Parallel Connection of Piezoelectric Transformers

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Abstract - **The behavior of parallel connected Piezoelectric Transformers (PTs) was analyzed by applying the generic PT model to represent the combined network in the most general way.**

The influence of a deviation in each of the PT parameters was examined with respect to total output and losses in each PT. It was found that the requirement for identity in the equivalent circuit parameters R_m and C_o in the two PTs is not strong. The most critical parameter for equal loss in the PTs is ω_r – the mechanical resonant frequency of PTs, which has to be equal in the parallel units, even though the values of the parameters C_r and L_r can be slightly different. Since the resonant LCC type inverters are practically identical to the equivalent electrical circuit of PTs, the results of this study are relevant to combined connected LCC inverters.

The analytical results of this study were verified by SPICE simulation and experiments. The experiments were done on parallel-connected PTs (T1-2, Face Co., VA, USA) and LCC inverters built by passive elements.

I. INTRODUCTION

As Piezoelectric Transformer (PT) technology is developing, PTs may become a viable alternative to magnetic transformers in various applications. Presently, however, the power that can be handled through commercial PTs is still limited. Possible solution to this limitation is joint connection of a number of PTs [1-2]. However, since two different PTs are never absolutely identical, the main problem of joint connection of PTs is ensuring current sharing when the PTs are feeding the same load.

The objective of the study was to delineate the requirements for joint connection of PTs, and in particular, to identify and quantify the effect on current sharing and overall efficiency due to non-identities in the parameters of the PTs. This is necessary to avoid the overheating of one or more of the joint connected PTs.

The methodology, applied in this study, follows the concept of Generic Model to represent the behavior of the combined PTs in the most general way [3]. The influence of each parameter of PT on the operation of the parallel-parallel connected PTs is examined and tested by simulation and experiment.

II. PARAMETERS OF A SINGLE PT

The equivalent circuit of the PT, reflected to the primary side, is given in Fig.1, where C_{in} is the input capacitance of PT, $C_o^{'} - R_L^{'}$ are the reflected values of the PT output capacitance and the load resistance, $C_r - L_r - R_m$ are the equivalent electrical parameters that represent the mechanical resonant circuit of the PT.

This study applies the principles of the Generic Model. All parameters of the model are normalized. It is assumed that the PT inverter operates at the frequency of the maximum output voltage [3-4]. The normalized parameters of the base PT are defined as follows:

$$\begin{cases} a = \dfrac{C_o^{'}}{C_r} \\ \omega_r = \dfrac{1}{\sqrt{L_r C_r}} \\ Q = \omega_r C_o^{'} R_L^{'} \\ Q_m = \dfrac{1}{\omega_r C_r R_m} \end{cases} \tag{1}$$

where ω_r is the mechanical resonant frequency of the PT, Q is the normalized load factor, and Q_m is the mechanical quality factor.

Following [3-4], the frequency corresponding to the maximum output voltage ω_m, the efficiency η, the voltage transfer ratio k_{21}, and the output power P_o are obtained as:

$$\omega_m = \omega_r \left[1 + \frac{1}{2a\left(1 + Q^{-2}\right)} \right] \tag{2}$$

$$\eta = \frac{1}{1 + \dfrac{a}{Q_m}\left[\dfrac{1}{Q} + \left(\dfrac{\omega_m}{\omega_r} \right)^2 Q \right]} \tag{3}$$

$$k_{21} = \frac{1}{\sqrt{\left\{ 1 + \dfrac{a}{QQ_m} - ax \right\}^2 + \left\{ \dfrac{\omega_r}{\omega_m}\dfrac{a}{Q}x + \dfrac{\omega_m}{\omega_r}\dfrac{a}{Q_m} \right\}^2}} \tag{4}$$

$$P_o = \frac{V_o^2}{R_L} = k_{21}^2 \frac{V_{in}^2}{R_L} \tag{5}$$

where $x = \left[\left(\dfrac{\omega_m}{\omega_r} \right)^2 - 1 \right]$

III. Possible Connection of PTs for Combined Operation (Identical PTs)

Generally, the PT can be considered as a two port network. There are four possible ways of interconnection PTs for combined operation [5]: parallel input – parallel output, series input – series output, series input – parallel output, and parallel input – series output (Fig. 2, a-d). It should be noted that last two types of connections are realizable only in the case of ground isolated PTs. Otherwise input or output circuit will be shorted [5].

