

Segmentation with Shape Priors



Tammy Riklin-Raviv

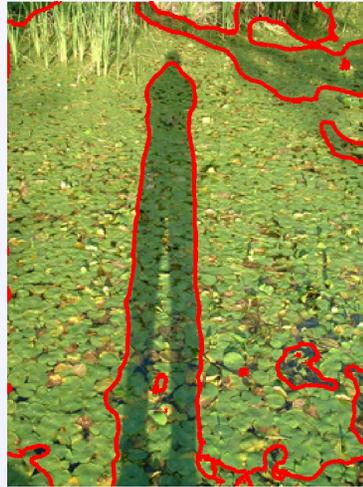
Joint work with Nahum Kiryati and Nir Sochen*

School of Electrical Engineering

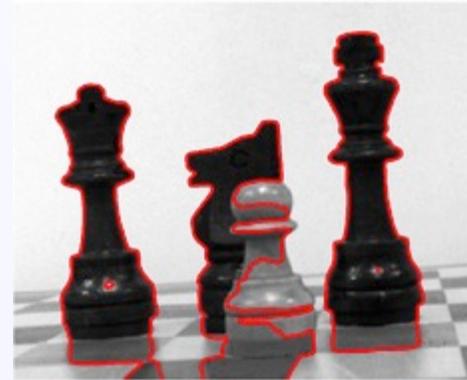
*Department of Applied Mathematics

