

# Frequency Tracking to Maximum Power of Piezoelectric Transformer HV Converters under Load Variations

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**Abstract**—The problem of maximum power point tracking of high output DC voltage converters that apply Piezoelectric Transformers (PT) and voltage doublers was studied theoretically and experimentally. It was shown that the operating frequency of the PT, at which maximum voltage gain is reached, is a function of the load. Hence, under load variations, and to overcome parameters instability, there is a need for some mechanism of frequency tracking that will help to lock the operating frequency to the optimum one. The proposed method to achieve frequency tracking is based on a Phase Locked Loop (PLL). The PLL inputs are the phase of the input voltage driving the PT and the phase of the current flowing through one of the voltage doubler diodes. Theoretical analysis, verified by experiments, shows that when the phase shift of the diode current relative the phase of the input voltage is zero, the voltage gain of the system is at its maximum point. By applying this approach, the system operation can be made independent on input voltage, load variations, temperature (within permitted range), and the spread and non-linearity of the PT parameters, as well their drift with time.

## I. INTRODUCTION

The main advantages of Piezoelectric Transformers (PTs) are potential low cost, small size, low profile, good insulation capability and the absent of windings and hence magnetic field. In some specific applications, PTs are superior to electromagnetic transformers, making the PT a good design choice. Among these is the generation of a low power, High DC Voltage (HV). Rosen type (Fig. 1) PTs have a high gain ratio that, when combined with excellent insulation properties of the PT, make it a good candidate for the construction of compact HV converters – up to few kV. Since the PT is a resonant element, its output to input voltage gain is strongly dependent on the operating frequency [1].

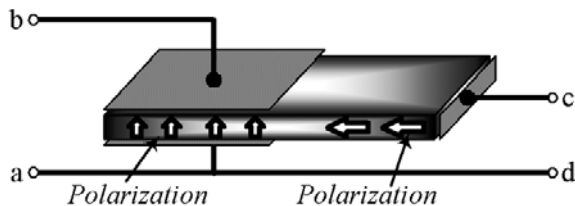


Fig. 1. Rosen type piezoelectric transformer.

Hence, to maintain maximum output voltage under variable operating conditions (load variation, temperature changes and components tolerances) it is necessary to lock the operating frequency to the one that will ensure highest possible output voltage for any given load.

As a prerequisite for solving the frequency-tracking problem, one needs first to find a parameter that can be used as a measure of the deviation from the desired frequency. Different approaches have been suggested for frequency tracking of PTs drivers. In [2] the phase angel between input voltage and input current was used as a criterion while in [3], the phase angel between input and output voltage was used as a measure for the deviation from the frequency of maximum gain. Unfortunately, these criteria are load dependent and could be used only over a narrow load resistance range.

The difficulty in locating the optimum tracking parameter can be appreciated by considering the equivalent circuit of a typical PT (Fig. 2a) and its series resonance representation (Fig. 2b).

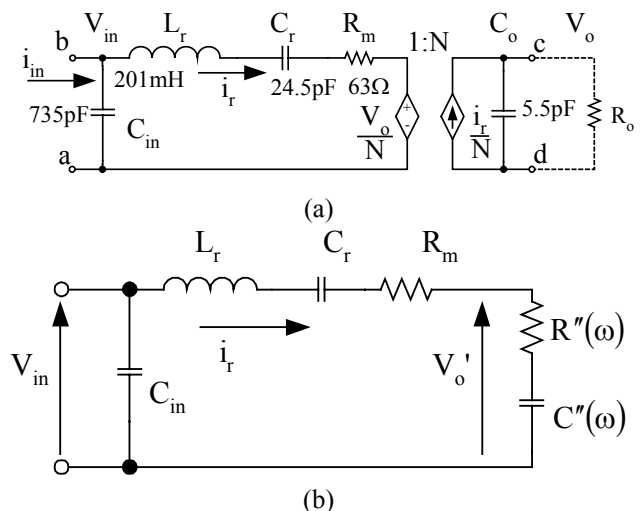


Fig. 2. Equivalent circuit of a PT (a) Original equivalent circuit (b) Simplified equivalent circuit reflecting secondary to the primary side. Values are for PXE43, Philips, operating around 73 kHz.