We consider these ways of connection of N PTs under the assumption that all PTs are identical. In order to achieve the multiple power in the output, the following conditions for the load resistance R_{LT}, input V_{inT} and output V_{oT} voltages of the total (combined) circuit were derived for each type of connection:

a) Parallel-parallel connection (Fig. 2, a).

In this case $V_{inT}=V_{in1}=\ldots=V_{inN}$ and $V_{oT}=V_{o1}=\ldots=V_{oN}$. The voltage gain $k_{21T}=k_{21}$. The load resistance and the output power of the combined circuit:

$$R_{LT}={R_L}/{N} \tag{6}$$

$$P_{oT}=\frac{N(V_o)^2}{R_L}=NP_o \tag{7}$$

b) Series-series connection (Fig. 2, b).

The input and output voltages of the total circuit: V_{inT} and V_{oT}, and the load resistance R_{LT} are distributed equally between the PTs. Therefore:

$$\begin{aligned} V_{inT}&=NV_{in}\\ V_{oT}&=NV_o\\ R_{LT}&=NR_L \end{aligned} \tag{8}$$

The voltage gain of the total circuit k_{21T} is equal to k_{21}:

$$k_{21T}=\frac{V_{oT}}{V_{inT}}=\frac{NV_o}{NR_L}=k_{21} \tag{9}$$

The output power of the combined circuit:

$$P_{oT}=\frac{(NV_o)^2}{NR_L}=NP_o \tag{10}$$

c) Series-parallel connection (Fig. 2, c).

In a similar way as above, the input voltage for N PTs has to be N times higher than of single unit, the output voltage is the same as for a single PT, and the load resistance will be $R_{LT}={R_L}/{N}$.

d) Parallel-series connection (Fig. 2, d).

The input conditions in this case are the same as for parallel-parallel connection and the output – are the same as in series-series.

Table 1 summarizes results indicated above.

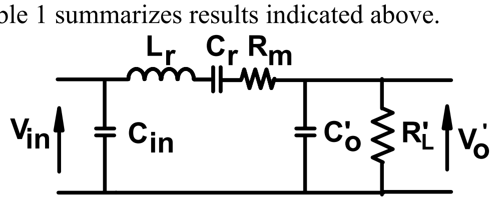


Fig. 1. The equivalent circuit of a PT after reflecting the secondary to the primary side.

Table 1: Load and voltage ratio conditions for different connections of combined PT inverters.

Topology	Figure	R_{LT}/R_L	V_{inT}/V_{in}	V_{oT}/V_o
Parallel-Parallel	2a	1/N	1	1
Series-Series	2b	N	N	N
Series-Parallel	2c	1/N	N	1
Parallel-Series	2d	N	1	N

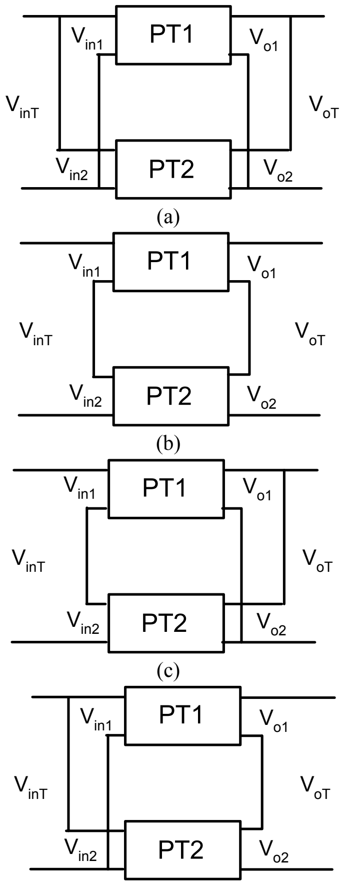
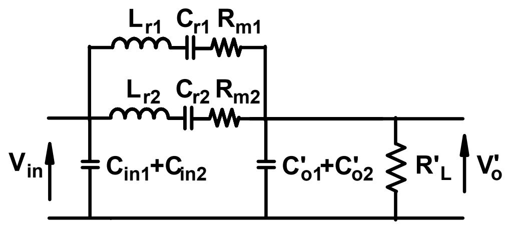