Tel-Aviv University

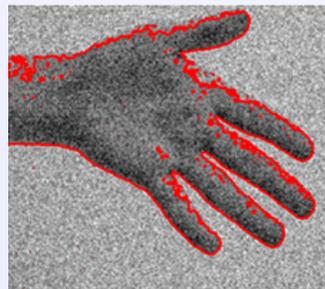
Bottom-up Segmentation



Clutter



Occlusion



Noise



Reflection

Prior based Segmentation

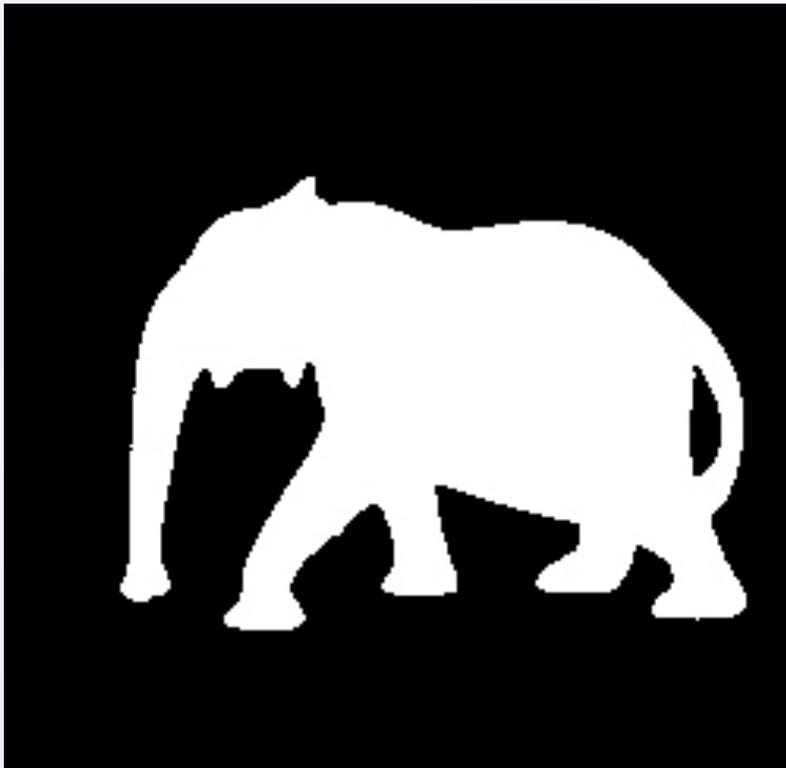


Prior image



To Segment

Prior based Segmentation

