Average Modeling Technique for Switched Capacitor Converters Including Large Signal Dynamics and Small Signal Responses

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***Abstract* — A generic average model that is capable of predicting the static, large signal dynamics and small signal response of Switched Capacitor Converters (SCC) was developed and tested. The proposed model was verified by full circuit simulation and experimentally, and good agreement was found between the results. The model was used to study dynamic behavior of SCC systems including the small signal responses, which are required for designing control loops. The model can be of great design value as it can be used to optimize the dynamic design of SCC systems and to design the compensator for closed loop operation.**

***Index Terms* — Switched Capacitor Converter, SCC, modeling, dynamic behavior, power converter, small signal, large signal.**

I. Introduction

Switched capacitor converters (referred further as SCC, for singular and plural), also known as a charge pumps, are a family of DC-DC converters that transfer a charge between input and output of the converter employing one, or a number of flying capacitors. The flying capacitor is first connected to the input charging the capacitor, and then connected to an output, or the next flying capacitor in chain to transfer the charge towards the output, or the load. SCC are preferred in a number of power management systems due to their small size, the absence of inductors and integration compatibility.

The static behavior of SCC systems was analyzed in numerous earlier studies (e.g. [1-13]), in which the expressions of the voltage transfer ratios and the expected losses were derived. The objective of this study was to investigate the dynamic behavior of SCC systems including the small signal responses, which are required for designing control loops. This was accomplished by developing a generic average model of SCC systems that is capable of predicting not only the static, but also the large signal dynamic behavior and small signal responses of SCC converters. The model is an extension of the analytical findings of [12], [13] and includes the dynamic aspects. The proposed model covers hard switched SCC topologies and charge pumps that are built around active switches, or diodes, or both. The model is also applicable to multi phase, multi capacitors SCC if it can be assumed that each of the subcircuits of the modeled SCC can be described, or approximated, by a first order RC circuit which, as described in [12], is possible in many practical cases. The model covers all operational modes of SCC, Complete Charge – CC (Fig. 1a), Partial Charge – PC (Fig. 1b), and No Charge – NC (Fig. 1c) as discussed in [13].

II. Basic aspects of the proposed SCC modeling approach

For the sake of brevity, the model development is presented by considering an inverting 1:1 charge pump converter (Fig. 2). The converter consists of a flying capacitor C_f, with a lossy component R_{ESR}, two switches S_1 and S_2 with "ON" resistance of R_{s1} and R_{s2} respectively, an output filter capacitor C_o with R_{ESRo} and load resistance R_o. It should be noted that the model treats the switches as a resistive elements when "ON" and conducting, and discontinuity while "OFF", any possible non-linearity of the switches, like entering saturation mode for example during the turn on of the converter, due to the high inrush current, are not taken into account in this model. The switches are operated complementary to each other, running at switching frequency, f_s. Switches operation brings to periodical charge of the flying capacitor during T_1, when S_1 is "ON", and discharge of the flying capacitor during T_2, when S_2 is "ON". The equivalent charge and discharge circuits are presented in Fig. 3 as the charge subcircuit (i = 1, Fig. 3a), and discharge subcircuit (i = 2, Fig. 3b).

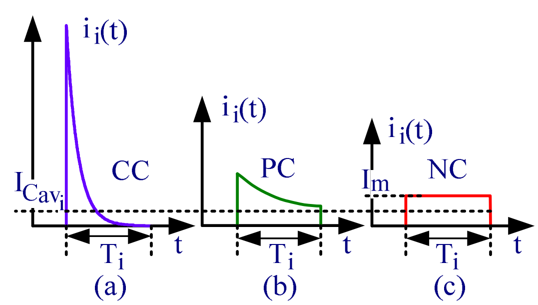

Fig. 1. Possible charge current shapes: (a) Complete charge-CC; (b) Partial charge-PC; (c) No charge-NC.

