Envelope Impedance: Theory and Applications

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Abstract - **Various power electronics systems, such as resonant converters, motor drives, and electronic ballasts for fluorescent lamps, are driven by carriers of relatively high frequency as compared to the control bandwidth. In these cases, the small signal response relevant to the feedback network design is embedded in the envelope of the signals rather then in the carrier. In this study, a systematic procedure for deriving the Envelope Impedances (EI) of elements and sub systems was developed. The proposed method is based on phasor transformation by which the carrier is eliminated, leaving only the low frequency components. It is demonstrated, by analyzing passive elements and sub circuits, that the EI parameter could help to better understand and analyze carrier driven systems and may lead to new applications. The predictions of proposed method were verified against experimental measurements.**

I. INTRODUCTION

Various power electronics systems, such as resonant converters, motor drives, and electronic ballasts for fluorescent lamps, are driven by carriers of relatively high frequency as compared to the control bandwidth. In these cases, the small signal response relevant to the feedback network design is embedded in the envelope of the signals rather then in the carrier. To illustrate this point we consider a simple system shown in Fig. 1 that includes a modulated source that feeds a load via a reactive network. A feedback signal is taken off the voltage across the load, rectified, compared to a reference and the error signal is used to control the source by AM, FM or PM. A key parameter in the behavior of such a system is the Envelope Impedance (EI) defined as the ratio between the voltage envelope and the current envelope for a given modulating frequency f_m. Since the EI of reactive elements and resonant networks play a major role in the behavior of carrier driven system, a characterization of the EI could help to better understand, analyze and design such carrier driven systems. The objective of this study was thus to develop analytical tools for extracting the EI of basic elements (L, C) and sub systems (series and parallel resonant networks) and to study their behavior.

II. THE ENVELOPE IMPEDANCE

Any analog modulated signal (AM, FM, and PM) can be described by the following general expression:

$$u(t) = U_1(t)\cos(\omega_c t) + U_2(t)\sin(\omega_c t) \tag{1}$$

where $U_1(t)$ and $U_2(t)$ are modulation signals and ω_c is the angular frequency of the carrier signal and t is time.

Expression (1) can also be written in complex form as [2]:

$$u(t) = \mathrm{Re}\left[(U_1(t) - jU_2(t))e^{j\omega_c t}\right] \tag{2}$$

The envelope equivalent circuit model [1] - [5] can be used to derive the EI of circuit elements and sub systems. In the followings, we derive the EI of an inductor, capacitor, and series and parallel resonant networks. For the sake clarity we first present, step-by-step, the procedure for deriving the PI of an inductor.

EI of an Inductor. The state space equation of an inductor is given by:

$$u_L = L\frac{di_L}{dt} \tag{3}$$

Applying (2):

$$(U_1(t) - jU_2(t))e^{-j\omega_c t} = L\frac{d}{dt}(I_1(t) - jI_2(t))e^{-j\omega_c t} \tag{4}$$

Assuming that the current is also of the form:

$$i(t) = (I_1(t) - jI_2(t))e^{-j\omega_c t} \tag{5}$$

we find

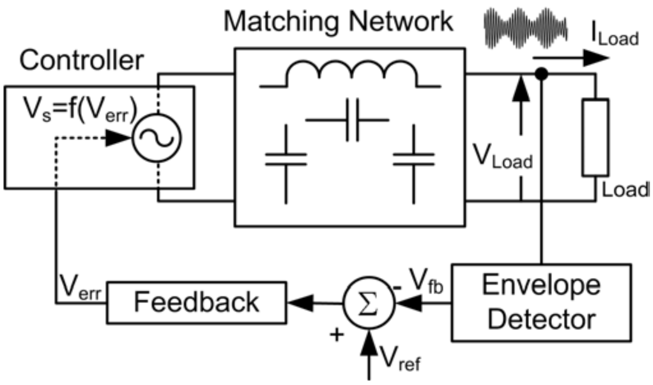

Fig. 1. A general representation of a carrier driven system.