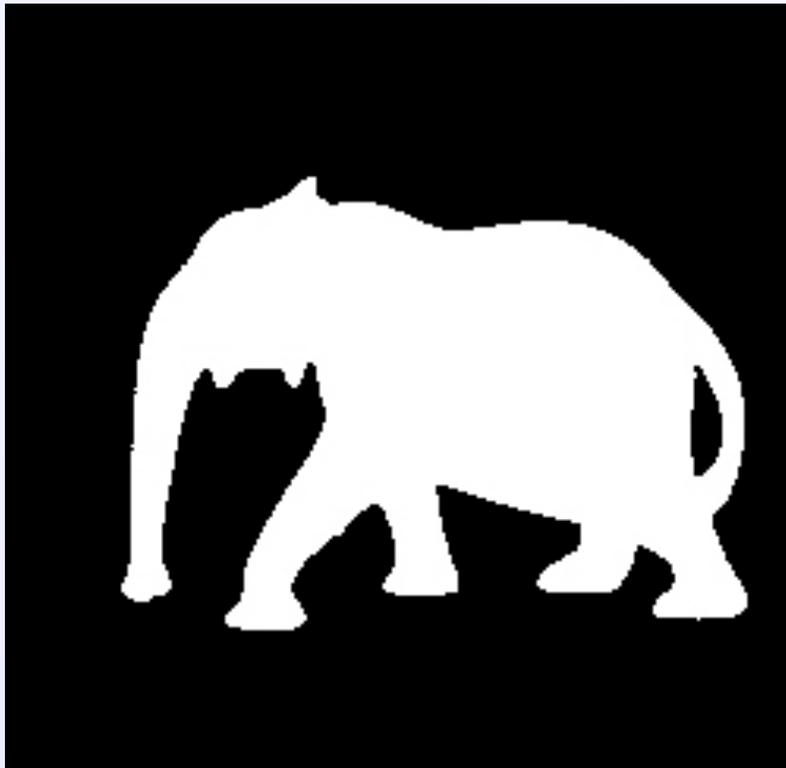


Prior shape



To Segment

Prior based Segmentation



Prior shape



Segmentation

Mutual Segmentation



Mutual Segmentation



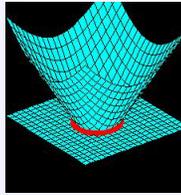
Symmetry-based Segmentation



Symmetry-based Segmentation