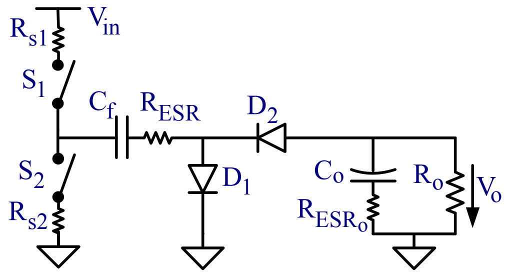

Fig. 2. Inverting 1:1 charge-pump.

Each of the subcircuits can be further simplified to a simple RC equivalent circuit of Fig. 4, where R_i and C_i are the total charging/discharging loop resistance and capacitance respectively. V_i is the voltage source that represents voltage across the corresponding switch before closure and a total loop capacitor C_i. The values of R_i and C_i for the inverting SCC/Charge-pump are calculated in (1).

$$R_1 = R_{s1} + R_{ESR}; \quad R_2 = R_{s2} + R_{ESR} + R_{ESR_o}$$
$$C_1 = C_f; \quad C_2 = \frac{C_f \cdot C_o}{C_f + C_o} \tag{1}$$

Once a switch is turned on, the equivalent capacitor will start charging (by a positive or negative ΔV_i) and a current $i_i(t)$ will build up. Depending on the relationship between the duration T_i and the time constant R_iC_i, the current can take one of three possible shapes. For $T_i >> R_iC_i$, the charging will be completed within T_i (Fig. 1a); this case is denoted as CC. For $T_i \cong R_iC_i$, the charging will be partial (PC, Fig. 1b). For $T_i << R_iC_i$, there will be no effective charging (NC,) and the current will be practically constant (Fig. 1c). In this latter case, the capacitor voltage will stay about constant within T_i.

As was proven in an earlier publication [12], the average power P_i dissipated by a given subcircuit i during a switching phase duration T_i can be expressed as a function of the average current I_{avi} in the subcircuit (averaged over the switching period T_s=1/f_s):

$$P_i = \left(I_{av_i}\right)^2 \cdot \left\{\frac{1}{2f_sC_i} \cdot \frac{(1+e^{-\beta_i})}{(1-e^{-\beta_i})}\right\}; \quad \beta_i = \frac{T_i}{R_iC_i} \tag{2}$$

The term in brackets can be defined as the equivalent resistance R_{ei} of subcircuit i:

$$R_{e_i} = \left\{\frac{1}{2f_sC_i} \cdot \frac{(1+e^{-\beta_i})}{(1-e^{-\beta_i})}\right\} \tag{3}$$

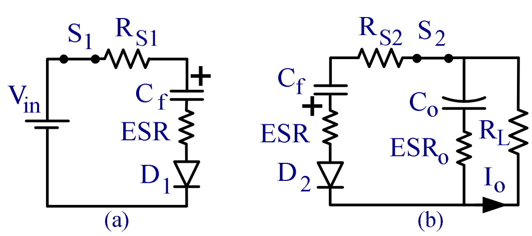

Fig. 3. Inverting 1:1 charge-pump, instantaneous subcircuits: (a) Charging (i = 1), (b) Discharging (i = 2).

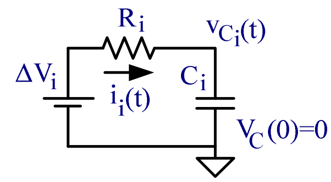

Fig. 4. Simple RC equivalent charge/discharge circuit.

Based on the above results, an average circuit can be constructed reflecting the average behavior of the relevant subcircuit (Fig. 5). At this point it is important to emphasize the difference between the circuit of Fig. 4, which represent the momentary current i(t), and the average circuit of Fig. 5, which represent average voltages and currents (averaged over the full switching cycle).