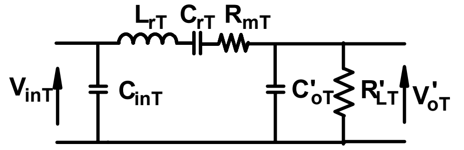
Fig. 2. Possible ways of PTs interconnection (illustrated for two units): a) parallel-parallel, b) series-series, c) series-parallel, d) parallel-series

IV. Parallel-Parallel Connection of Non-Identical PTs

In the detailed analysis of this study we consider the combined operation of two PTs. The equivalent circuit of two parallel-parallel connected PTs, after reflecting the secondary to the primary, is given in Fig. 3a. If the combined network operates at the frequency closed to its mechanical resonant frequency this circuit can be simplified to the one shown in Fig. 3b replacing the two networks by a single one. In the following analysis we replace the two parallel branches Z_1 and Z_2 (that is L_r-C_r-R_m networks) by an equivalent impedance Z_T and derive the parameters of the 'new' PT from the expression of Z_T. This is used to quantify the effect of non-equality in the PT parameters on the overall operation of the parallel-connected PTs.



(a)



(b)

Fig. 3. The equivalent circuits of parallel-parallel connected PTs: (a) common case, (b) operation close to the resonant frequency.

We compare the output power P_{oT} and efficiency η_T of the combined PT with the output power P_o and efficiency η of a single PT.

A. The Influence of the Non-Equality of C_o

The deviation in C_o is presented as follows:

$$C'_{o1} = C'_o$$

$$C'_{o2} = C'_o + \Delta C'_o = C'_o(1+\delta_c) \tag{11}$$

$$C'_{oT} = C'_{o1} + C'_{o2} = 2C'_o\left(1+\frac{\delta_c}{2}\right) \tag{12}$$

where C'_{o1} and C'_{o2} are the output capacitances of the parallel PTs reflected to the primary side, ΔC_o and $\delta_c=\Delta C_o/C_o$ are the absolute and relative deviation of the parameter C_o from the nominal value, and C'_{oT} is the reflected capacitance of total PT. We analyze the influence of δ_c in the case when the impedances of the RLC networks Z_1 and Z_2 are the same:

$$Z_1 = Z_2 = R_{m1} + j\omega L_{r1} + \frac{1}{j\omega C_{r1}} = R_{m2} + j\omega L_{r2} + \frac{1}{j\omega C_{r2}} \tag{13}$$

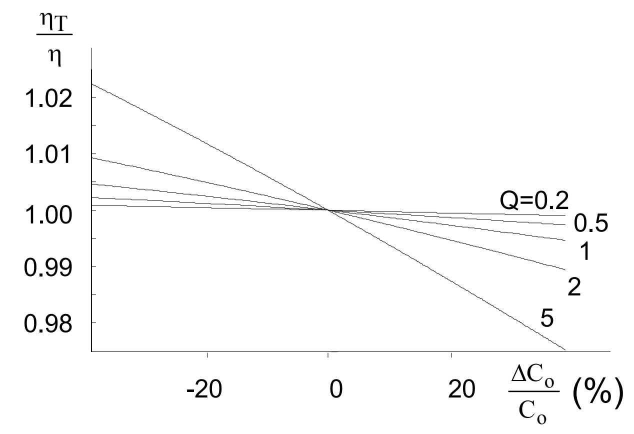


Fig. 4. The relation of efficiency of the combined circuit to the nominal efficiency as a function of the relative deviation of the output capacitance C_o for Q range 0.2 - 5 and Q_m=900.

If the capacitance deviation is not too great, one can assume that the reflected load resistance of the circuit under consideration is the same as in ideal case (Table 1): $R'_{LT} = 0.5R'_L$. The impedance and the parameters of the total network are:

$$Z_T = Z_1 || Z_2 = 0.5Z_1 \tag{14}$$

$$\begin{cases} R_{mT} = 0.5R_m \\ C_{rT} = 2C_r \\ L_{rT} = 0.5L_r \end{cases} \tag{15}$$

The normalized parameters (1) for this case are:

$$\begin{cases} a_T = \dfrac{C'_{oT}}{C_{rT}} = a\left(1+\dfrac{\delta_c}{2}\right) \\ \omega_{rT} = \omega_r \\ Q_T = \omega_{rT}C'_{oT}R'_{LT} = Q\left(1+\dfrac{\delta_c}{2}\right) \\ Q_{mT} = Q_m \end{cases} \tag{16}$$

By substituting these normalized parameters into equations (2)-(5) we calculate the optimal operating frequency for maximum power transfer ω_{mT}, the efficiency η_T, the voltage transfer ratio k_{21T}, and the output power P_{oT}.