$$\left(U_1(t) - jU_2(t)\right)e^{-j\omega_c t} = \left(\left(L\frac{d}{dt}I_1(t) - \omega_c L I_2(t)\right) + j\left(L\frac{d}{dt}I_2(t) + \omega_c L I_1(t)\right)\right)e^{-j\omega_c t} \quad (6)$$

Dividing out the phasor term from both sides of the equation we obtain:

$$U_1(t) - jU_2(t) = \left(L\frac{d}{dt}I_1(t) - \omega_c L I_2(t)\right) + j\left(L\frac{d}{dt}I_2(t) + \omega_c L I_1(t)\right) \quad (7)$$

Separating out the equation into the real part and imaginary part we find:

$$\begin{cases} \mathrm{Re}:\ U_1(t) = L\frac{d}{dt}I_1(t) - \omega_c L I_2(t) \\ \mathrm{Im}:\ U_2(t) = L\frac{d}{dt}I_2(t) + \omega_c L I_1(t) \end{cases} \quad (8)$$

A two-port network (Fig. 2) excited by the real and imaginary portions of the modulating signal (1) can represent these two cross-coupled equations.

The Z matrix of the network is defined by:

$$\begin{bmatrix} U_1(t) \\ U_2(t) \end{bmatrix} = Z_{LE}\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} \quad (9)$$

where:

$$Z_{LE} = \begin{bmatrix} sL & -\omega_c L \\ \omega_c L & sL \end{bmatrix} \quad (10)$$

The absolute envelope voltage and current are:

$$\begin{aligned} \left|\vec{U}(t)\right| &= \sqrt{U_1^2(t) + U_2^2(t)} \\ \left|\vec{I}(t)\right| &= \sqrt{I_1^2(t) + I_2^2(t)} \end{aligned} \quad (11)$$

Solving for the real and imaginary parts of the currents we find:

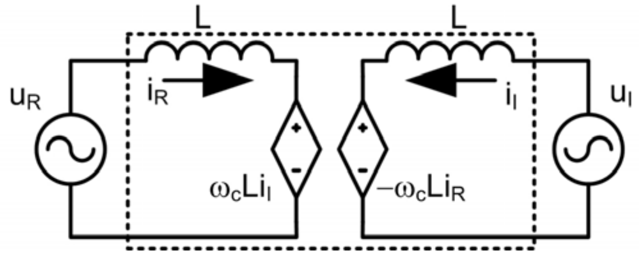

Fig. 2. Equivalent circuit of an inductor for envelope impedance (EI) analysis.

$$\begin{cases} I_1(t) = \dfrac{s U_1(t) + \omega_c U_2(t)}{L\left(s^2 + \omega_c^2\right)} \\ I_2(t) = \dfrac{s U_2(t) - \omega_c U_1(t)}{L\left(s^2 + \omega_c^2\right)} \end{cases} \quad (12)$$

And hence:

$$\left|\vec{I}(t)\right| = \sqrt{I_1^2(t) + I_2^2(t)} = \sqrt{\frac{U_1^2(t) + U_2^2(t)}{L^2\left(s^2 + \omega_c^2\right)}} \quad (13)$$

For steady state $s = j\omega$

$$\left|\vec{I}(t)\right| = \frac{\left|\vec{U}(t)\right|}{L\sqrt{\left(\omega_c^2 - \omega^2\right)}} = \frac{\left|\vec{U}(t)\right|}{L\omega_c\sqrt{(1-d)}} \quad (14)$$

where

$$d = \left(\frac{\omega}{\omega_c}\right)^2 \quad (15)$$

The EI of the inductor is then found to be:

$$Z_L = \frac{\left|\vec{U}(t)\right|}{\left|\vec{I}(t)\right|} = L\omega_c\sqrt{1-d} \quad (16)$$

