{31oc} \\
& + T_{3oc} - T_{3oc}T_{12oc}T_{21oc} + T_{1oc}T_{2oc} + T_{1oc}T_{3oc} \\
& + T_{2oc}T_{3oc} - T_{12oc}T_{21oc} - T_{13oc}T_{31oc} - T_{23oc}T_{32oc} \\
& + T_{1oc}T_{2oc}T_{3oc} + T_{12oc}T_{23oc}T_{31oc} \\
& + T_{13oc}T_{21oc}T_{32oc}
\end{aligned} \tag{3.36}$$

According to (3.23) and (3.27), the first term in (3.32) is

$$\frac{G_{3i}}{G_{3v}} = Z_t \Big|_{\substack{a_j=0 \\ j=1,2,3}} \tag{3.37}$$

As in the previous section, it can also be shown that the term B_{oc3} / B_{sc3} in (3.32) equals unity. As a result, the impedance seen from a pair of arbitrary terminals of a linear feedback network with three dependent sources can be obtained by (3.32), where the open-loop impedance is given by (3.37) and $T_{\Sigma_{sc}}$ and $T_{\Sigma_{oc}}$ are given by (3.35) and (3.36) respectively.

We can write equations (3.30) and (3.31) in a matrix form through return difference matrix:

$$v_t \begin{bmatrix} G_{1v} \\ G_{2v} \\ G_{3v} \end{bmatrix} = \begin{bmatrix} 1+T_{1sc} & T_{21sc} & T_{31sc} \\ T_{12sc} & 1+T_{2sc} & T_{32sc} \\ T_{13sc} & T_{23sc} & 1+T_{3sc} \end{bmatrix} \begin{bmatrix} S_{\varepsilon 1} \\ S_{\varepsilon 2} \\ S_{\varepsilon 3} \end{bmatrix} \tag{3.38}$$

$$i_t \begin{bmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \end{bmatrix} = \begin{bmatrix} 1+T_{1oc} & T_{21oc} & T_{31oc} \\ T_{12oc} & 1+T_{2oc} & T_{32oc} \\ T_{13oc} & T_{23oc} & 1+T_{3oc} \end{bmatrix} \begin{bmatrix} S_{\varepsilon 1} \\ S_{\varepsilon 2} \\ S_{\varepsilon 3} \end{bmatrix} \tag{3.39}$$

After combining (3.38) and (3.39) we became:

$$v_t \cdot \text{adj}F(I_{sc}) \begin{bmatrix} G_{1v} \\ G_{2v} \\ G_{3v} \end{bmatrix} = i_t \cdot \frac{\det F(I_{sc})}{\det F(I_{oc})} \text{adj}F(I_{oc}) \begin{bmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \end{bmatrix} \quad (3.40)$$

$$\text{adj}F(I_{oc}) \begin{bmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \end{bmatrix} / \text{adj}F(I_{sc}) \begin{bmatrix} G_{1v} \\ G_{2v} \\ G_{3v} \end{bmatrix} = \begin{bmatrix} \frac{G_{1i}}{G_{1v}} \frac{B_{oc1}}{B_{sc1}} \\ \frac{G_{2i}}{G_{2v}} \frac{B_{oc2}}{B_{sc2}} \\ \frac{G_{3i}}{G_{3v}} \frac{B_{oc3}}{B_{sc3}} \end{bmatrix} \quad (3.41)$$

It can be verified that all the terms B_{oc1} / B_{sc1} , B_{oc2} / B_{sc2} and B_{oc3} / B_{sc3} are equal to unity and for the same reason as (3.27) we have that $G_{1v} / G_{1i} = G_{2v} / G_{2i} = G_{3v} / G_{3i} = Z_t |_{a_j=0}$, $j=1,2,3$.

As a result if we take one of the rows from equation (3.41) we become Blackman's formula in a matrix form:

$$Z_t = Z_t |_{a_j=0} \frac{\det F(I_{sc})}{\det F(I_{oc})} \quad (3.42)$$

$F(I_{sc})$ – is a return ratio matrix with the short-circuited terminal and $F(I_{oc})$ – is a return ratio matrix with the open-circuited terminal. We have checked for networks with more than three dependent sources that the formula (3.42) is valid.

3.5 Conclusion

It has been shown that the Blackman's formula can be extended to the case of feedback networks with multiple dependent sources. The obtained equations provide exact solutions for feedback networks up to five dependent sources. It can be checked that this formula can be applied to networks with a greater number of dependent sources.

Transistor small-signal models in many cases include only one dependent source. Double and multiple transistor feedback circuits are used widely. As a result, the proposed theory could serve a useful reference to do quick calculations by hand and, thus, to obtain a better intuitive insight into the effect of feedback on the circuit impedances.

In contrast to the canonical approach [7] and [33], the described theoretical treatment involves no approximations to simplify the analysis.

To validate the proposed theory, theoretical results were compared against simulations and experimental measurements. In both the theoretical calculations and simulations, the transistors were modeled by equivalent small-signal circuits with a single dependent source $h_{fe}i_b$. A perfect matching has been obtained between the theory and simulations. The matching between the theoretical and experimental results was very close. The difference between these results is due to the approximate transistor models and the measurement uncertainties of the transistor small-signal parameters.

Chapter 4

Simulating Return Ratio in Linear Feedback Networks by Middlebrook's Method

4.1 Introduction

The procedure of finding the return ratio that was depicted previously is not suitable for simulating electronic circuits, where there is no access to circuit's dependent sources, or for testing real electronic circuits.

An alternative approach was suggested by Middlebrook in [15], [16] and consists in connecting test sources to the accessible terminals of a transistor or operational amplifier. According to this method, two partial return ratios are measured first, one for a current injection and the other for a voltage injection, and then they are translated into the circuit's return ratio. The methods on determining loop gain through circuit simulation based on Middlebrook's approach are suggested in [34]-[36].

Unfortunately, the proof of Middlebrook's method is based on an idealized feedback model, does not account for nonzero reverse loop gain and for multiple feedback loops.

Another method is introduced in [37] of measuring loop gain is also based on an idealized feedback network.

In this chapter, we revisit the proof of Middlebrook's method and extend it to linear networks with multiple bilateral loops. Instead of an idealized feedback model, our proof is based on a generic feedback model.

4.2 Return ratio and the closed-loop gains

Let us consider in Figure 4.1 (a) a generic linear feedback network with a single dependent source. It is important to note that the generic network in Figure 4.1 is not necessarily a single-loop feedback one and can include a number of bilateral feedback paths connecting between the dependent source and its controlling terminals.

We have seen in (2.4) that the network's return ratio is defined as follows:

$$T \equiv - \frac{s_{\varepsilon} \Big|_{s_s=0}}{s_{\varepsilon}} \quad (4.1)$$

where s_s is the input signal source, and s_{ε} is the signal controlling the dependent source.

In Figures 4.1 (b) and (c), where $s_s = 0$, we assume that the entire network seen by the dependent sources can be replaced by equivalent impedance composed of Z_o , representing the output impedance of a transistor or operational amplifier, and Z representing the rest of the network's total equivalent impedance.

Injecting either a test current in Figures 4.2 (a), (b) or a test voltage in Figures 4.2 (c), (d) right at the output terminals of the dependent source, allows calculating T in

each test. The closed-loop gains, i_y / i_x and v_y / v_x in Figures 4.2 (a), (c) can be found as follows:

$$A_{CL} \equiv \frac{s_o}{s_t} = G \cdot \frac{A_{OL}}{1+T} + D \quad (4.2)$$

Where s_o denotes either i_y or v_y output signals, s_t denotes either i_t or v_t test sources,

$$G \equiv \frac{s_e \Big|_{a=0}}{s_t} \quad (4.3)$$

is the input transmission, s_e denotes either i_e for the current injection or v_e for the voltage injection,

$$D \equiv \frac{s_o \Big|_{a=0}}{s_t} \quad (4.4)$$

is the direct transmission, s_o denotes either i_y or v_y output signals, and

$$A_{OL} \equiv \frac{s_o \Big|_{s_t=0}}{s_e} \quad (4.5)$$

is the open-loop gain.