Talk Overview



■ Level-sets formulation

- Shape representation
- Segmentation



■ Prior based segmentation

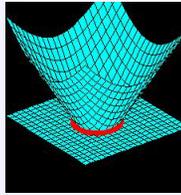


■ Mutual segmentation



■ Symmetry based segmentation

Talk Overview



- Level-sets formulation

- Shape representation

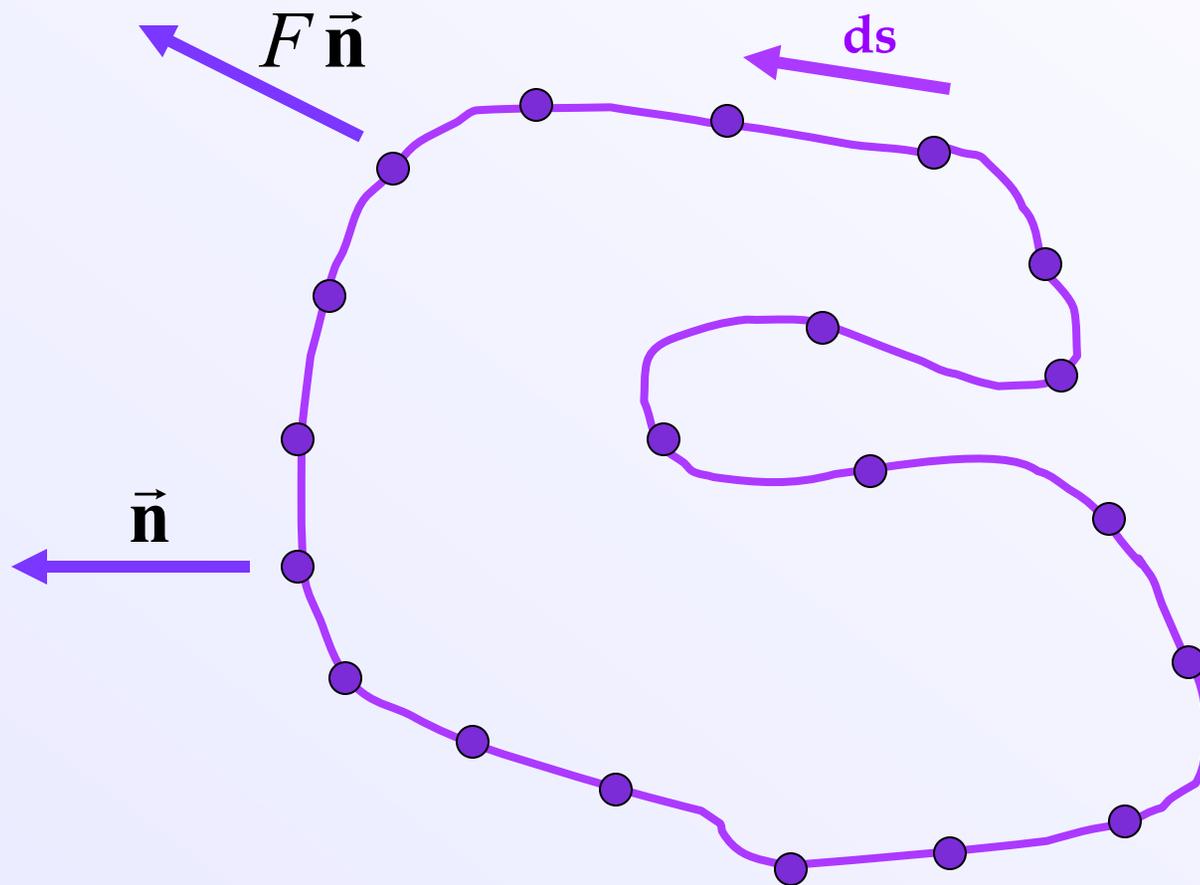
- Segmentation

- Prior based segmentation

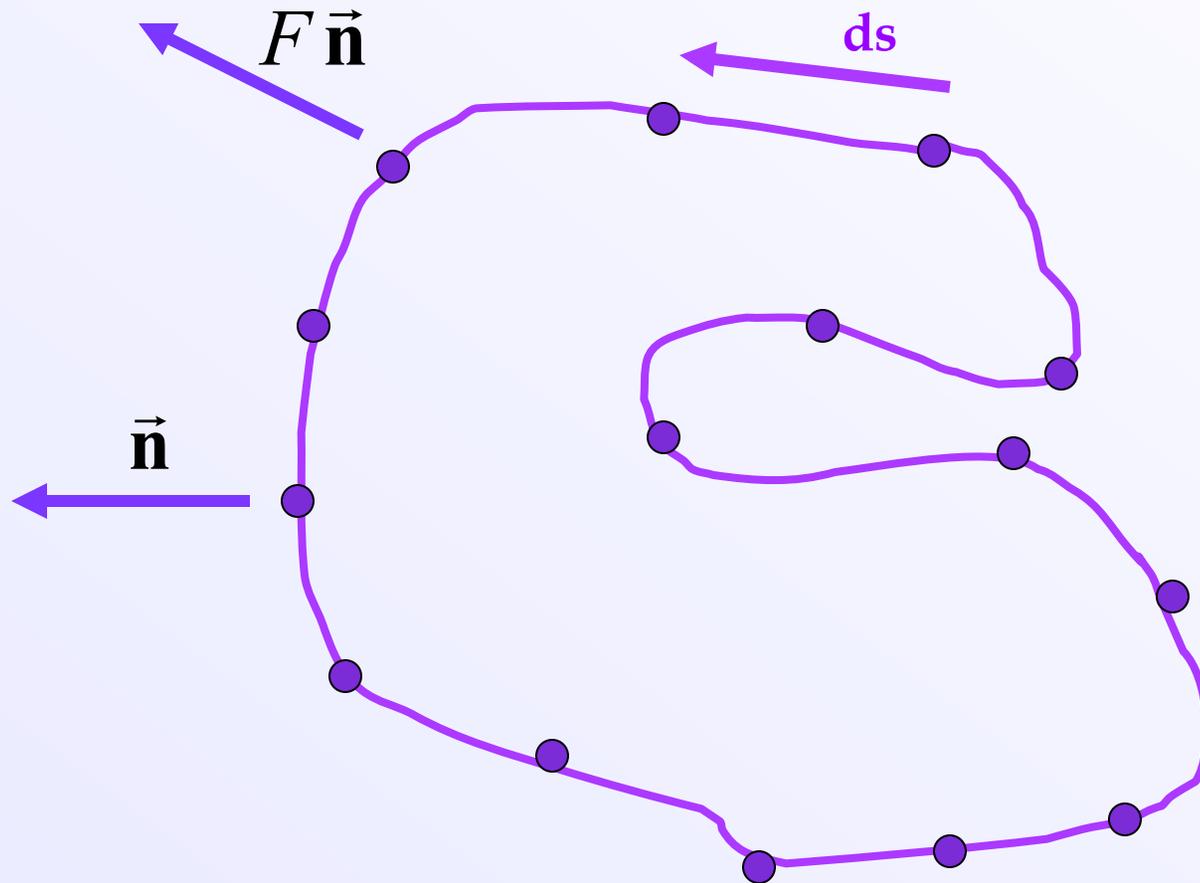
- Mutual segmentation

- Symmetry based segmentation

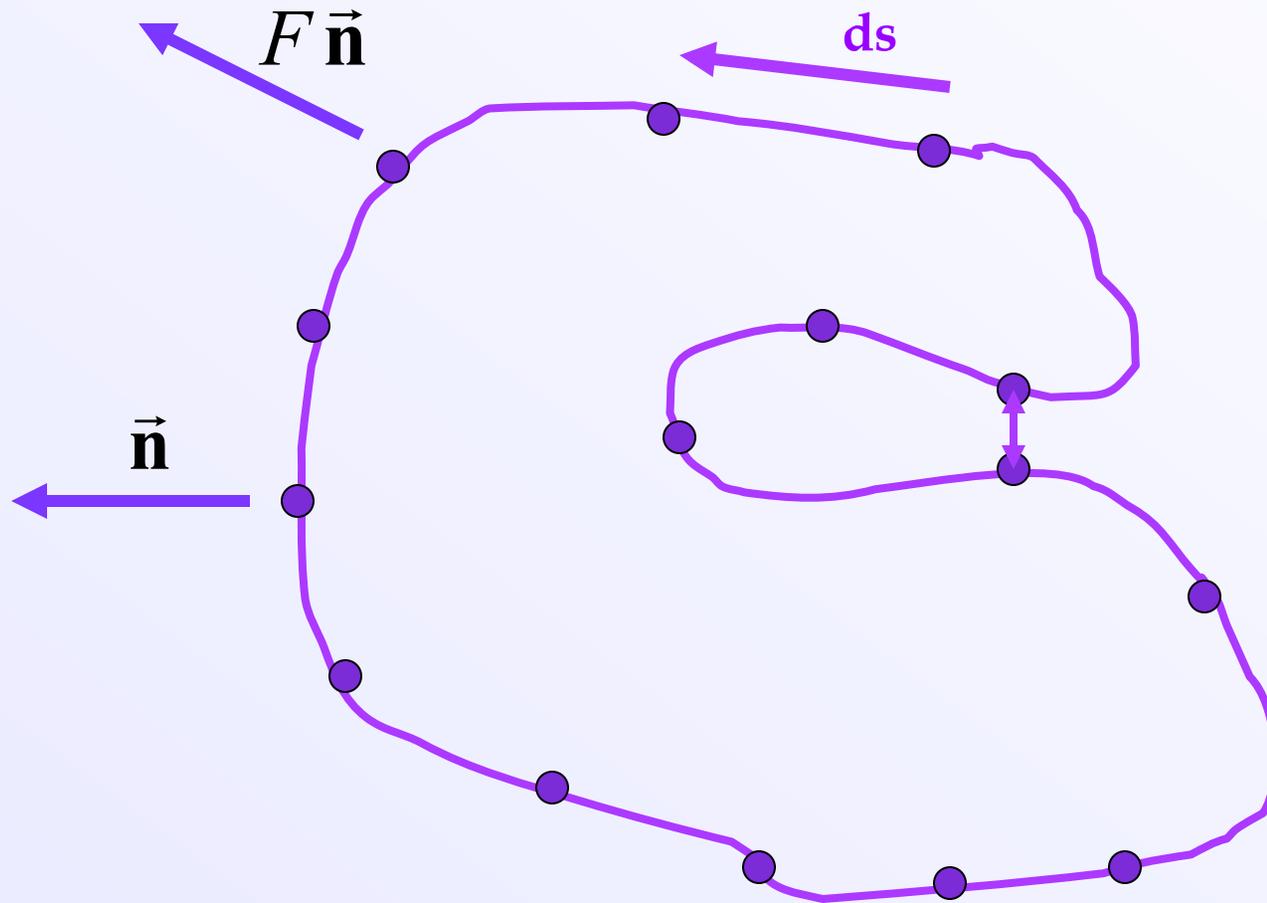
Parametric Representation