If $\omega_m << \omega_c$

$$\lim_{d\to 0}\left(Z_L\right) = \omega_c L \quad (17)$$

EI of a Capacitor. Following the above approach, we find for a capacitor C:

$$Y_C = \frac{\left|\vec{I}(t)\right|}{\left|\vec{U}(t)\right|} = C\omega_c\sqrt{1-d} \quad (18)$$

$$\lim_{d\to 0}\left(Y_C\right) = \omega_c C \quad (19)$$

EI of a Series Resonant Network. Following the above approach and applying the equivalent circuit of Fig. 3, we find for a series resonant network:

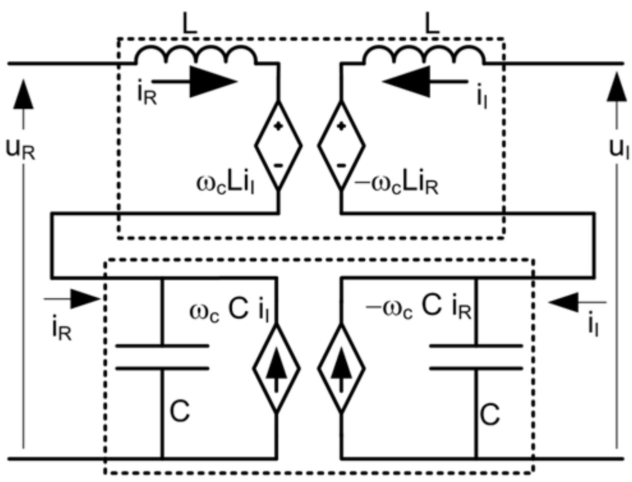

Fig. 3. Equivalent circuit of a series resonant circuit for envelope impedance ($EI = Z_{CLser}$) analysis.

$$Z_{CLser} = L\sqrt{\frac{\left(\omega^2-(\omega_c+\omega_r)^2\right)\left(\omega^2-(\omega_c-\omega_r)^2\right)}{\omega_c^2-\omega^2}} \quad (20)$$

$$\omega_r = \frac{1}{\sqrt{CL}} \quad (21)$$

for $\omega << \omega_c$

$$Z_{CLser} = \frac{\omega_c+\omega_r}{\omega_c} L\sqrt{(\omega_c-\omega_r)^2-\omega^2} \quad (22)$$

The frequency dependence of this envelope impedance Z_{CLser} is depicted in Fig. 4

EI of a Parallel Resonant Network. Applying the equivalent circuit (Fig. 5), we find:

$$Y_{CLpar} = C\sqrt{\frac{\left(\omega^2-(\omega_c+\omega_r)^2\right)\left(\omega^2-(\omega_c-\omega_r)^2\right)}{\omega_c^2-\omega^2}} \quad (23)$$

for $\omega << \omega_c$

$$Y_{CLpar} = \frac{\omega_c+\omega_r}{\omega_c} C\sqrt{(\omega_c-\omega_r)^2-\omega^2} \quad (24)$$

The frequency dependence of the envelope admittance Y_{CLpar} is shown in Fig. 6.

III. EXPERIMENTAL

The proposed EI simulation method was verified experimentally by testing a series LR network (Fig. 7a), that was excited by an amplitude modulated signal, where u_c is the carrier wave and u_m is the modulating signal. The behavior of the resistor's envelope voltage v_p (amplitude and phase) was measured (Fig. 8, dots). Fig. 7b depicts the electrical equivalent circuit used for running small signal envelope simulation of the LR circuit by proposed method.