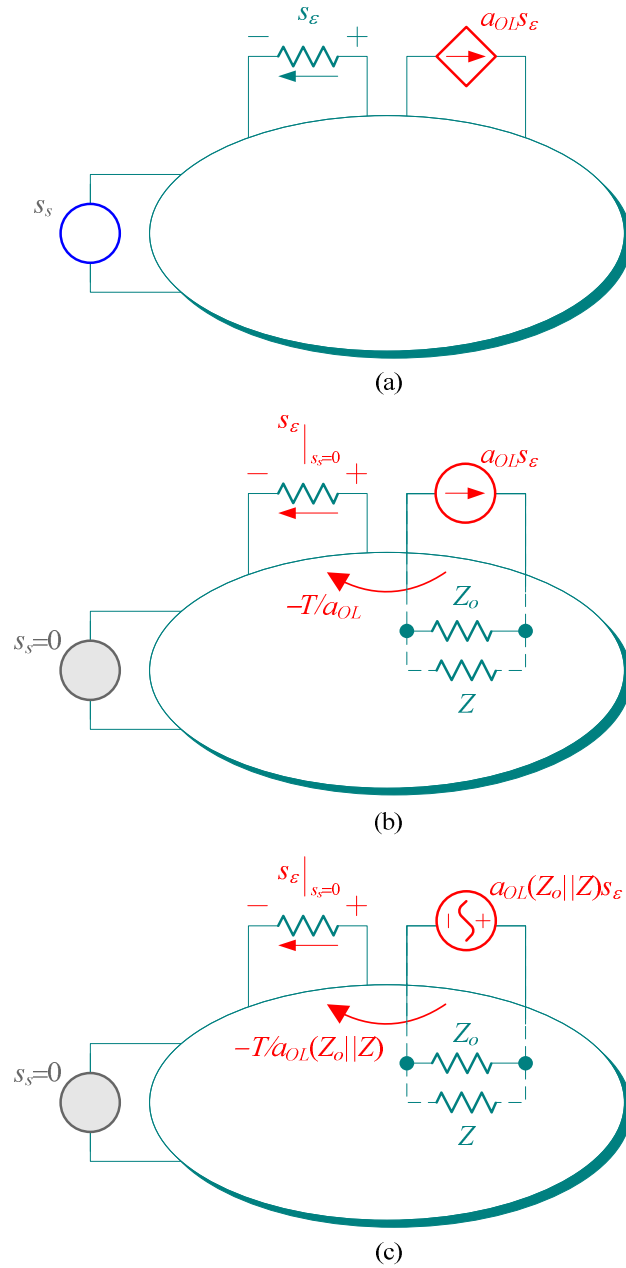


Figure 4.1: Finding the return ratio for a generic linear feedback network: (a) original network, (b) and (c) suppressing the signal source and replacing the dependent source with an equivalent independent one. Z_o denotes the output impedance of a transistor or operational amplifier, and Z denotes the rest of the total equivalent impedance seen by the dependent sources when $s_s = 0$.

4.3 Measuring return ratio directly at the dependent source terminals

If a test source can be connected directly to the dependent source, as shown in Figure 4.2 (a) and (c), a single test is enough to calculate the return ratio. According to (4.2)

$$\frac{i_y}{i_x} = \frac{G_i \cdot \frac{A_{OLiy}}{1+T} + D_{iy}}{G_i \cdot \frac{A_{OLix}}{1+T} + D_{ix}} = \frac{-\frac{T}{a_{OL}} \cdot \frac{a_{OL}}{1+T} + 0}{-\frac{T}{a_{OL}} \cdot \frac{a_{OL}}{1+T} + 1} = -T \quad (4.6)$$

$$\frac{v_y}{v_x} = \frac{G_v \cdot \frac{A_{OLvy}}{1+T} + D_{vy}}{G_v \cdot \frac{A_{OLvx}}{1+T} + D_{vx}} = \frac{-\frac{T}{a_{OL}(Z_o \parallel Z)} \cdot \frac{a_{OL}(Z_o \parallel Z)}{1+T} + 0}{-\frac{T}{a_{OL}(Z_o \parallel Z)} \cdot \frac{a_{OL}(Z_o \parallel Z)}{1+T} + 1} = -T \quad (4.7)$$

where A_{OLiy} , A_{OLix} , A_{OLvy} and A_{OLvx} are the open-loop gains for output signals i_y , i_x , v_y and v_x respectively; G_i and G_v are the input transmissions from i_t and v_t sources; D_{iy} and D_{ix} are the direct transmissions from i_t source to output signals i_y and i_x , D_{vy} and D_{vx} are the direct transmissions from v_t source to output signals v_y and v_x , and a_{OL} is the dependent source gain.

Note that the only difference between calculating T in Figure 4.1 (b) and calculating G_i in Figure 4.2 (b) is in the active independent source value: $a_{OL}s_\varepsilon$ against i_t . Therefore, according to (4.1) and (4.3) $G_i = -T/a_{OL}$ in (4.6). Similarly, the active independent sources in Figure 4.1 (c) and Figure 4.2 (d) are $a_{OL}(Z_o \parallel Z)s_\varepsilon$ and v_t , therefore, $G_v = -T/[a_{OL}(Z_o \parallel Z)]$ in (4.7).

Note also that suppressing the dependent sources in Figures 4.2 (b) and (d) zeroes D_{iy} and D_{vy} and forces $i_x = i_t$ and $v_x = v_t$, therefore, according to (4.4), $D_{iy} = 0$ and

$D_{ix} = 1$ in (4.6), and $D_{vy} = 0$ and $D_{vx} = 1$ in (4.7).

Note as well, that suppressing the test sources in Figures 4.2 (a) and (c) to calculate the open-loop gains in accordance to (4.5) results in $i_y = i_x$ in Figure 4.2 (a) and $v_y = v_x$ in Figure 4.2 (c). Therefore, $A_{OLiy} = A_{OLix}$ in (4.6) and $A_{OLvy} = A_{OLvx}$ in (4.7).

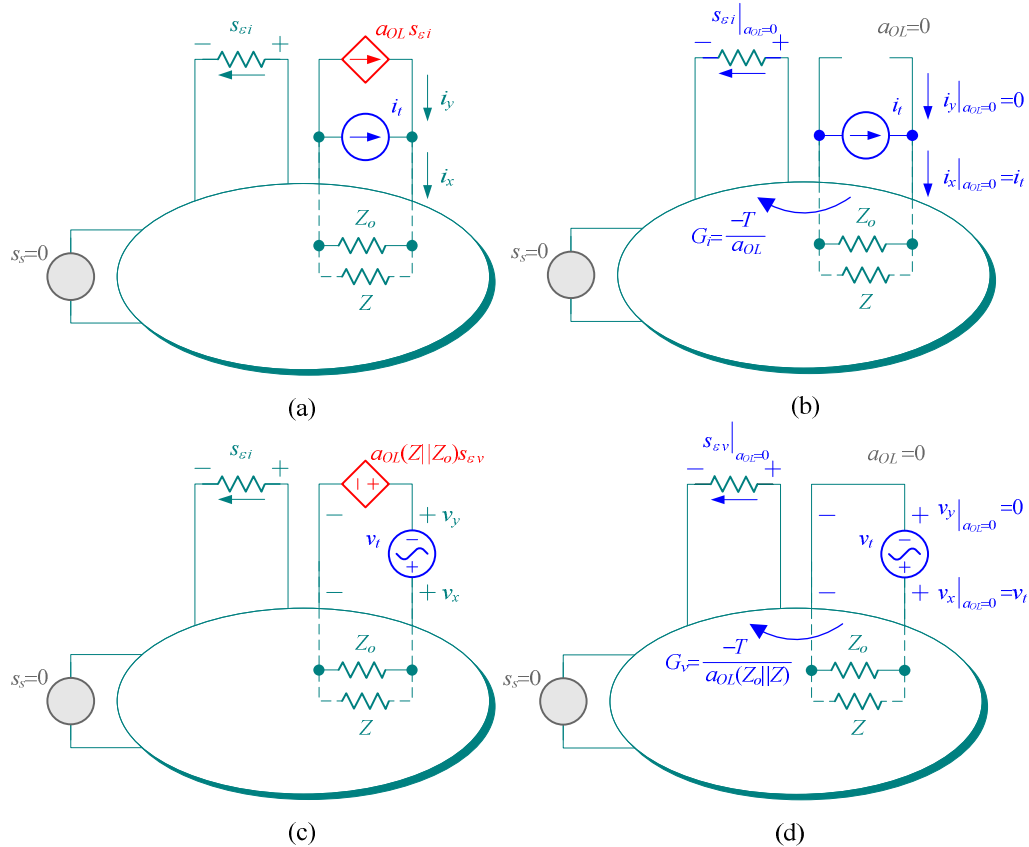


Figure 4.2: Finding the return ratio for a generic linear feedback network: (a) by current injection and (c) by voltage injection, (b) finding input and direct transmissions G_i , and D_i , (d) finding input and direct transmissions G_v and D_v .