Fig. 4 represents the relation of the efficiencies (η_T/η) of the combined and single PTs as a function of the normalized non-equality $\Delta C_o/C_o$ for different values of the normalized load parameter Q while the mechanical quality factor is assumed to be Q_m=900. From these plots one can see that the efficiency is decreasing with an increase in $\Delta C_o/C_o$ This can be explained by the following: an increase of the output capacitance C_o causes a larger current in the L_r-C_r-R_m network, which in turn, causes a rise of the losses in R_m and hence decreasing the efficiency. This effect is similar to case of a single PT with larger C_o. The larger the output capacitance the lower will be the PT's efficiency. This study shows, however, that substantial deviation in the values of the output capacitances of the PTs (up to 30%) results in minor change in the overall efficiency.

B. The Influence of Non-Equality of R_m

The deviation in R_m is presented as follows:

$$\begin{cases} R_{m1} = R_m \\ R_{m2} = R_m + \Delta R_m = R_m(1+\delta_m) \end{cases} \tag{17}$$

where ΔR_m and $\delta_m=\Delta R_m/R_m$ are the absolute and the relative deviation of the losses resistance from its nominal value. Assuming the rest of parameters to be identical:

$$\begin{cases} C'_{o1} = C'_{o2} \\ L_{r1} = L_{r2} \\ C_{r1} = C_{r2} \end{cases} \tag{18}$$

we find the total impedance of the network $Z_T=Z_1||Z_2$:

$$Z_T \approx \frac{R_m}{2}\left(1+\frac{\delta_m}{2}\right) + \frac{1}{2}j\omega L_r + \frac{1}{2j\omega C_r} + \varphi\left(R_m, C_r, \omega, \delta_m^2\right) \tag{19}$$

The function φ (R_m, C_r, ω,δ_m^2) in (19) has a relatively small value and can be neglected. Therefore the equivalent series components of the total PT are:

$$\begin{cases} R_{mT} = 0.5R_m(1+0.5\delta_m) \\ L_{rT} = 0.5L_r \\ C_{rT} = 2C_r \end{cases} \tag{20}$$

The total output capacitance C_{oT}=2C_o. The load resistance is the same as in 4.1. Using these parameters, we recalculate the normalized parameters of the total circuit:

$$\begin{cases} a_T = a \\ \omega_{rT} = \dfrac{1}{\sqrt{L_{rT}C_{rT}}} = \omega_r \\ Q_T = \omega_{rT}C_{rT}R_{LT} = Q \\ Q_{mT} = \dfrac{1}{\omega_{rT}C_{rT}R_{mT}} = \dfrac{Q_m}{1+0.5\delta_m} \end{cases} \tag{21}$$

Substituting these normalized parameters into (2)-(5), we obtain the expressions for the operating frequency, efficiency, voltage transfer ratio, and output power of the equivalent inverter.

The plots of the output power and efficiency ratios of the combined PT to the single PT P_{oT}/P_o and η_T/η (Fig. 5, a, b) as a function of the non-equality $\Delta R_m/R_m$ for different values of load resistance (parameter Q) show that fairly large changes in the parameter R_m (up to 20%) cause rather small changes in the output characteristics.

Since the impedances of the parallel networks are different, we compare the power losses in the networks Z_1 and Z_2. The loss relation P_{Loss1}/P_{Loss2} can be calculated using the PTs parameters:

$$\begin{aligned} P_{Loss1} &= I_1^2 R_m = \left(\frac{V_{z1}}{Z_1}\right)^2 R_m \\ P_{Loss2} &= I_2^2 R_m(1+\delta_m) = \left(\frac{V_{z2}}{Z_2}\right)^2 R_m(1+\delta_m) \end{aligned} \tag{22}$$

where I_1 and I_2 are the rms currents, V_{z1} and V_{z2} are the rms voltages across the series parts (Z_1 and Z_2) of PTs. Since $V_{z1} = V_{z2}$, the losses relation λ_m is:

$$\lambda_m = \frac{P_{Loss1}}{P_{Loss2}} = \frac{1}{1+\delta_m}\left(\frac{Z_2}{Z_1}\right)^2 \tag{23}$$

where:

$$\left(\frac{Z_2}{Z_1}\right)^2 = \frac{(1+\delta_m)^2 + \left\{Q_m \dfrac{\omega_r}{\omega}\left[\left(\dfrac{\omega}{\omega_r}\right)^2 - 1\right]\right\}^2}{1 + \left\{Q_m \dfrac{\omega_r}{\omega}\left[\left(\dfrac{\omega}{\omega_r}\right)^2 - 1\right]\right\}^2} \tag{24}$$

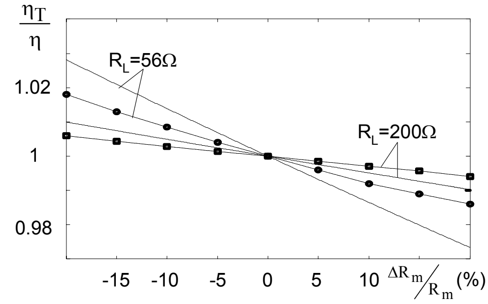
When R_m increases the current through RLC network decreases. Hence, the effect on the product I^2R_m (that is, the losses) could be positive or negative. In order to examine the behavior of the loss sharing, we take the derivative of (23) with respect to δ_m:

$$\frac{\partial\lambda_m}{\partial\delta_m} = \frac{(1+A)\left\{(1+\delta_m)^2 - A\right\}}{(1+\delta_m)^2(1+A)^2} \tag{25}$$

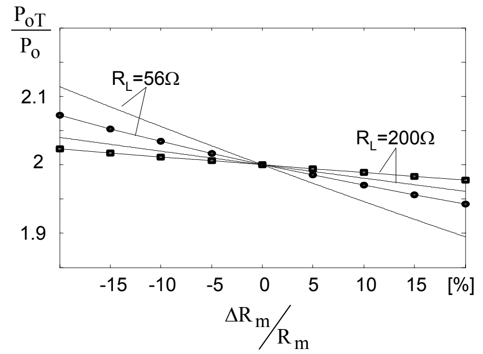
where:

$$A = \left\{Q_m \frac{\omega_r}{\omega}\left[\left(\frac{\omega}{\omega_r}\right)^2 - 1\right]\right\}^2 \tag{26}$$

It should be noted that a change in R_m has practically no impact on the maximum output voltage frequency [3-4]. Hence, A is independent of δ_m. The sign of the derivative $\frac{\partial\lambda_m}{\partial\delta_m}$, which is defined by the sign of the expression $(1+\delta_m)^2 - Q_m^2\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)^2$, determines the slope of the losses ratio curves.



(a)



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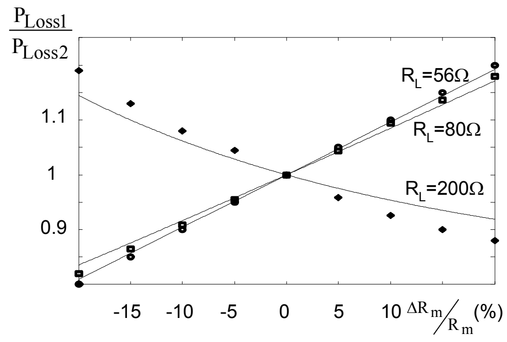

(c)

Fig. 5. The efficiency, (a), output power, (b), and losses ratio, (c) as a function of the relative deviation of the loss resistance R_m for different values of the load resistances R_L. Solid line – theoretical prediction, dots – PSPICE simulation results. Simulation was carried out assuming that PTs nominal parameters are identical to those of PT2-1, Face Co., VA, USA.

If Q_m is very high, then the derivative is negative and the plots of losses ratio will go down.

If Q_m is very low, then the derivative is positive and the plots of losses ratio will go up. Otherwise, the slope of the losses ratio dependent on the load factor (the load resistance) and can even change the polarity for a given δ_m.