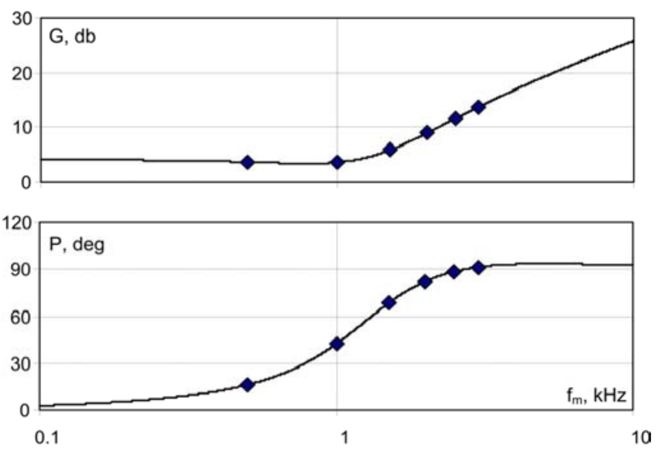

Fig. 4. Envelope impedance (EI) of a series resonant network as a function of modulation frequency. f_c = 1MHz, f_r = 1.001 MHz. Points show result of cycle-by-cycle simulation, line - small signal envelope analysis.

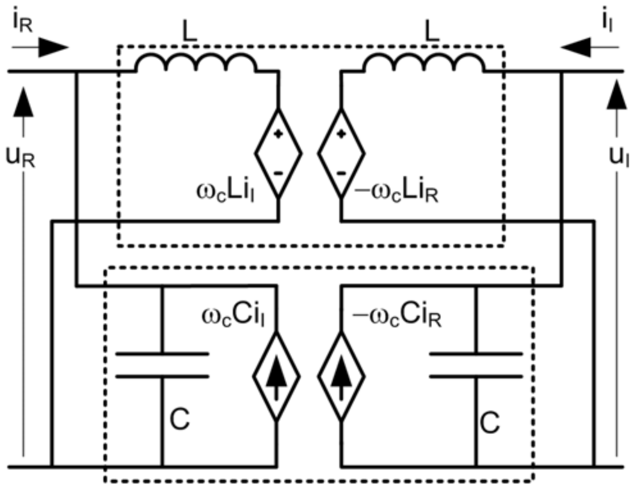

Fig. 5. Equivalent circuit of a parallel resonant network for envelope impedance (EI) analysis.

The circuit was excited by sine-wave signal. The root mean square of real and imaginary components v_{outR} and v_{outI} (Fig. 7b) emulate v_p. The simulation results (solid line on Fig. 8) were found to duplicate the experimental results (dots).

IV. RESULT AND DISCUSSION

The results obtained in this study show that for $\omega_m << \omega_c$ the EI of an inductor and capacitor are constant and independent of ω_m. In this sense these elements can be considered "resistive" for a modulating signal. However, when an inductor and a resistor are in series, the total EI need to be summed vectorially. That is, the phase shift between the carrier voltages across the reactive element and

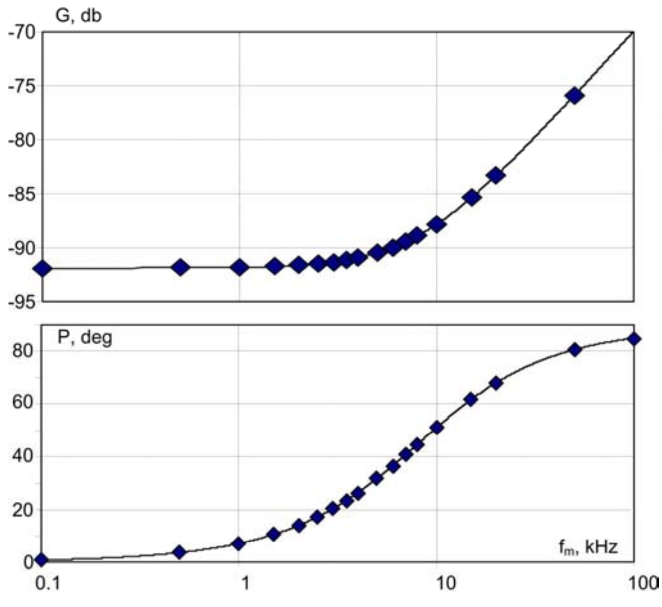

Fig. 6. Envelope admittance Y_{CLpar}=1/(EI) of a parallel resonant network as a function of modulation frequency. f_c= 1MHz, f_r = 1.001MHz, Quality factor = 62800. Points show result of cycle-by-cycle simulation, line - small signal envelope analysis.