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As can be easily observed from Fig. 2a [1], the angular frequency of the maximum output to input voltage ratio will be between  $\sqrt{(C_r + C'_o)/(L_r C_r C'_o)}$  for high resistance loads (close to the open circuit situation, load is negligible) and  $1/\sqrt{L_r C_c}$  for low resistance loads, when  $C_o$  is practically shorted. Thus, the frequency of the maximum output to input voltage ratio is a strong function of the load resistance. However, since the series resonance branch (Fig. 2b) is responsible for the input to output power transfer, it stands to reason that maximum voltage gain will be obtained when the operating frequency is locked to this series resonance. In this study we explored this possible criterion for frequency locking to maximum power and propose a novel method for obtaining a reliable bipolar signal that is a measure of the deviation from the optimal frequency. It is then demonstrated how this signal can be used to lock the frequency to the optimal one.

## II. THE ROSEN TYPE PT IN HV APPLICATIONS

The Rosen-type piezoelectric transformer 48x12x2mm (Phillips [4]) was chosen for present research (Fig. 1). This unit has a large output to input voltage ratio at the frequency of the maximum output voltage, relatively high power (up to 5 W), and high input to output insulation (tens of kilovolts). The equivalent circuit of a PT operating near its resonance point (Fig. 2a) includes a resonant network ( $L_r, C_r, R_m$ ) that emulates the effect of the mechanical vibration and dependent sources that express the mechanical to electrical energy transformation [1]. The model also comprises the physical dielectric capacitors ( $C_{in}, C_o$ ) that are formed by the input and output electrodes. Since the network is highly selective it will pass, with reasonable gain, only frequencies that are in the vicinity of the resonant frequency.

To simplify the analysis some equivalent transformations of the equivalent circuit can be applied. First, reflection of the output part (right-hand side of Fig. 2a) of the equivalent circuit to the input side.  $R_o$  and  $C_o$  of the equivalent circuit are transformed to  $R'_o$  and  $C'_o$ . Output voltage  $V_o$  is transformed to  $V'_o$ .

$$R'_o = \frac{R_o}{N^2} \quad (1)$$

$$C'_o = N^2 C_o \quad (2)$$

$$V'_o = \frac{V_o}{N} \quad (3)$$

Secondly, the parallel network  $R'_o, C'_o$  is transformed to a series frequency dependent network  $R''(\omega), C''(\omega)$  (Fig. 2b), where:

$$R''(\omega) = \frac{R_o}{N^2(1 + C_o^2 R_o^2 \omega^2)} \quad (4)$$

$$C''(\omega) = \frac{N^2(1 + C_o^2 R_o^2 \omega^2)}{C_o R_o^2 \omega} \quad (5)$$

The voltage transfer function of a PT (in terms of equivalent circuit parameters) is

$$\frac{V_o}{V_{in}}(s) = \frac{N C_r R_o s}{C_r R_o s + N^2(1 + C_o R_o s)(1 + C_r(R_m + L_r s)s)} \quad (6)$$

As one can see, the system is of third order. An analysis of this system is given in [1].

In a typical HV application, one would use a voltage doubler to boost the output voltage. Hence, the output section will be constructed as shown in Fig. 3a, where  $C_f$  is the output filter capacitor and  $R_L$  is the load resistance.

Assuming that the quality factor (Q) of the PT is high, the current through the primary series resonant circuit  $i_r$  will be sinusoidal. This current, after being transferred to the secondary, charges and discharges the capacitor  $C_o$  of the output section (Fig. 3b).

As shown in [5], the voltage doubler and load section can be represented as an equivalent reactive load (a resistor in parallel to a capacitor). Consequently, the gain function (6) is valid for this case too except that the load resistance  $R_o$  and  $C_o$  in Fig. 2a need to be replaced by an equivalent resistor and capacitor respectively.