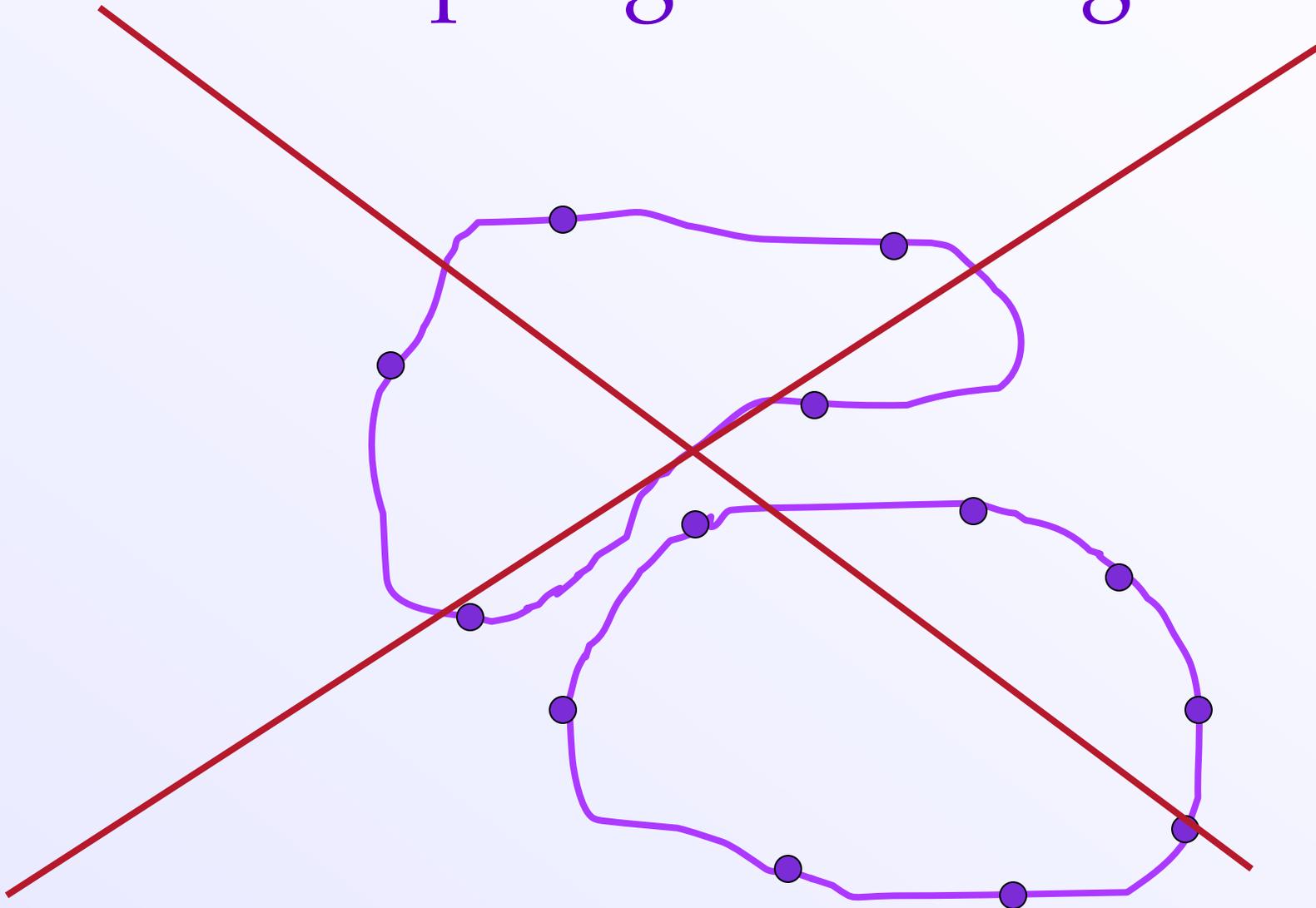
Parameterization - Dependency



Self Intersections



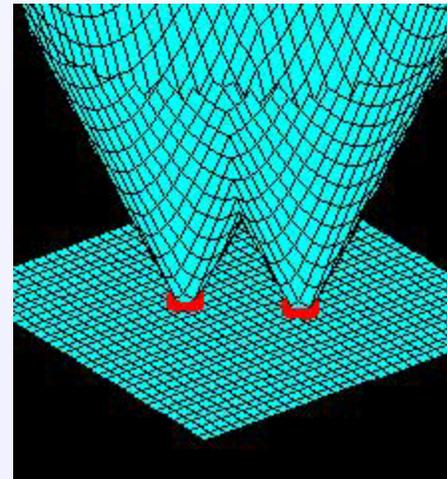
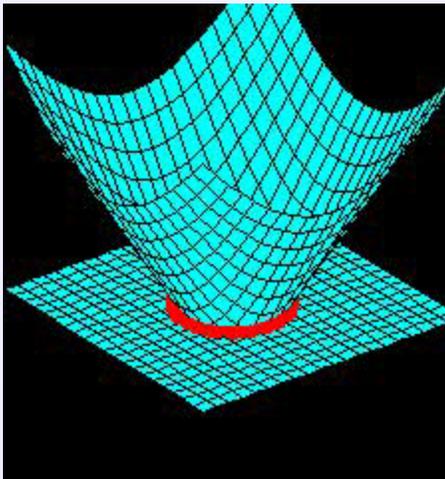
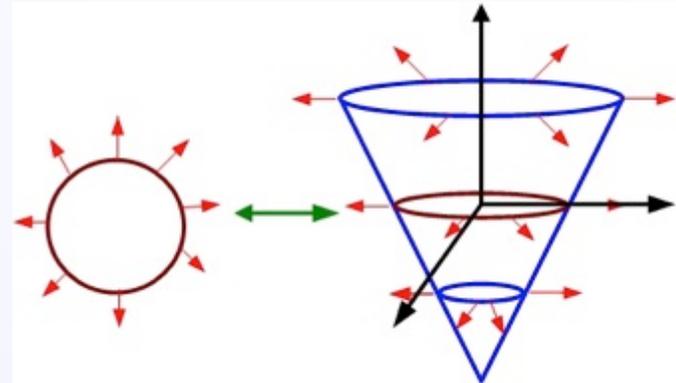
Cannot Handle Automatic Topological Changes