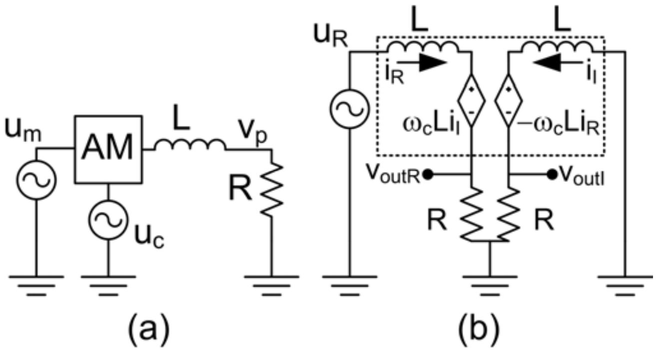

Fig. 7. Experimental setup (a) and its equivalent circuit after phasor transformation (b). The mean square of v_{outR} and v_{outI} emulate the envelope of the v_p.

the resistor needs to be taken into account. This was verified experimentally and compared to envelop simulation results.

The EI of the resonant networks are rather complex and a number of features of these EI are unique. For example for $\omega_m < (\omega_c - \omega_r)$, the EI are resistive, whereas for $\omega_m > (\omega_c - \omega_r)$ the EI are reactive. This is depicted in the plots of Fig. 4 and Fig. 6. It should further be noticed that the break points are located at low frequency $\{\omega_m = \omega_c - \omega_r\}$. This implies that in carrier driven systems, signal interactions could take place at low frequencies much below the frequency of the carrier and might affect the dynamics and stability of the system. For example, the minimum in the EI of a Piezoelectric Transformer [5] could cause instability when driving a fluorescent lamp.

The EI results of this study could lead to new and exciting technologies. For example, examination of (22) and (24) reveal that for the range $\omega_m > \omega_c - \omega_r$, one could emulate a variable inductor or capacitor, the value of which can be adjusted by ω_c.

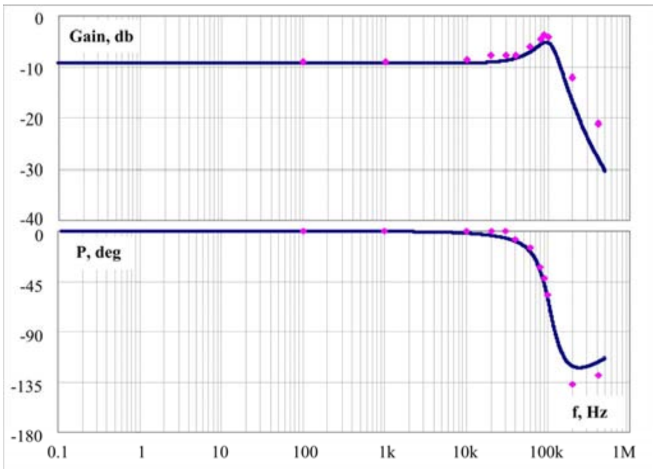

Fig. 8. Experimental results. Measured envelope of v_p in Fig. 7(a) (dots) compared to small-signal envelope simulation results (lines). Upper plot: amplitude. Lower plot: phase

V. Conclusions

In this study we developed a systematic procedure for deriving the Envelope Impedances (EI) of elements and sub systems. It was shown that the EI behavior is far from being simple and/or intuitively inferred. It is further demonstrated that the EI parameter could help to better understand and analyze carrier driven systems and may lead to new applications.

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