## III. CRITERIA OF MAXIMUM POWER POINT FREQUENCY

As shown in [1], taking the derivative of the absolute value of (6) and equating it to zero can be used to derive the frequency of maximum voltage gain. This results in a third order equation of the form of (7).

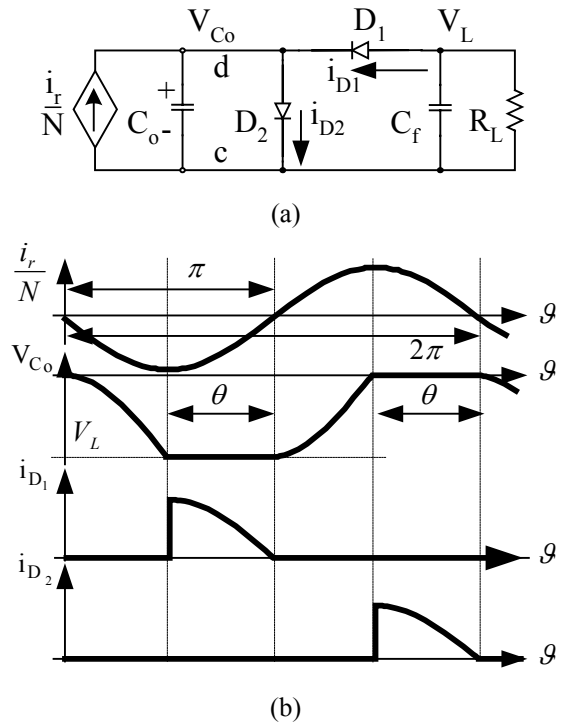


Fig. 3. (a) Output stage of PT with voltage-doubler rectifier. (b) Voltage-doubler waveforms.

$$2C_r^2 N^6 R_o^2 - \left( 2C_r^4 L_r^2 N^6 R_o^2 + 4C_o C_r^4 L_r N^4 R_o^4 + 4C_o^2 C_r^3 L_r N^6 R_o^4 - 2C_o^2 C_r^4 N^6 R_m^2 R_o^4 \right) x^2 - 4C_o^2 C_r^4 L_r^2 N^6 R_o^4 x^3 = 0 \quad (7)$$

where:  $x = (2\pi f_{\max})^2$ ,  $f_{\max}$  – frequency of the maximum of PT's transfer function.

The proposed criterion for maximum voltage gain is zero phase-shift between input voltage  $V_{in}$  and virtual current  $i_r$  of resonant branch of the equivalent circuit (Fig. 2b). The rationale of this hypothesis is that maximum output power will be obtained when the PT is driven at the series resonance frequency. This is further supported by the observation made in [1] that the behavior of (6) is like that of second order RLC band-pass filter, because within the narrow band around the resonant frequency,  $R''(\omega)$  and  $C''(\omega)$  change slowly.

Based on the simplified equivalent circuit of Fig. 2 the transfer function  $\frac{i_r}{V_{in}}(j\omega)$  is (8).

$$\frac{i_r}{V_{in}}(j\omega) = \frac{j\omega C_{eq}(\omega)}{1 - L_r C_{eq}(\omega)\omega^2 + j\omega R_{eq}(\omega)C_{eq}(\omega)} \quad (8)$$

where  $R_{eq}(\omega)$  and  $C_{eq}(\omega)$  are expressed in (9) and (10) as a function of the parameters shown in Fig. 2:

$$R_{eq}(\omega) = R_m + R''(\omega) \quad (9)$$

$$C_{eq}(\omega) = \frac{C_r C''(\omega)}{C_r + C''(\omega)} \quad (10)$$

From (8) one can see that zero phase-shift between  $i_r$  and  $V_{in}$  occurs when the imaginary part is zero, i. e:

$$1 - L_r C_{eq}(\omega_{opt})\omega_{opt}^2 = 0 \quad (11)$$

or:

$$\omega_{opt} = \frac{1}{\sqrt{L_r C_{eq}(\omega_{opt})}} \quad (12)$$

Applying (4), (5), (9), (10) and (12) the frequency of zero phase-shift  $f_{opt}$ :

$$f_{opt} = \frac{\sqrt{C_o C_r R_o^2 + C_o^2 N^2 R_o^2 - C_r L_r N^2 + \sqrt{4C_o^2 C_r L_r N^4 R_o^2 + (C_r L_r N^2 - C_o C_r R_o^2 - C_o^2 N^2 R_o^2)^2}}}{\pi C_o N R_o \sqrt{8C_r L_r}} \quad (13)$$

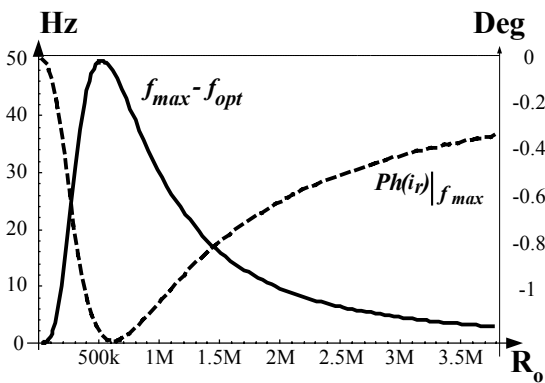


Fig. 4. Phase of  $i_r$  when  $f = f_{\max}$  (dashed-line). And difference between  $f_{\max}$  and  $f_{opt}$  (solid line)

This frequency of series resonance of equivalent circuit is not identical to  $f_{\max}$  but is very close to it [1]. For the experimental high voltage PT used in this study, the maximum difference between  $f_{\max}$  and  $f_{opt}$  is less than 50Hz (less than 0.1% of operation frequency) and phase-shift of the  $i_r$  at  $f_{\max}$  is less than  $1.2^\circ$  (Fig. 4). Fig. 5 shows the ratio of PT's output power at  $f_{opt}$  to the output power at  $f_{\max}$ . These numerical values clearly point out to the fact that  $f_{opt}$  is a very good approximation of  $f_{\max}$ . Hence, the phase shift of  $i_r$  can be used for all practical purposes as a sense signal for deviation from  $f_{\max}$ . The objective of the tracking system will thus to zero this phase shift.

#### IV. PROPOSED TRACKING METHOD

The results of section III suggest that the maximum gain is reached when the phase angle between the input voltage  $V_{in}$  and  $i_r$  is zero.

Unfortunately, there is no direct way to measure this current or its phase since there is no physical access to the (virtual) series branch. This is overcome, in proposed method, by an indirect measurement that makes use of the observation that current of diode  $D_2$  is in fact a sample of the current  $i_r$  (Fig. 3b). Note, in particular, that  $D_2$  will stop conducting when the polarity of the current  $i_r$  is reversed. Hence, this polarity reversal instant can be used as an indicator for the phase of the series current. This proposed sensing method is demonstrated in Fig. 6. Comp1 and Comp2 are used to generate two square waves; one is synchronized to  $V_{in}$  while the second one is synchronized with  $i_r$ . The phase angle between  $V_{in}$  and  $i_r$  can thus be measured by feeding these two signals to a phase detector.

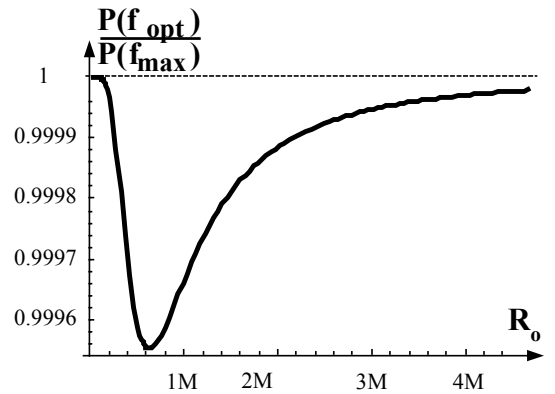


Fig. 5. The ratio between the output power of PT driven at  $f_{opt}$  to the output power at  $f_{\max}$  as a function of  $R_o$ .