III. SCC Large signal Average Model Construction

Each of the equivalent circuits of Fig. 3 is translated into its **average** equivalent circuit, according to Fig. 5. The resulting equivalent subcircuits are shown in Fig. 6. Based on (3), the equivalent resistances are calculated to be:

$$R_{e_1} = \frac{1}{2f_sC_f} \cdot \coth\left(\frac{\beta_1}{2}\right)$$
$$R_{e_2} = \frac{1}{2f_s\left(\frac{C_f \cdot C_o}{C_f + C_o}\right)} \cdot \coth\left(\frac{\beta_2}{2}\right) \tag{4}$$

where

$$\beta_1 = \frac{1}{2f_s \cdot (R_{S_1} + ESR) \cdot C_f}$$
$$\beta_2 = \frac{1}{2f_s \cdot (R_{S_2} + ESR + ESR_o) \cdot \left(\frac{C_f \cdot C_o}{C_f + C_o}\right)}$$

The subcircuits of Fig. 6 also include voltage sources V_{D1} and V_{D2} that represent the diodes of the SCC to retain the correct power dissipation of the circuit. The magnitude of these voltage sources are set to represent the average voltage of the diodes such that the product of the average current of the subcircuit times the voltage of the diode emulators is equal to the correct power dissipated by the diodes.

The average equivalent subcircuits of the charge and discharge phases given in Fig. 6 are linear and emulate the

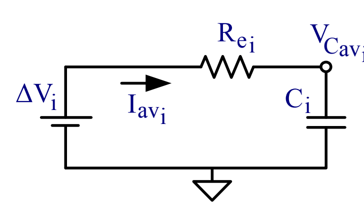

Fig. 5. SCC equivalent average subcircuit.

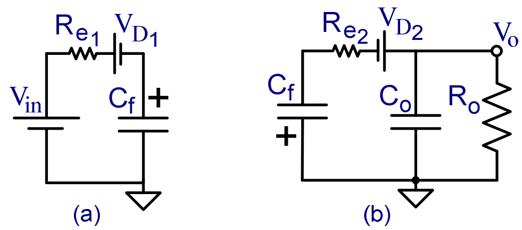

Fig. 6. Inverting 1:1 charge-pump average dynamic equivalent circuits: (a) Charging subcircuit (i=1), (b) Discharging subcircuit (i=2).

average, static and dynamic, large signal behavior of the original SCC circuit (Fig. 2), divided according to its switching phases (Fig. 3).

The subcircuits of Fig. 6 can be used to calculate the static average currents and voltages by simple KVL equations. It should be noted that the average voltage on C_f in the two circuits is identical (the average voltage within the switching period). However, these two circuits cannot be run as is on a circuit simulator since there is no mechanism to retain the charge balance of the flying capacitor C_f. This can be remedied by introducing an ideal DC "transformer" to reflect the capacitor from one subcircuit to the other. The transformer can be realized by dependent voltage source, E_T, and dependent current source, G_T, (Fig. 7). This transformer carries out the inverting action, and overcomes the problem of subcircuits interconnection when there is no common reference point for the capacitor in the two circuits.

The coupled circuit of Fig. 7 can help explain an apparent ambiguity of the "average subcircuit current" concept used in this work. The averaging method applied in this model, and the meaning of the phrase "averaged over the full switching cycle" is based on the integration of the instantaneous current (solid trace in Fig. 8) to find the total charge Q_i (crossed area in Fig. 8), passing during the time slot, T_i, and dividing this total charge by the switching period, T_s. The result is an average current of the subcircuit, averaged over the full switching cycle (dashed line in Fig. 8). The resulting average circuits (Fig. 5, Fig. 6) represent only one phase of the operation (charge or discharge). Since the same capacitor will be involved in the complementary phase, the capacitor will be charged and discharged and the net charge transferred to the capacitor, at steady state, will be zero. This coupling of the capacitor between the two subcircuits is accomplished by the DC "transformer" (Fig. 7).

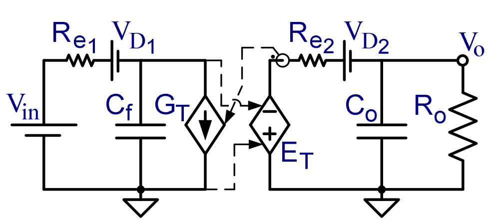

Fig. 7. Inverting 1:1 charge-pump average dynamic equivalent circuit.