Plots of the losses ratio as a function of the ratio $\Delta R_m/R_m$ for different values of the load resistance (parameters Q varied from 0.056 to 0.2) and Q_m=370 (Fig. 5, c) show that the losses relation in the parallel parts varies less than 5-10% due to more than 15% of change in the ratio $\Delta R_m/R_m$. Since the mechanical quality factor of the PT is intermediate (370) one can see that the slopes of the curves change polarity from one load resistance to another.

The boundaries of losses ratio deviation can be obtained by analyzing (23):

$$\lambda_m = \frac{1}{1+\delta_m}\frac{R_m^2(1+\delta_m)^2+X^2}{R_m^2+X^2} = \frac{\left(\frac{R_m}{X}\right)^2(1+\delta_m)^2+1}{\left[\left(\frac{R_m}{X}\right)^2+1\right](1+\delta_m)} \qquad (27)$$

where $X = \omega L_r - \frac{1}{\omega C_r}$.

If $\left(\frac{R_m}{X}\right)^2 << 1$, then $\frac{P_{Loss1}}{P_{Loss2}} \approx \frac{1}{1+\delta_m}$.

If $\left(\frac{R_m}{X}\right)^2 >> 1$, then $\frac{P_{Loss1}}{P_{Loss2}} \approx 1+\delta_m$.

Therefore:

$$\frac{1}{1+\delta_m} < \frac{P_{Loss1}}{P_{Loss2}} < 1+\delta_m \qquad (28)$$

C. *The Influence of Non-Equality of ω_r*

The main problem of this case is that the currents in the parallel branches of the PTs are strongly unequal, causing a large difference in the losses. The losses ratio λ_ω:

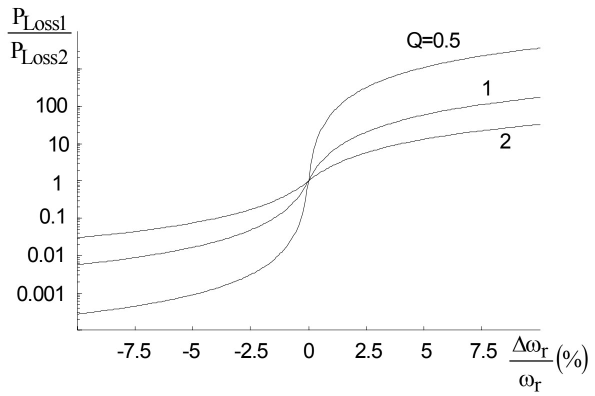

Fig. 6. The losses ratio in the parallel connected PTs as a function of the relative change of the mechanical resonant frequencies of PTs for different values of Q while Q_m=900.

$$\lambda_\omega = \frac{1+\left\{Q_m\frac{\omega_r}{\omega}\left[\frac{\left(\frac{\omega}{\omega_r}\right)^2(1+\delta_\omega)-1}{(1+\delta_\omega)}\right]\right\}^2}{1+\left\{Q_m\frac{\omega_r}{\omega}\left[\left(\frac{\omega}{\omega_r}\right)^2-1\right]\right\}^2} \qquad (29)$$

where δ_ω is the normalized mechanical resonant frequency deviation ($\delta_\omega=\Delta\omega_r/\omega_r$).

Fig. 6 shows the losses ratio as a function of the frequency deviation for different values of the parameter Q while Q_m=900. One can see that even minor deviation in the resonant frequency of the PTs (less than 2-3%) causes great difference in the losses (up to ten times and more). That renders the combined operation impractical.

D. *The Influence of Non-Equality of C_r and L_r while ω_r is Kept Constant*

In order to examine how non-equality of the parameters C_r and L_r in parallel parts (when ω_r are the same) impacts the output characteristics of the combined operated PTs we assume the following conditions: the series capacitance and inductance of the second PT are different from the parameters of the first (nominal) PT while the series resonant frequency of the parallel parts remains constant (δ_{Cr} and δ_{Lr} are relative variations of C_r and L_r):

$$C_{r2} = C_{r1}(1-\delta_{Cr}) \qquad L_{r2} = L_{r1}(1+\delta_{Lr}) \qquad (30)$$

while: $\omega_{r1}=\omega_{r2}$, which means:

$$C_{r1}L_{r1} = C_{r2}L_{r2} \qquad (31)$$

or:

$$\delta_{Lr} = \frac{\delta_{Cr}}{1-\delta_{Cr}} \qquad (32)$$

The total series impedance is:

$$Z_T \approx \frac{(2+2\delta_{Lr}+\delta_{Lr}^2)R_m}{(2+\delta_{Lr})^2} + \frac{1}{2j\omega C_r(1-0.5\delta_{Cr})} + \frac{1}{2}j\omega L_r\frac{1+\delta_{Lr}}{1+0.5\delta_{Lr}} \qquad (33)$$

which implies that:

$$\begin{cases} R_{mT} \approx 0.5R_m \\ L_{rT} \approx 0.5L_r(1+0.5\delta_{Lr}) \\ C_{rT} \approx 2C_r(1-0.5\delta_{Cr}) \end{cases} \qquad (34)$$

The losses ratio λ_r:

$$\lambda_r \approx \frac{\frac{1}{Q_m^2}+\left[\frac{\frac{\omega}{\omega_r}-\frac{\omega_r}{\omega}}{1-\delta_{Cr}}\right]^2}{\frac{1}{Q_m^2}+\left[\frac{\omega}{\omega_r}-\frac{\omega_r}{\omega}\right]^2} \qquad (35)$$

Analyzing (33) we obtain the boundary of the losses ratio deviation:

$$\text{If } \frac{1}{Q_m^2} << \left[\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right]^2, \text{ then } \frac{P_{Loss1}}{P_{Loss2}} \approx \frac{1}{(1-\delta_{Cr})^2} \quad (36)$$

$$\text{If } \frac{1}{Q_m^2} >> \left[\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right]^2, \text{ then } \frac{P_{Loss1}}{P_{Loss2}} \approx 1 \quad (37)$$

Losses relation as a function of relative non-equality of C_r and L_r for two load resistances is shown in Fig. 7. The plots reveal that the losses ratios is lower than 1.5 times for a non-equality of the parameters C_r and L_r of up to 15%.

V. SIMULATION AND EXPERIMENT

The equivalent circuit parameters of the experimental PTs, (PT2-1 Face Co., VA, USA) were measured [6] to be as follows: R_m=21Ω, C_r=172.5pF, L_r=10.5mH, C_o=1.33nF, C_{in}=1.78nF, n=1.08. (Q_m=370, a=8.33). The influence of deviation of different PT parameters on the output power, efficiency and losses sharing in the combined circuit were tested by time domain PSPICE (simulation and found to be in a good agreement with the theoretical predictions (Figs. 5, 7).

Experiments were carried out on two radial vibration mode PTs. The mechanical resonant frequencies of PTs were found to be 118.43kHz and 118.38kHz respectively (less than 0.06% difference). The maximum efficiency achieved in both individual and combined operation of the PTs was 96.5%. Parallel operation provided double output power as compared to the single PT.

For the experimental verification of the theoretical study, the imitation of two PTs was built in the form of two LCC series-parallel resonant networks with no output transformer.

The experimental set up comprised the following data:

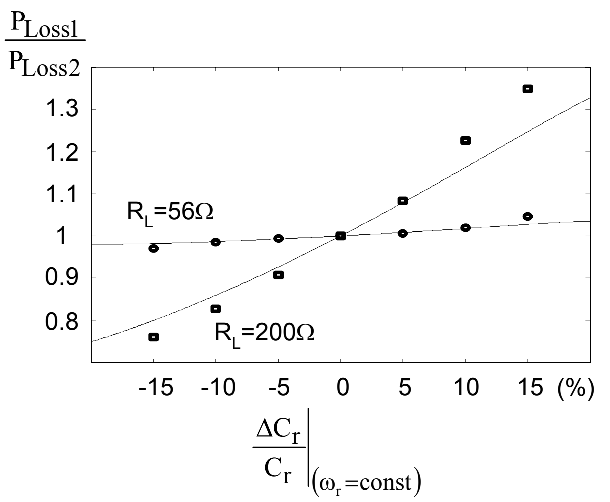

Fig. 7. The losses ratio in the parallel-connected PT as a function of the relative change of the parameters C_r (complementary to L_r) while mechanical resonant frequencies ω_r=118.4kHz are identical.. Solid line – theory, dots – simulation results. Simulation was carried out assuming that PTs nominal parameters are identical to those of PT2-1, Face Co., VA, USA