4.4 Measuring return ratio at the terminals of a dependent source linked to Z_o

If a test source can be connected only to the dependent source linked to its output impedance Z_o , as shown in Figures 4.3 (a) or (c), two tests are needed to calculate

the return ratio.

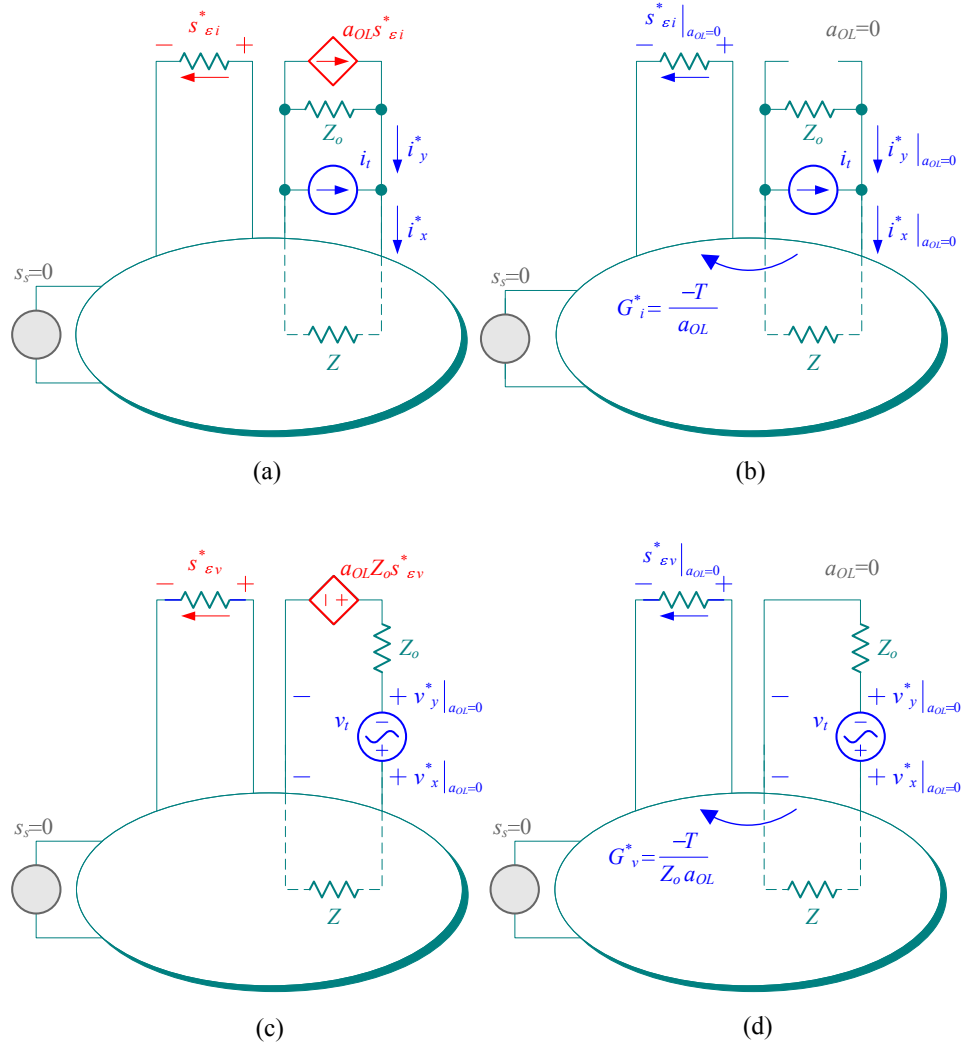


Figure 4.3: Finding partial return ratios in a generic feedback network: (a) injection of the test current, (b) finding G_i^* , D_{iy}^* and D_{ix}^* , (c) injection of the test voltage, and (d) finding G_v^* , D_{vy}^* and D_{vx}^* .

In each test, we will find the circuit's partial return ratios, either T_i for the current injection in Figure 4.3 (a), or T_v for the voltage injection in Figure 4.3 (c), and then translate them into T .

According to Figures 4.3 (a) and (b),

$$G_i^* = \frac{s_{\mathcal{E}i}^* \Big|_{a=0}}{s_t} = \frac{s_{\mathcal{E}i}^* \Big|_{s_s=0}}{a_{OL} s_{\mathcal{E}i}^*} = -\frac{T}{a_{OL}} \quad (4.8)$$

$$A_{OLiy}^* = \frac{s_o \Big|_{s_t=0}}{s_{\mathcal{E}}} = \frac{i_y \Big|_{i_t=0}}{s_{\mathcal{E}i}^*} = a_{OL} \cdot \frac{Z_o}{Z_o + Z} \quad (4.9)$$

$$A_{OLix}^* = \frac{s_o \Big|_{s_t=0}}{s_{\mathcal{E}}} = \frac{i_x \Big|_{i_t=0}}{s_{\mathcal{E}i}^*} = a_{OL} \cdot \frac{Z_o}{Z_o + Z} \quad (4.10)$$

$$D_{iy}^* = \frac{s_o \Big|_{a=0}}{s_t} = \frac{i_y \Big|_{a_{OL}=0}}{i_t} = -\frac{Z}{Z_o + Z} \quad (4.11)$$

$$D_{ix}^* = \frac{s_o \Big|_{a=0}}{s_t} = \frac{i_x \Big|_{a_{OL}=0}}{i_t} = \frac{Z_o}{Z_o + Z} \quad (4.12)$$

$$\begin{aligned} T_i &\equiv -\frac{i_y^*}{i_x^*} = -\frac{G_i^* \cdot \frac{A_{OLiy}^*}{1+T} + D_{iy}^*}{G_i^* \cdot \frac{A_{OLix}^*}{1+T} + D_{ix}^*} \\ &= -\frac{\frac{T}{a_{OL}} \cdot \frac{Z_o}{Z_o + Z} \cdot \frac{a_{OL}}{1+T} - \frac{Z}{Z_o + Z}}{-\frac{T}{a_{OL}} \cdot \frac{Z_o}{Z_o + Z} \cdot \frac{a_{OL}}{1+T} + \frac{Z_o}{Z_o + Z}} \\ &= \frac{T(Z_o + Z) + Z}{Z_o} \end{aligned} \quad (4.13)$$

$$G_v^* = \frac{s_{\mathcal{E}v}^* \Big|_{a=0}}{s_t} = \frac{s_{\mathcal{E}v}^* \Big|_{v_t=0}}{a_{OL} \cdot Z_o s_{\mathcal{E}v}^*} = -\frac{T}{a_{OL} Z_o} \quad (4.14)$$

$$A_{OLvy}^* = \frac{s_o \Big|_{s_t=0}}{s_{\mathcal{E}}} = \frac{v_y \Big|_{v_t=0}}{s_{\mathcal{E}v}^*} = a_{OL} Z_o \cdot \frac{Z}{Z_o + Z} \quad (4.15)$$

$$A_{OLvx}^* = \frac{s_o \Big|_{s_t=0}}{s_{\mathcal{E}}} = \frac{v_x \Big|_{v_t=0}}{s_{\mathcal{E}v}^*} = a_{OL} Z_o \cdot \frac{Z}{Z_o + Z} \quad (4.16)$$

$$D_{vy}^* = \frac{s_o \Big|_{a=0}}{s_t} = \frac{v_y \Big|_{a_{OL}=0}}{v_t} = -\frac{Z_o}{Z_o + Z} \quad (4.17)$$

$$D_{vx}^* = \frac{s_o \Big|_{a=0}}{s_t} = \frac{v_x \Big|_{a_{OL}=0}}{v_t} = \frac{Z}{Z_o + Z} \quad (4.18)$$

$$\begin{aligned}
T_v &\equiv -\frac{v_y^*}{v_x^*} = -\frac{G_v^* \cdot \frac{A_{OLvy}^*}{1+T} + D_{vy}^*}{G_v^* \cdot \frac{A_{OLvx}^*}{1+T} + D_{vx}^*} \\
&= -\frac{-\frac{T}{a_{OL}Z_o} \cdot \frac{Z}{Z_o+Z} \cdot \frac{a_{OL}Z_o}{1+T} - \frac{Z_o}{Z_o+Z}}{-\frac{T}{a_{OL}Z_o} \cdot \frac{Z}{Z_o+Z} \cdot \frac{a_{OL}Z_o}{1+T} + \frac{Z}{Z_o+Z}} \\
&= \frac{T(Z_o+Z) + Z_o}{Z}
\end{aligned} \tag{4.19}$$