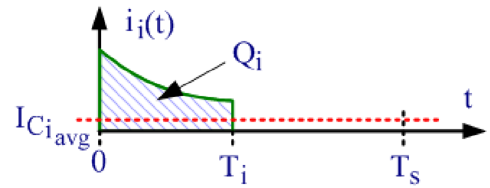

Fig. 8. Averaging of a subcircuit's current.

IV. SIMULATION AND EXPERIMENTAL RESULTS

The proposed SCC generic behavioral average circuit model was verified against full circuit simulation (Cycle-By-Cycle) carried out on two different software packages – PSIM and OrCAD PSpice, and against experimental results obtained by a laboratory breadboard. Verification was carried out on the inverting SCC of Fig. 2, while circuit parameters and experimental devices were selected as follows: Duty Cycle = 50%; Dead Time = 120ns; Input voltage V_{in} = 12V; Switches S_1 - SMU10P05, S_2 - SMU15N05; Diodes V_{D1} and V_{D2} - MBR320P; Flying capacitor C_f = 22μF and R_{ESR} = 0.1•; Output capacitor C_o = 560μF and R_{ESRo} = 33m•. Experiments and simulations covered all operational modes: CC with f_s = 5kHz, PC with f_s = 50kHz, and NC with f_s = 250kHz.

Experimental results were conducted such as to cover both large and small signal response of the converter, in an attempt to cover and validate most of the model abilities. Large signal response includes a load step in PC mode (Fig. 9), and turn on transients at CC (Fig. 10) and NC (Fig. 11) modes. Fig. 12 shows the small signal response $\{v_o/f_s(f)\}$ in CC and NC operation mode, obtained by the average model simulation and experimentally. The small deviations of the experimental results from average model simulations are probably due to experimental uncertainties such as errors in the exact values of the capacitances and resistances (ESR), nonlinearities, etc.

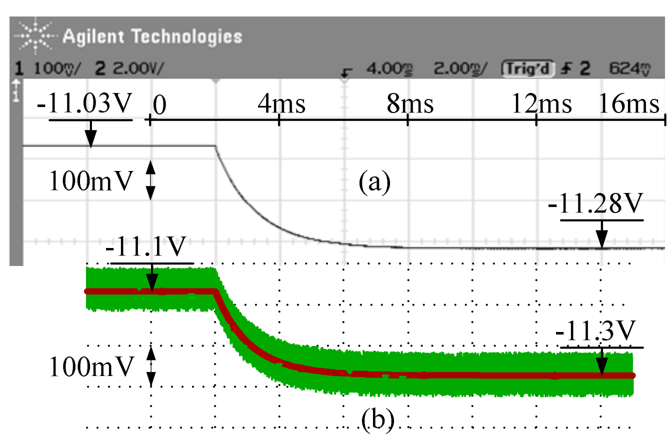

Fig. 9. Load-step response of the inverting 1:1 SCC - PC Mode (a) Experimental, (b) Wide trace – full circuit simulation; Solid thin trace – Average model simulation.

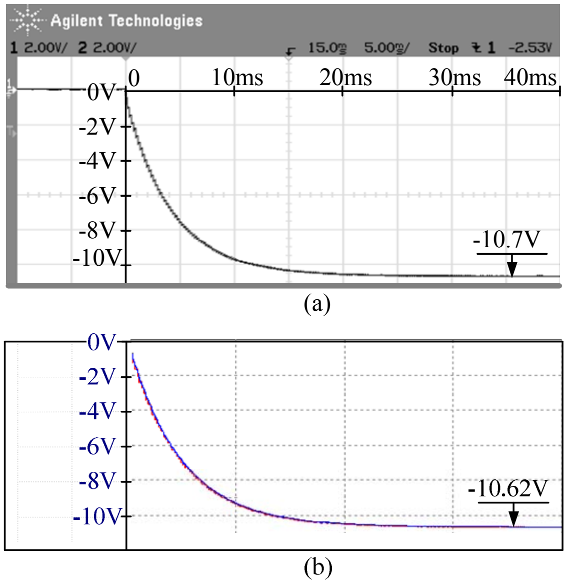

Fig. 10. Start-up transients of the inverting 1:1 charge-pump in a CC mode: (a) Experimental, (b) Full Simulation and model based average simulation. Please note: in (b) the two traces are on top of each other.