1) The series inductances and capacitances were: C_{r1}=1.011nF, L_{r1}=2.087mH, C_{r2}=0.945nF, L_{r2}=2.25mH. Measured series resonant frequencies were: f_{r1}=108.6kHz and f_{r2}=109.3kHz. Thus, the parameters deviations were $\delta_{Cr}\approx-6.5\%$, $\delta_{Lr}\approx7.8\%$, and mechanical frequency deviation δ_ω was about 0.65%.
2) The parallel capacitances were equal: $C_{o1}=C_{o2}$=10nF (capacitances ratio a=9.9).
3) The measured ac resistances of the inductances and the capacitances in the experimental setup were: R_{m1}=13.67Ω, R_{m2}=17Ω which means the parameter deviation δ_m=24.4%.
4) Mechanical quality factor was $Q_m\approx105$.
5) Experimental resistances were 56Ω and 200Ω (load factor Q was equal to 0.77 and 2.75), the capacitances ratio, a, was about 10 (similar to the one of the experimental PT).

The experiment involved low input power (up to 5W). The two LCC networks were connected in parallel-parallel scheme.

The results of the measurements were as follows:

1. R_L=56Ω. Currents in the RLC networks were: I_{r1}=185mA, I_{r2}=153mA, that caused the losses P_{Loss1}=0.468W and P_{Loss2}=0.4W, and hence the losses ratio λ was 1.17
2. R_L=200Ω. The measured currents were I_{r1}=152mA, I_{r2}=132mA, losses: P_{Loss1}=0.317W, P_{Loss2}=0.296W, and losses' ratio λ=1.07.

It can be shown that for small deviations, $\lambda \approx 1$, the total losses ratio λ_T can be approximated by:

$$\lambda_T = 1-(1-\lambda_m)-(1-\lambda_\omega)-(1-\lambda_{Cr}) \quad (38)$$

Using expressions (23), (29) and (35) the theoretical predictions were found to be as following:

For R_L=56Ω:

λ_T=1–(1–1.37)–(1–0.835)–(1–0.89)=1.095

For R_L=200Ω:

λ_T=1–(1–1.2)–(1–0.85)–(1–0.883)=0.94

As can be seen, the experimental results are in good agreement with theoretical prediction. These results also demonstrate the high sensitivity to δ_ω, especially when the load factor Q is small (Fig. 6).

VI. DISCUSSION AND CONCLUSIONS

This study carried out detailed analysis of the influence of each PT parameter deviation on the effectiveness of parallel-parallel operation of PTs. It was found that the requirements for equality of R_m and C_o in two parallel-connected PTs are not strong. The most critical parameter that can harm the parallel operation is ω_r – the mechanical resonant frequencies of PTs. For good current sharing the mechanical resonance of the parallel-connected PTs has to be very close while C_r and L_r could be slightly different.

The main conclusion of this study is that effective parallel connection of PTs can be achieved if the mechanical resonant frequencies of the parallel units are matched. This could be explained by the fact that current sharing in parallel parts is dependent on the impedances of the parallel branches. The impedance of the L_r-C_r-R_m network, Z, is a function of the characteristic impedance Z_r of the network and its series resonant frequency:

$$Z \approx \sqrt{R_m^2 + \left(Z_r \frac{2\Delta\omega}{\omega_r} \right)^2} \qquad (39)$$

where $\Delta\omega = \omega_m - \omega_r$ and $Z_r = \sqrt{\frac{L_r}{C_r}}$

Z_r in practical PTs is very high. For the experimental units Z_r is about 7.5kΩ while in others it may reach 100kΩ [3] and above. Consequently, a small deviation in ω_r, that will cause a corresponding shift in $\Delta\omega$, will result in a large difference in Z and hence in the losses ratio.

As discussed in [7] the equivalent parameters of PT are dependent on the transformer dimensions and ceramic constants. When two PT are produced from the same ceramic under the same technological process, their dimensions are equal with high accuracy, the sound velocity in the PTs are practically the same (that is, the same mechanical resonant frequencies), and their masses and all mechanical properties are the same, thus their equivalent parameters L_rC_r are probably very close (which suggest equal characteristic impedances). That means that the main requirement for proper combined operation could be fulfilled in practical piezoelectric transformers.

Since discrete resonant LCC type inverters follow the same network relationships as the PT equivalent circuit, the results of this study are relevant to the case of parallel connected LCC inverters.

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