Solving the (4.13) and (4.19) for T yields the same formula as received Middlebrook

$$T = \frac{T_i \cdot T_v - 1}{T_i + T_v + 2} \tag{4.20}$$

The formula for measuring return ratio (4.20) from measurements at a point of arbitrary impedance ratio has inaccuracy in the cases of small T . The problem is described in [15]. From equations (4.13) and (4.19) we can see that when $T \rightarrow 0$ we receive $T_v \rightarrow \frac{Z_o}{Z}$ and $T_i \rightarrow \frac{Z}{Z_o}$ and therefore T_v and T_i becomes insensitive to T , and as a consequence equation (4.20) became inaccurate. Furthermore it can also be seen by numerical methods that sensitivity function of T to T_v and T_i (4.21) see Appendix B, takes the maximum values under $T_v = \frac{1}{T_i}$ curve. That means that in the case when $T_v = \frac{Z_o}{Z}$ and $T_i = \frac{Z}{Z_o}$ the function of T is sensitive to errors which are due to measurements of T_v and T_i , such that small error in them influence on return ratio function.

$$\Delta T \cong \frac{\partial T}{\partial T_v} \Delta \partial T_v + \frac{\partial T}{\partial T_i} \Delta \partial T_i \quad (4.21)$$

An improved method for measuring of T that was introduced by Middelbrook in [15] overcomes this inaccuracy. The proof of the method for general circuits is given in the next section.

4.5 Improved measurement of return ratio at the terminals of a dependent source linked to Z_0 by null double injection

In this section, we revisit the proof of Middlebrook's method of measurement of return ratio at a point when a test sources can be connected to the dependent source linked to its output impedance Z_0 by null double injection and extend it to linear networks with multiple bilateral loops. As in the previous section, instead of an idealized feedback model, the proof is based on a generic feedback model. An impedance ratio Z_0 / Z can be arbitrary.

Voltage and current test sources are connected as shown in Figure 4.4 to the dependent source linked to its output impedance. Two tests are needed in order to calculate the return ratio each involving simultaneous current and voltage injection. If a voltage and current sources have there own impedances the proof is similar, we can only use v_i the voltage on a test voltage source and its impedance instead of v_t and i_i the current of the test source and its impedance instead of i_t .

From superposition each voltage and current in Figure 4.4 can be represented as a sum of two injected sources, therefore each current and voltage in the circuit can be set to zero by adjusting injected current and voltage. In the first test injected current

and voltage in Figure 4.4 (a) are adjusted with respect to each other such that v_y is equal to zero and a partial null return ratio T_i^n is found. Then in another test injected current and voltage in Figure 4.4 (c) are adjusted with respect to each other such that i_y is equal to zero and a partial null return ratio T_v^n is found, then they are translated into T .

In order to obtain the ratio of injected voltage to injected current when v_y is set to zero Figure 4.4 (a), we write v_y by superposition as a function of two test sources:

$$A_{CL\,vy}^1 \cdot i_t + A_{CL\,vy}^2 \cdot v_t = v_y = 0 \quad (4.22)$$

By superposition with $v_t = 0$ from Figure 4.4 (a) and (b):

$$A_{CL\,vy}^1 \equiv \frac{v_y}{i_t} = G_{vy}^1 \cdot \frac{A_{OL\,vy}^1}{1+T} + D_{vy}^1 \quad (4.23)$$

$$G_{vy}^1 = \frac{s_\varepsilon|_{a=0}}{s_t} = \frac{s_\varepsilon|_{a=0}}{i_t} = \frac{s_\varepsilon|_{s_t=0}}{a_{OL} \cdot s_\varepsilon} = -\frac{T}{a_{OL}} \quad (4.24)$$

$$A_{OL\,vy}^1 = \frac{s_0|_{s_t=0}}{s_\varepsilon} = \frac{v_y|_{i_t=0}}{s_\varepsilon} = a_{OL} \cdot \frac{Z_0 \cdot Z}{Z + Z_0} \quad (4.25)$$

$$D_{vy}^1 = \frac{s_0|_{a=0}}{s_t} = \frac{v_y|_{a=0}}{s_t} = \frac{Z_0 \cdot Z}{Z + Z_0} \quad (4.26)$$

If we set $i_t = 0$ from Figure 4.4 (a) and (b):

$$A_{CL\,vy}^2 \equiv \frac{v_y}{v_t} = G_{vy}^2 \cdot \frac{A_{OL\,vy}^2}{1+T} + D_{vy}^2 \quad (4.27)$$