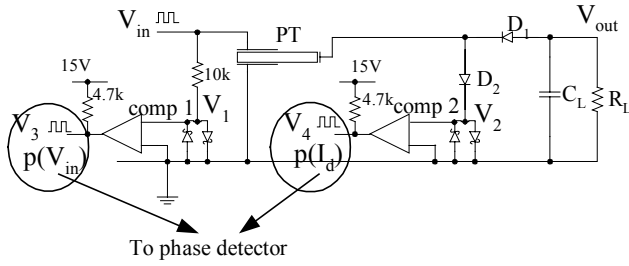


Fig. 6. Proposed method for extracting the phase signals of  $V_{in}$  and  $i_r$ .

### V. EXPERIMENTAL RESULTS

The experimental circuit (Fig. 7) included a Phase Lock Loop (PLL) fed by the two phase signals. The digital frequency phase detector of the PLL (CD4046A) compares these rectangular waveforms and feeds the VCO by the phase error signal. Typical experimental waveforms are shown in Figs. 8-10. A bias network (BN) was used to slightly shift the zero point such that frequency locking is obtained when there is a small phase shift between the

Comp1 and Comp2 signal. This was found necessary for the compensation of the phase shift caused by the parasitic capacitances of the diodes used as clamps (Fig. 7). This phase offset was found to be constant and independent of load resistance. Practical designs of this tracking system should attempt to minimize this parasitic effect by choosing low capacitance diodes.

The frequency tracking system was tested by subjecting it to a load variation (Fig. 11). First, the open loop output voltage as a function of the drive frequency was measured for each load (Fig. 12). The plot clearly shows that maximum output voltage is obtained at different frequencies  $f_1$  and  $f_2$ , corresponding to  $R_L=1.2M\Omega$  and  $R_L=1.76M\Omega$  respectively. It is thus expected that when the load is varied periodically between the two values, under closed loop, the output voltage should vary between peaks of the curves with amplitude  $A_1$ , Fig. 12 (from 150.5V to 178.5V).

Fig. 13 shows the operation under closed loop conditions. The output voltage variations as well as the frequency hopping matched the expected ones (Fig. 12).