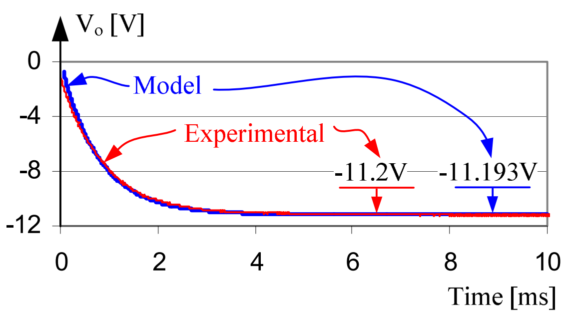

Fig. 11. Start-up transients of the inverting 1:1 charge-pump in NC mode, experimental and average model simulation. Please note: the two traces are on top of each other.

V. Discussion and Conclusions

The main attributes of the new modeling approach is that it is based on average modeling and that the resulting SPICE compatible equivalent circuits emulate the large and small signal responses of the modeled SCC. For a given , the model is linear while the non-linear relationships between the components of (T_i, C_i, R_i) and R_{ei} are accurately retained and can be replicated by large and small signal simulation. The model covers all operational modes (CC, PC, and NC) and can be used to examine the effects of individual elements such as the resistance of each switch, the ESR of each capacitor, the influence of capacitors' values and the effect of the switching frequency and duty cycle. A powerful feature of the model is its seamless compatibility with SPICE based AC simulation in which the linearization (when required) is done by the simulator. As in the case with the Switched Inductor Model

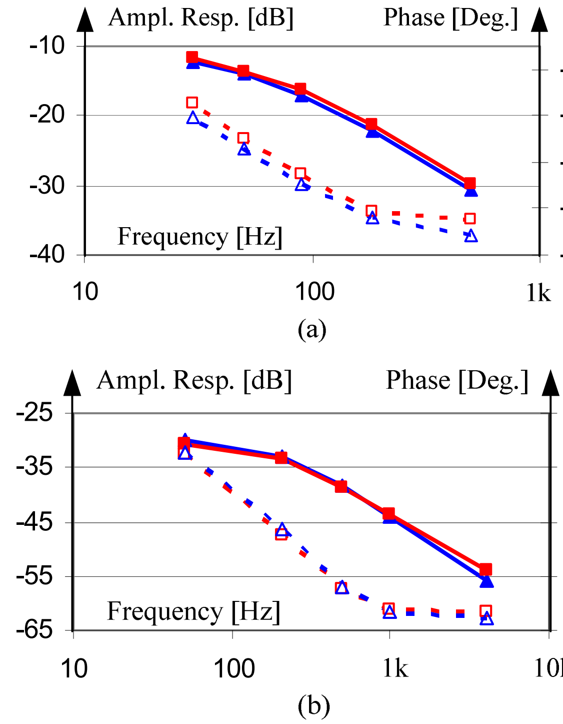

Fig. 12. Small signal, control to output responses {vo/fs(f)} of the inverting 1:1 SCC, obtained by the model (AC analysis) and experimentally: (a) CC mode, (b) PC mode; Triangles – Average model simulation results; Squares - Experimental results; Solid traces – Amplitude response; Dashed traces – Phase response.

(SIM) [14-17], this can be used conveniently to obtain the small signal control to output responses for various control methods such as duty cycle control or frequency control. These results can also be of a great educational value, and for general understanding of SCC dynamics.

Although discussed and demonstrated by a simple inverting 1:1 SCC/Charge-pump, the proposed modeling methodology can be easily applied to multi-capacitor and multi-phase SCC systems such as those described in [18-20] by applying the concepts introduced in [12] and [13].

Acknowledgement

This research was supported by The Israel Science Foundation (grant 476/08)

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