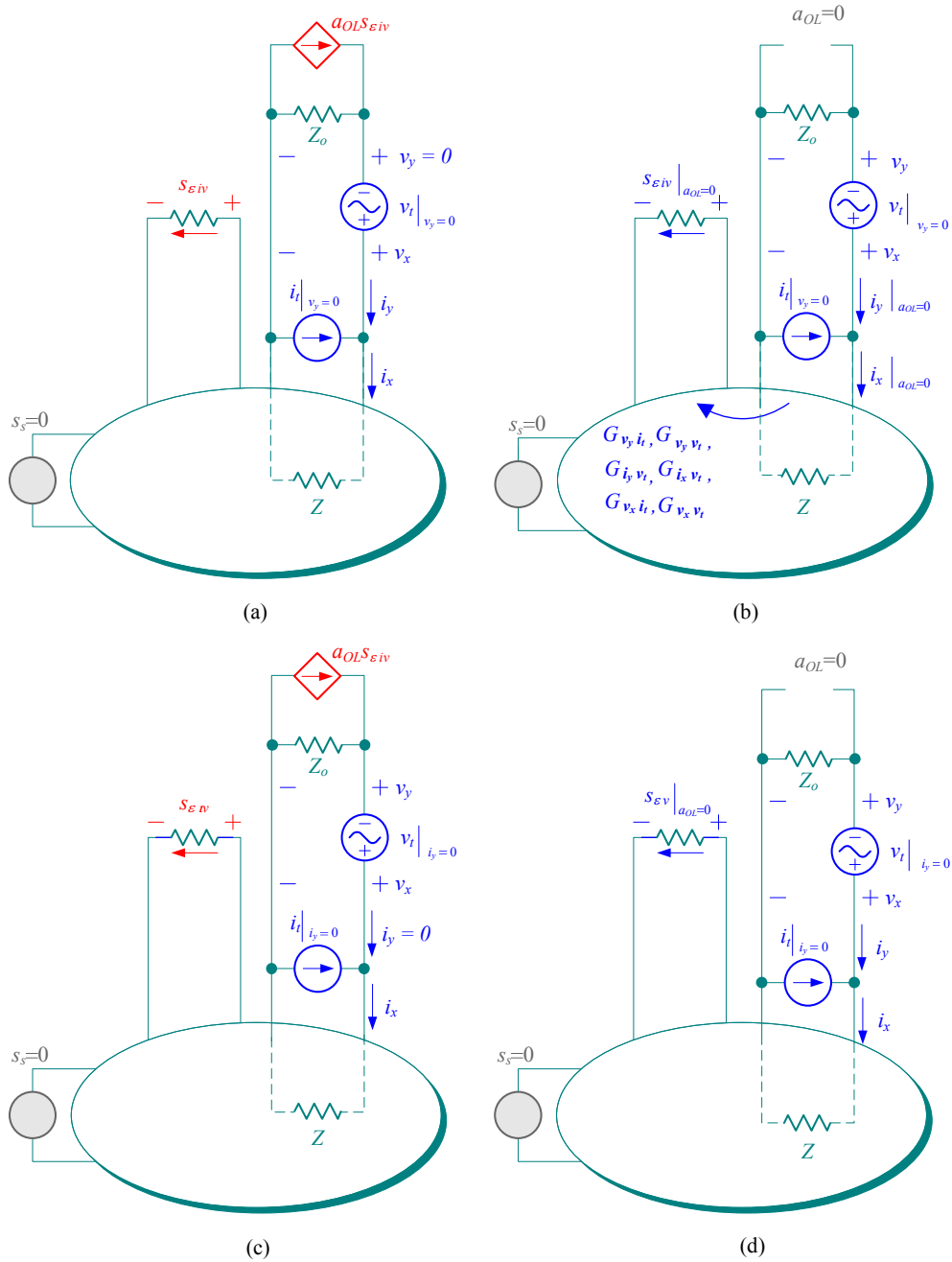


Figure 4.4: Finding partial null return ratios in a generic linear feedback network: (a) injection of the test current and voltage in order to null v_y (b) finding $G_{v_y i_i}^1, D_{v_y}^1, G_{v_y v_i}^2, D_{v_y}^2, G_{i_y}^2, G_{i_x}^4, D_{i_y}^2,$ and $D_{i_x}^4$, $G_{v_x}^3, D_{v_x}^3, G_{v_x}^4, D_{v_x}^4$. (c) injection of the test current and voltage in order to null i_y , and (d) the same network as previous with $a_{OL} = 0$.

$$G_{vy}^2 = \frac{s_\varepsilon|_{a=0}}{s_t} = \frac{s_\varepsilon|_{a=0}}{v_t} = \frac{s_\varepsilon|_{s_t=0}}{Z_0 \cdot a_{OL} s_\varepsilon} = -\frac{T}{Z_0 \cdot a_{OL}} \quad (4.28)$$

$$A_{OLvy}^2 = \frac{s_0|_{s_t=0}}{s_\varepsilon} = \frac{v_y|_{v_t=0}}{s_\varepsilon} = a_{OL} \cdot \frac{Z_0 Z}{Z + Z_0} \quad (4.29)$$

$$D_{vy}^2 = \frac{s_0|_{a=0}}{s_t} = \frac{v_y|_{a=0}}{v_t} = -\frac{Z_0}{Z + Z_0} \quad (4.30)$$

Substituting all the values into (4.23), (4.27) and (4.22) we became:

$$A_{CLvy}^1 = \frac{Z \cdot Z_0}{(Z + Z_0) \cdot (1 + T)} \quad A_{CLvy}^2 = \frac{-T \cdot Z - Z_0 \cdot (1 + T)}{(Z + Z_0) \cdot (1 + T)} \quad (4.31)$$

$$v_t = i_t \frac{Z \cdot Z_0}{T \cdot Z + Z_0 \cdot (1 + T)} \quad (4.32)$$

We define

$$T_i^n \equiv -\frac{i_y}{i_x} \Big|_{v_y=0} = -\frac{A_{CLiy}^1 \cdot i_t + A_{CLiy}^2 \cdot v_t}{A_{CLix}^3 \cdot i_t + A_{CLix}^4 \cdot v_t} \quad (4.33)$$

We already found $A_{CLiy}^1 \equiv i_y / i_t$ and $A_{CLix}^3 \equiv i_x / i_t$ in (4.8) - (4.13).

By superposition with $i_t = 0$ we have $i_y = i_x$ from Figure 4.4 (a) and (b):

$$A_{CLiy}^2 \equiv \frac{i_y}{v_t} = G_{iy}^2 \cdot \frac{A_{OLiy}^2}{1 + T} + D_{iy}^2 \quad (4.34)$$

$$A_{CLix}^4 \equiv \frac{i_x}{v_t} = G_{ix}^4 \cdot \frac{A_{OLix}^4}{1 + T} + D_{ix}^4 \quad (4.35)$$

$$G_{iy}^2 = G_{ix}^4 = \frac{s_\varepsilon|_{a=0}}{s_t} = \frac{s_\varepsilon|_{a=0}}{v_t} = \frac{s_\varepsilon|_{s_t=0}}{a_{OL} \cdot s_\varepsilon Z_0} = -\frac{T}{a_{OL} Z_0} \quad (4.36)$$

$$D_{iy}^2 = D_{ix}^4 = \frac{s_0|_{a=0}}{s_t} = \frac{i_y|_{a=0}}{v_t} = \frac{1}{Z_0 + Z} \quad (4.37)$$

$$A_{OLiy}^2 = A_{OLix}^4 = \frac{i_y|_{v_t=0}}{s_e} = a_{OL} \frac{Z_0}{Z_0 + Z} \quad (4.38)$$

$$A_{CLiy}^2 = A_{CLix}^4 = \frac{1}{(Z_0 + Z) \cdot (1+T)} \quad (4.39)$$

Finally from (4.33) we obtain:

$$\begin{aligned} T_i^n &\equiv - \left. \frac{i_y}{i_x} \right|_{v_y=0} = - \frac{A_{CLiy}^1 \cdot i_t + A_{CLiy}^2 \cdot v_t}{A_{CLix}^3 \cdot i_t + A_{CLix}^4 \cdot v_t} \\ &= - \frac{\frac{-T \cdot Z_0 - Z \cdot (1+T)}{(Z + Z_0) \cdot (1+T)} + \frac{1}{(Z + Z_0) \cdot (1+T)} \cdot \left[\frac{Z \cdot Z_0}{T \cdot Z + Z_0 \cdot (1+T)} \right]}{\frac{Z_0}{(Z + Z_0) \cdot (1+T)} + \frac{1}{(Z + Z_0) \cdot (1+T)} \cdot \left[\frac{Z \cdot Z_0}{T \cdot Z + Z_0 \cdot (1+T)} \right]} \quad (4.40) \\ &= \frac{T(Z + Z_0)}{Z_0} \end{aligned}$$

In order to obtain ratio of injected voltage to injected current when i_y is set to zero Figure 4.4 (c), we write i_y by superposition as a function of two test sources:

$$A_{CLiy}^1 \cdot i_t + A_{CLiy}^2 \cdot v_t = i_y = 0 \quad (4.41)$$

We found $A_{CLiy}^1 = i_y / i_t$ and $A_{CLiy}^2 = i_y / v_t$ previously.

$$v_t = i_t [T \cdot Z_0 + Z \cdot (1+T)] \quad (4.42)$$

We define

$$T_v^n \equiv - \left. \frac{v_y}{v_x} \right|_{i_y=0} = - \frac{A_{CLvy}^1 \cdot i_t + A_{CLvy}^2 \cdot v_t}{A_{CLvx}^3 \cdot i_t + A_{CLvx}^4 \cdot v_t} \quad (4.43)$$

By superposition with $v_t = 0$ from Figure 4.4 (a) and (b):

$$A_{CLvx}^3 \equiv \frac{v_x}{i_t} = G_{vx}^3 \cdot \frac{A_{OLvx}^3}{1+T} + D_{vx}^3 \quad (4.44)$$

$$G_{vx}^3 = \frac{s_\varepsilon \big|_{a=0}}{s_t} = \frac{s_\varepsilon \big|_{a=0}}{i_t} = \frac{s_\varepsilon \big|_{s_t=0}}{a_{OL} \cdot s_\varepsilon} = - \frac{T}{a_{OL}} \quad (4.45)$$

$$A_{OLvx}^3 = \frac{s_0 \big|_{s_t=0}}{s_\varepsilon} = \frac{v_x \big|_{i_t=0}}{s_\varepsilon} = a_{OL} \frac{Z \cdot Z_0}{Z_0 + Z} \quad (4.46)$$

$$D_{vx}^3 = \frac{s_0 \big|_{a=0}}{s_t} = \frac{v_x \big|_{a=0}}{i_t} = \frac{Z \cdot Z_0}{Z_0 + Z} \quad (4.47)$$