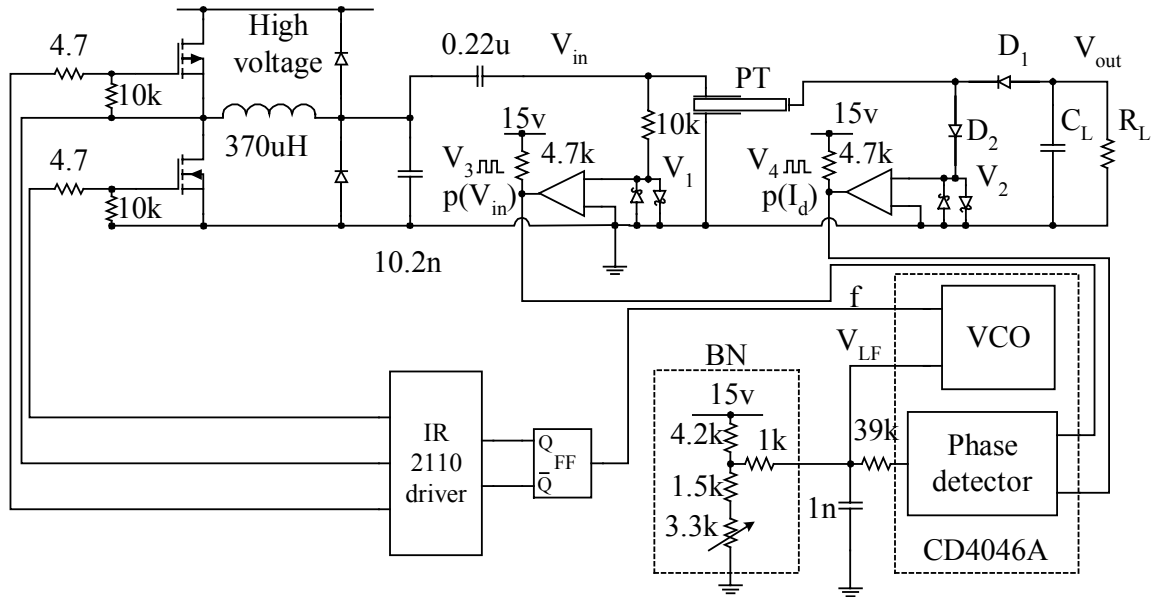


Fig. 7. Experimental setup of the proposed frequency tracking to maximum output voltage.

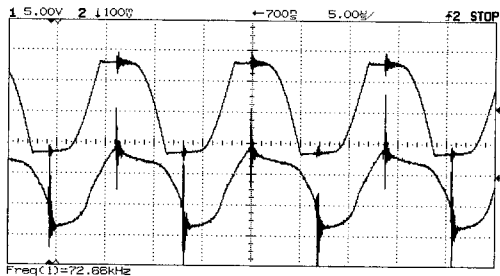


Fig. 8.  $V_{in}$  (upper trace) and  $V_2$  (lower trace) (see Fig. 6) at the series resonance frequency.

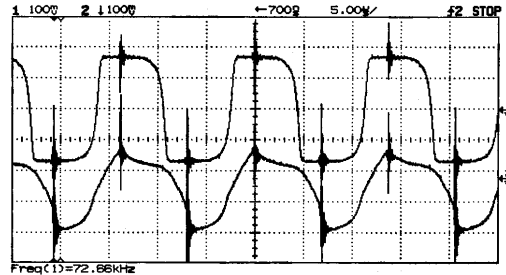


Fig. 9.  $V_1$  (upper trace) and  $V_2$  (lower trace) at the series resonant frequency.

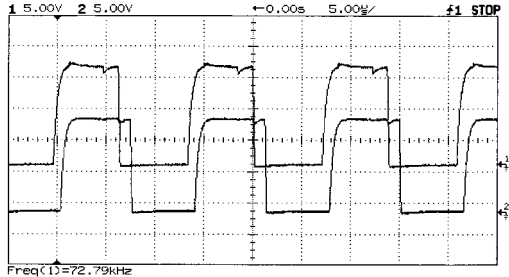


Fig. 10.  $V_3$  (upper trace) and  $V_4$  (lower trace) of the comparators outputs (Fig. 6).

In figures 14 and 15 the system was driven by fixed frequencies. In Fig. 14 - the frequency of operation was to  $f_1$  (Fig. 12) while in Fig. 15 - the frequency corresponds to  $f_2$ . The amplitudes of the output voltage steps were found to be equal to the expected one  $A_2$  and  $A_3$  (Fig. 12) respectively. The results of this experiment suggest that the experimental setup is functioning as expected.

### VI. DISCUSSIONS AND CONCLUSIONS

The proposed tracking method offers a way to lock to the frequency that provides the maximum output power for any load. This could be useful in various applications that need to generate high output voltage (e.g. ionization equipment). By applying the proposed approach, the system operation can be made independent on input voltage, load variations, temperature (within permitted range), and the spread and non-linearity of the PT parameters, as well their drift with time.

The phase detection method proposed here can also be used in cases that call for output voltage regulation. In such cases, a simple feedback loop via a Voltage Controlled Oscillator (VCO) would be ambiguous. If, say, the output voltage is too low, should the frequency be increased or decreased? This ambiguity can be resolved by applying the phase detection method proposed here that generates a clear unequivocal bi-polar signal.