If we set $i_t = 0$ from Figure 4.4 (a) and (b):

$$A_{CLvx}^4 \equiv \frac{v_x}{v_t} = G_{vx}^4 \cdot \frac{A_{OLvx}^4}{1+T} + D_{vx}^4 \quad (4.48)$$

$$G_{vx}^4 = \frac{s_\varepsilon \big|_{a=0}}{s_t} = \frac{s_\varepsilon \big|_{a=0}}{v_t} = \frac{s_\varepsilon \big|_{s_t=0}}{a_{OL} \cdot s_\varepsilon Z_0} = - \frac{T}{a_{OL} Z_0} \quad (4.49)$$

$$A_{OLvx}^4 = \frac{s_0 \big|_{s_t=0}}{s_\varepsilon} = \frac{v_x \big|_{i_t=0}}{s_\varepsilon} = a_{OL} \frac{Z \cdot Z_0}{Z_0 + Z} \quad (4.50)$$

$$D_{vx}^4 = \frac{s_0 \big|_{a=0}}{s_t} = \frac{v_x \big|_{a=0}}{v_t} = \frac{Z}{Z_0 + Z} \quad (4.51)$$

According to (4.43):

$$\begin{aligned}
T_v^n &\equiv - \left. \frac{v_y}{v_x} \right|_{i_y=0} = - \frac{A_{CLvy}^1 \cdot i_t + A_{CLvy}^2 \cdot v_t}{A_{CLvx}^3 \cdot i_t + A_{CLvx}^4 \cdot v_t} \\
&= - \frac{Z \cdot Z_0 + [T \cdot Z_0 + Z \cdot (1+T)] \cdot [-T \cdot Z - Z_0 \cdot (1+T)]}{Z \cdot Z_0 + [T \cdot Z_0 + Z \cdot (1+T)] \cdot Z} \\
&= \frac{T(Z+Z_0)}{Z}
\end{aligned} \tag{4.52}$$

Solving (4.40) and (4.52) for T yields Middlebrook's formula for null double injection

$$T = \frac{T_v^n \cdot T_i^n}{T_v^n + T_i^n} \tag{4.53}$$

and impedance ratio

$$\frac{Z_0}{Z} = \frac{T_v^n}{T_i^n} \tag{4.54}$$

4.6 Conclusion

Practical methods of measurement loop gain T in feedback systems have been extended to linear networks with multiple bilateral feedback loops. The methods allow measurement of loop gain without opening the loop. In two cases instead of an idealized feedback model that was used in the original proof by Middlebrook our proof is based on a generic feedback model. Thus, it is shown that Middlebrook's formulas for measurements of return ratio by single injection and by null double injection can be accurately applied to a much wider class of feedback networks.

The analytical and functional models carried out in the present work gives the solid base for analyzing and understanding of feedback networks with multiple dependent sources. In addition to that, the Middlebrook's formulas that were extended to general linear feedback networks allow finding closed-loop gain experimentally in real electronic circuits.

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Appendix

A. Finding a return ratio from SPICE simulation

To illustrate the accuracy of finding a return ratio by applying Middlebrook's method, let us consider in Figures A.1, A.2 and A.3 a SPICE simulation of a feedback network with two bilateral feedback loops.

The original circuit is shown in Figure A.1 (a). Figure A.1 (b) illustrates measuring return ratio by conventional method. It is worth to mention, this method does not suit for measurements of return ratio in a real circuit, because of the replacement the dependent source with an equivalent one. Figure A.2 (a) illustrates injection of the test current, and Figure A.2 (b) illustrates injection of the test voltage.

Figure A.3 compares between the return ratios found by conventional and Middlebrook's methods. Note that the relative difference is less than 0.2 ppm. This small difference is caused by the limited computing accuracy. The differences in circuit simulations between double injection and null double injection Middlebrooks methods can be found in his work. Because of this we only compared here the double injection method of Middlebrook and conventional method.

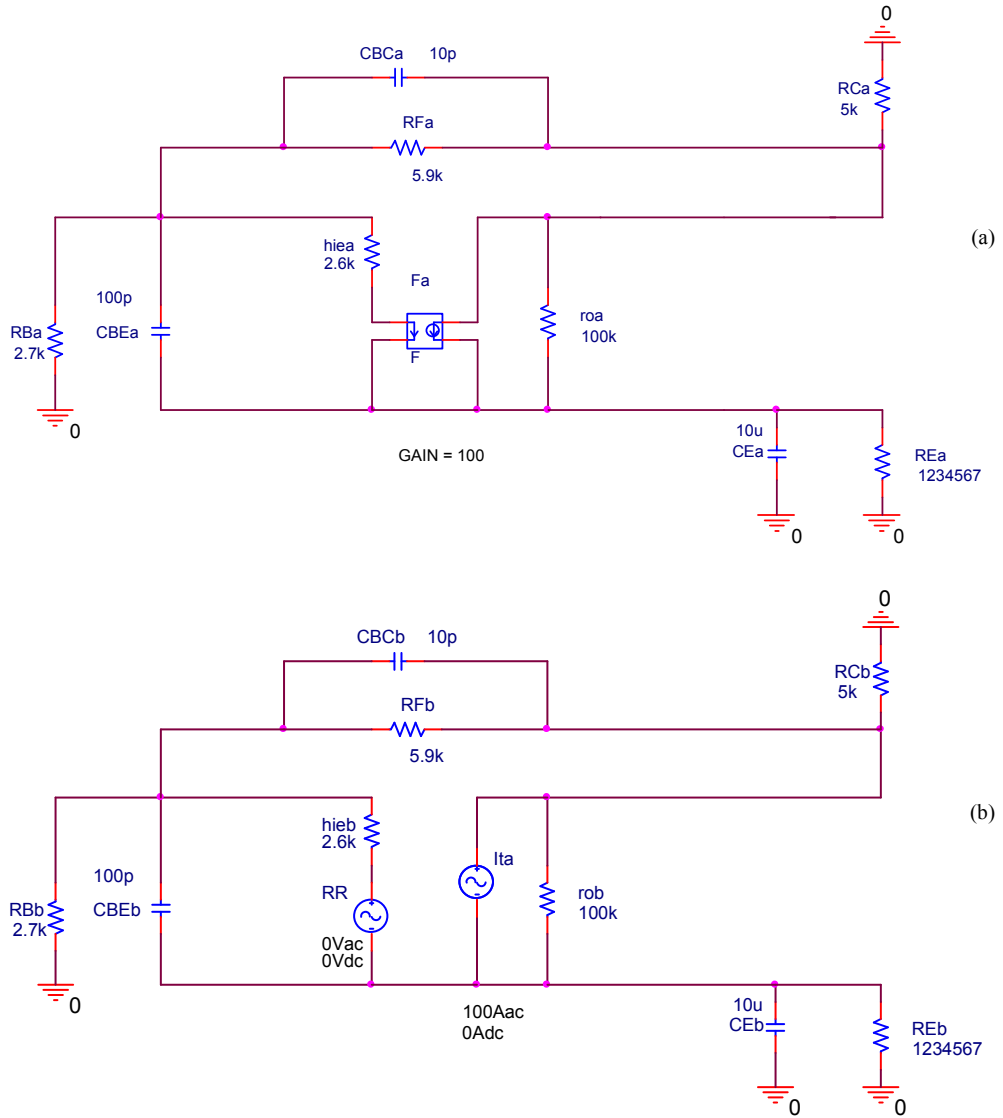


Figure A.1: Example circuit: (a) original circuit, (b) simulating return ratio by replacing the dependent source with an equivalent independent one.

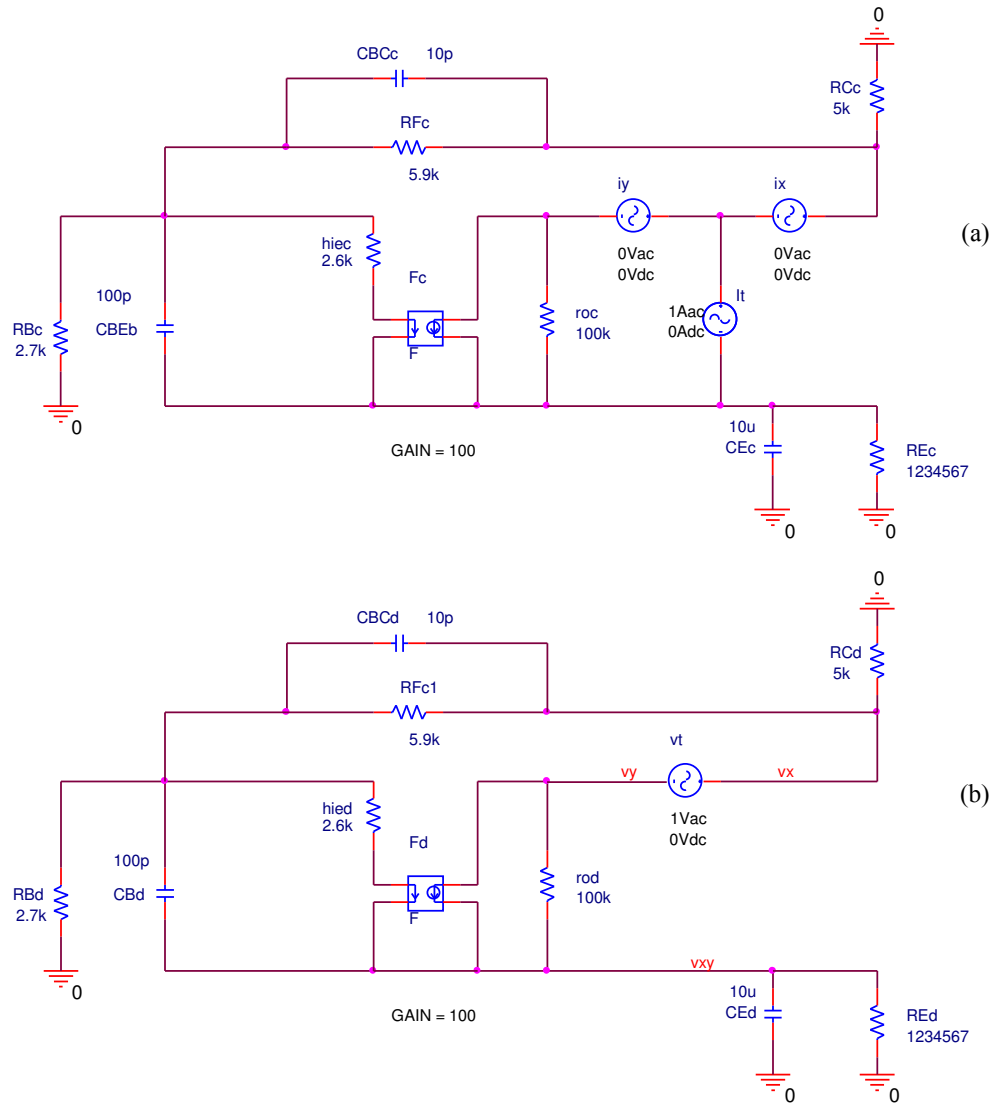


Figure A.2: Example circuit: (a) injection of the test current, (b) injection of the test voltage.

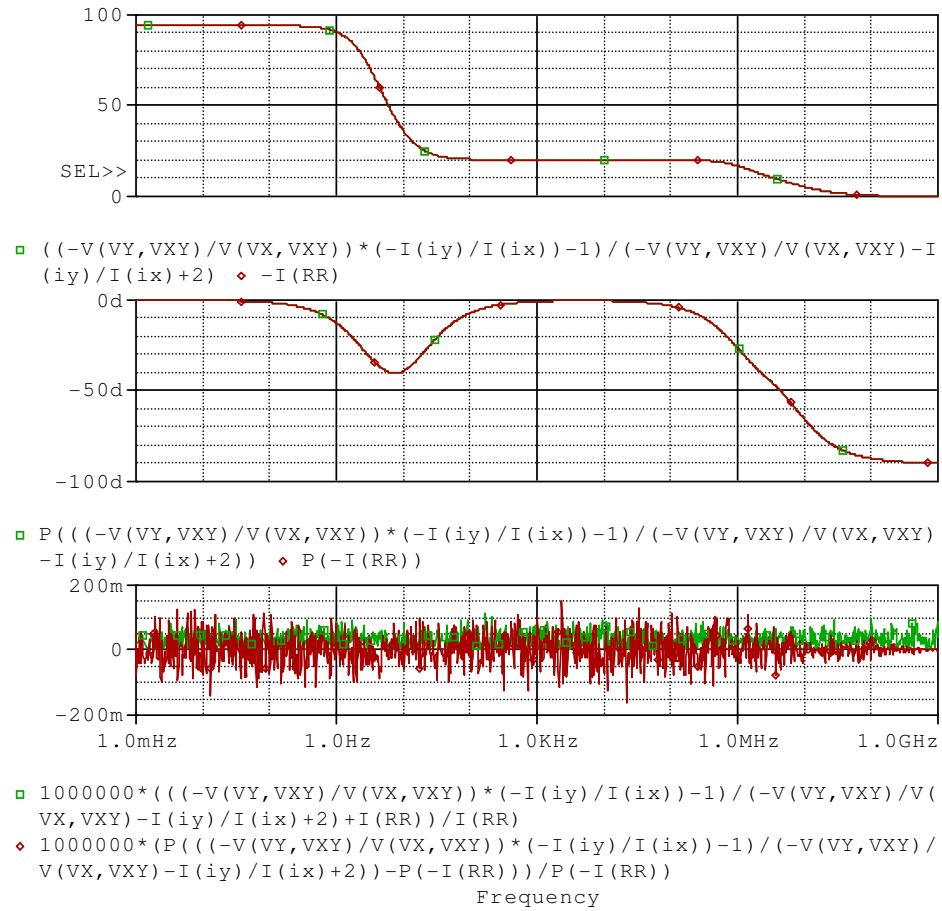


Figure A.3: Measuring and comparing return ratios: (top and middle) amplitude and phase of the return ratios measured by replacing the dependent source with an equivalent independent one (the green curves) and by Middlebrook's method (the red curves), (bottom) the relative difference (in ppm) between the return ratios amplitude (the green curve) and phase measurements (the red curve).

B. Sensitivity function of return ratio

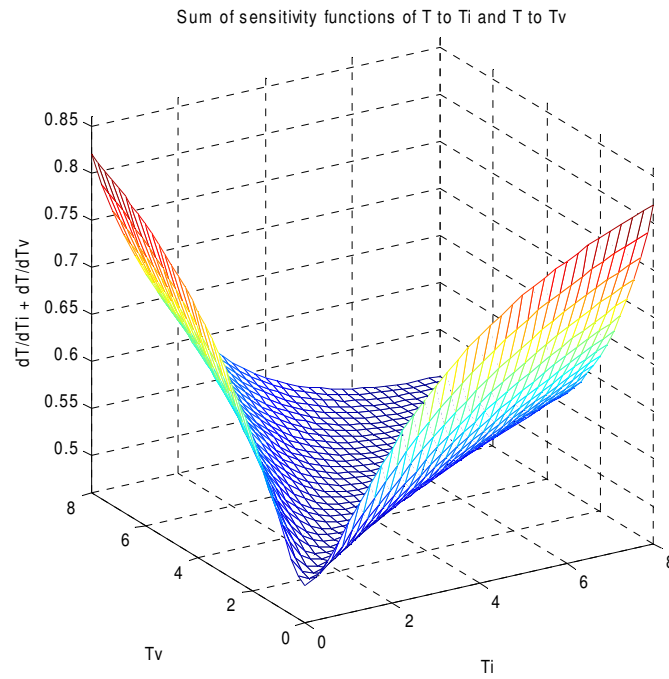


Figure B.1: Sensitivity function of $T = (T_i \cdot T_v - 1) / (T_i + T_v + 2)$ to T_v and T_i .