The proposed method was verified experimentally and it is demonstrated that the control circuitry needed for the implementation is simple and can be easily constructed from off the shelf components.

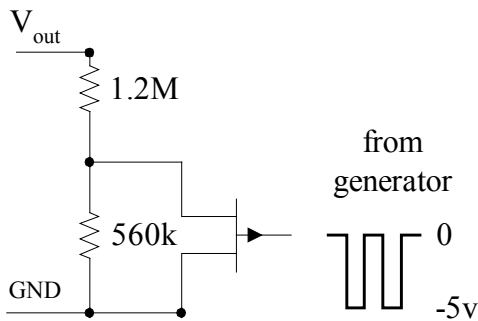


Fig. 11. The variable load.

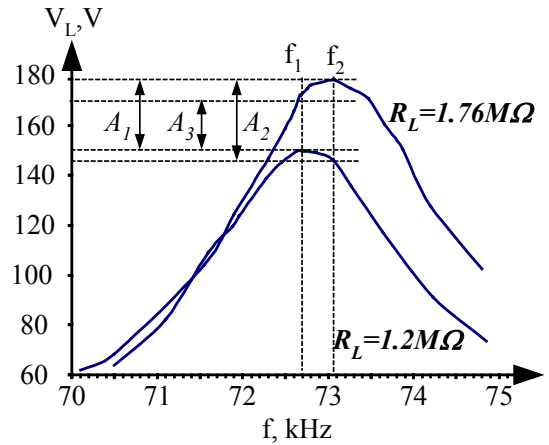


Fig. 12. Steady state output voltage as a function of the operation frequency  $f$ , for two values of the load resistance  $R_L$ . ( $V_{in}=20.3V$ ).

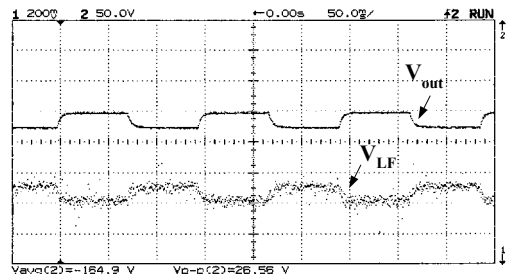


Fig. 13. The system with PLL control. Amplitude of  $V_{out}$  is equal to  $A_1$  of Fig. 12.  $V_{LF}$  is voltage on the loop filter, (VCO input, Fig. 7).

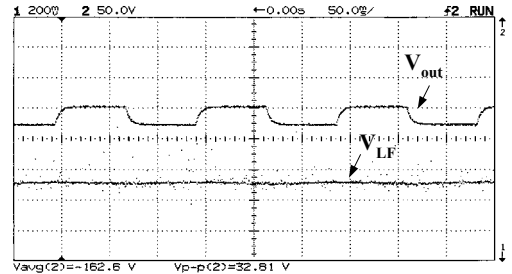


Fig. 14. The system without PLL control. Drive frequency is  $f_1$  - the frequency of resonance when  $R_L=1.2M$ . Amplitude of  $V_{out}$  is equal to  $A_2$  of Fig. 12.  $V_{LF}$  is the loop-filter voltage, (VCO input, Fig. 7).

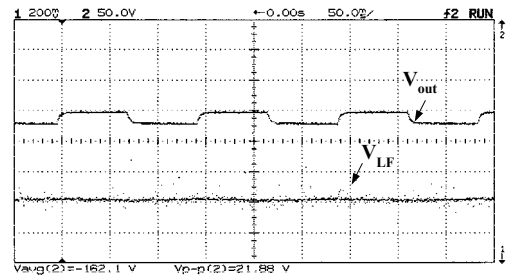


Fig. 15. The system without PLL control. Drive frequency is  $f_2$  - the frequency of resonance with  $R_L=1.76M$ . Amplitude of  $V_{out}$  is equal to  $A_3$  of Fig. 12.  $V_{LF}$  is the loop-filter voltage, (VCO input, Fig. 7).

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