תקציר

עבודה זו מוקדשת לניתוח רשתות ליניאריות עם משוב בעלות ריבוי של מקורות תלויים. במעגלים חשמליים עם משוב, תכונות המעגל ופונקציית ההגבר בחוג סגור הינם בלתי תלויים במידה רבה בתנאי העבודה של המעגל. לאור זאת, השימוש במעגלים אלה נפוץ מאוד בתכן אנלוגי. לרוב, מעגלים אלה מורכבים ממספר רב של טרנזיסטורים, וכן, במודל אות קטן הם יכללו מספר מקורות תלויים. מידול של רשתות עם משוב חשוב לצורך גילוי ההשפעה של המשוב על פונקציית התמסורת של הרשת.

במידה ויש ברשותנו מודל אנליטי ודיאגרמת בלוקים של הרשת, אנו יכולים להשתמש בכלים של תורת הבקרה לצורך ניתוח ההתנהגות שלה. בנוסף, מודל זה מספק לנו תובנות לגבי אופן הפעולה של הרשת. עם זאת, למרות שניתן למצע סימולציה של הרשת, התוצאות המתקבלות אינן נותנות אינדיקציה על יציבותו של המעגל. השיטות הקיימות לניתוח מעגלים לא מספקות מודלים אנליטיים ודיאגרמות לרשתות עם משוב בעלות ריבוי של מקורות תלויים.

המטרה העיקרית של עבודה זו היא לפתח מודל אנליטי ודיאגרמת בלוקים עבור פונקציית ההגבר בחוג סגור של רשתות עם משוב בעלות ריבוי מקורות תלויים. מטרה נוספת היא לבחון את ההשפעה של משוב שלילי על העכבה בין שתי נקודות שרירותיות במעגלים אלה. המטרה האחרונה היא להכליל את נוסחת Middlebrook עבור יחס ההחזרה (Return Ratio) עבור רשתות ליניאריות עם מספר משובים דו-צדדיים (Bilateral Feedbacks).

בעבודה זו פותח מודל אנליטי מדויק ומודל פונקציונלי עבור פונקציית ההגבר בחוג סגור של רשתות עם משוב בעלות ריבוי מקורות תלויים ומספר משובים דו-צדדיים. כתוצאה מכך, קיבלנו את פונקציית ההגבר בחוג סגור בצורה מטריציונית שמרכיביה הם יחסי ההחזרה (Return Ratios). במודל שפותח, רשתות ליניאריות עם משוב בעלות ריבוי מקורות תלויים מתוארים בצורה אנליטית בעזרת יחסי ההחזרה. ברשתות האלה, במקרה הכללי כל מקור תלוי תורם לא רק לאות ששולט בו (Control signal) אלה גם לאותות ששולטים על מקורות

תלויים אחרים. לשם עקביות, אנו מתייחסים בעבודה זו לתרומות מצולבות (Cross Contributions) האלה, מנורמלות לערך האות השליטה המוחלט (total control signal) שמופיע במקור התלוי, כיחסי החזרה צולב (Cross Return Ratios). אנו הראנו שאת יחסי ההחזרה ואת יחסי ההחזרה הצולבים ניתן ליחד ליחס ההחזרה הכללי (General Return Ratio). למיטב ידיעתנו, עבודה זו מראה בפעם הראשונה איך ניתן להשיג לרשתות לינאריות עם משוב בעלות ריבוי של מקורות תלויים יחס החזרה כללי כפונקציה של אלמנטי הרשת. את התוצאות האלו אנו השגנו על ידי שימוש בביטור (Dissection) והרכבה (Superposition) בניתוח של הרשתות. הפקת יחס ההחזרה הכללי כפונקציה של מרכיבי המעגל, בעל חשיבות רבה לניתוח יציבות הרשתות. זה גם הופך את בעיית היציבות לבעיית חישוב של עקומת הנייקויסט (Nyquist plot) ומאפשרת שימוש בכלים נוספים של תורת הבקרה.

על מנת לבחון את ההשפעה של משוב שלילי על העכבה בין שתי נקודות שרירותיות ברשתות לינאריות עם משוב אנו תארנו אותם בצורה אנליטית כפונקציה של יחסי ההחזרה. אנו הוכחנו שאת נוסחת בלקמן (Blackman) ניתן להרחיב למקרה של רשתות לינאריות עם משוב בעלות ריבוי של מקורות תלויים. על מנת להגיע למטרה זו, אנו שיחזרנו את ההוכחה של נוסחת בלקמן, באותו אופן הוכחנו אותה עבור רשתות עם משוב בעלות שני מקורות תלויים ולאחר מכן הוכחנו למקרה של רשתות בעלות ריבוי של מקורות תלויים. אנו גם כתבנו את הנוסחה הזאת בצורה מטריציאית.

כפי שהוזכר, מציאת יחס ההחזרה הינו בעל חשיבות רבה למציאת השפעת המשוב לפונקציית ההגבר בחוג סגור, עכבות ויציבות. בדרך הרגילה, את יחס ההחזרה מוצאים על ידי כיבוי של כל המקורות הבלתי-תלויים ברשת, הקצאת ערך קבוע למקור תלוי וחישוב של האות שמוחזר למסוף השליטה שלו (Control Terminal). אך, תהליך זה אינו מתאים לסימולציות של מעגלים אלקטרוניים או לאבחון מעגלים אלקטרוניים אמיתיים. גישה חלופית הוצעה על ידי מידלברוק (Middlebrook) ומורכבת מחיבור של מקורות בוחן לנקודות נגישות (Accessible terminals) של הטרנזיסטור או מגבר שרת. לפי שיטה זו, שני יחסי ההחזרה T_i ו- T_o נמדדים, אחד עבור הזרקת מתח (voltage injection) והשני עבור הזרקת זרם

(current injection) בתוך המעגל. אחר-כך הם מתורגמים ליחס ההחזרה אמיתי T . השיטה הזו אינה מדויקת עבור תדרים בהם יחס ההחזרה קרוב לאפס (Loop gain crossover) $|T| < 1$. השיטה המשופרת של מידלברוק מתגברת על חיסרון הזה. בשיטה הזו יחס-ההחזרה נמדד על ידי הזרקה בו זמנית של מתח וזרם לתוך המעגל. לפי שיטה זו, שני יחסי החזרה חלקיים נמדדים, אחד T_v^n נמדד עבור אותות מוזרקים מתואמים כדי לאפס את הזרם בנקודה מסוימת של המעגל, והשני T_i^n מתקבל כאשר המתח על המקור התלוי שווה לאפס. לאחר מכן, T_v^n ו- T_i^n מתורגמים ליחס ההחזרה של המעגל כולו. ההוכחות המקוריות של השיטות האלו משתמשות במודל משוב אידיאלי, שלא לוקח בחשבון מעגלים עם מספר משובים דו-צדדיים (Bilateral feedback loop). ההוכחה שלנו מבוססת על מודל משוב כללי (Generic feedback network). כתוצאה מכך, בעבודה הזו הוכח שאת שתי הנוסחאות של מידלברוק ניתן ליישם ללא קירובים לכל הרשתות הלינאריות עם משוב בעל מקור תלוי אחד. כתוצאה מהעבודה הזו, שתי תורות קלאסיות של בודה ושל בלקמן לניתוח מעגלים מורחבות לסוג הרבה יותר רחב של מעגלים בעלי ריבוי טרנזיסטורים. כמו כן התיאוריה המוצגת בעבודה הזו יכולה לשמש כהסתמכות שימושית כדי לעשות חישובים מהירים על הנייר, ובכך, להשיג הבחנה אינטואיטיבית טובה יותר על השפעת המשוב על פונקציית ההגבר בחוג סגור של הרשת ועכבות המעגל.

אוניברסיטת בן-גוריון בנגב
הפקולטה למדעי ההנדסה
המחלקה להנדסת חשמל ומחשבים

משוב כסופרפוזיציה

חיבור זה מהווה חלק מהדרישות לקבלת תואר מגיסטר בהנדסה

מאת : פבל שנין

09.2013

אלול תשע"ג

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מאת : פבל שנין

מנחה/ים : פרופ' יבגני פפרנו

.....תאריךחתימת המחבר
.....תאריךאישור המנחה/ים
.....תאריך
.....תאריךאישור יו"ר ועדת תואר שני מחלקתית