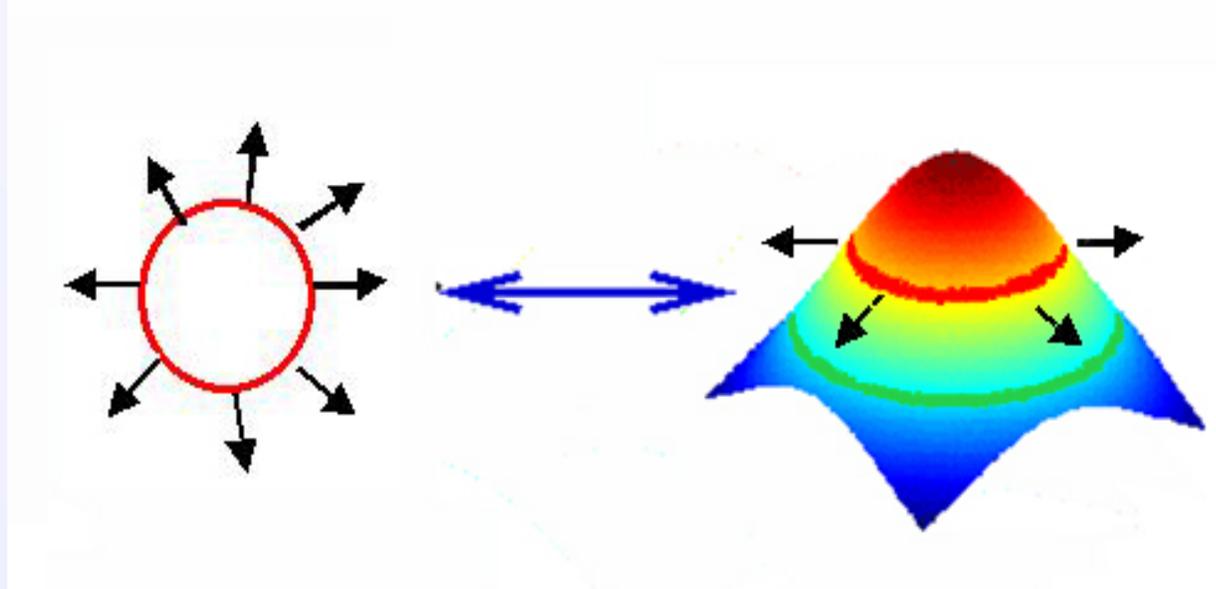
The Level-set Approach

Level-set function $\phi : \Omega \rightarrow \mathcal{R}$

Embedded contour $C = \{x \in \Omega \mid \phi(x) = 0\}$



The Level-set Approach

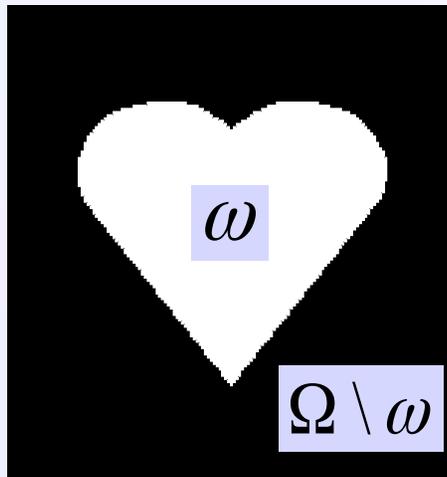


The original front.
Front lies in x-y plane

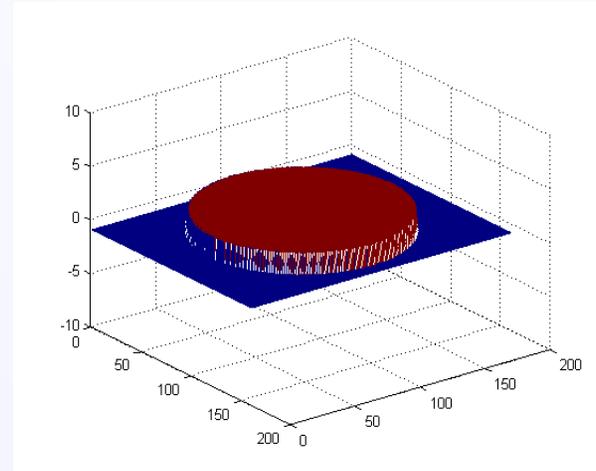
The level-set function.
Front is intersection of surface
and x-y plane

Osher-Sethian 1988

Dynamic Shape Representation



Active Contour



Evolving Level-Set Function

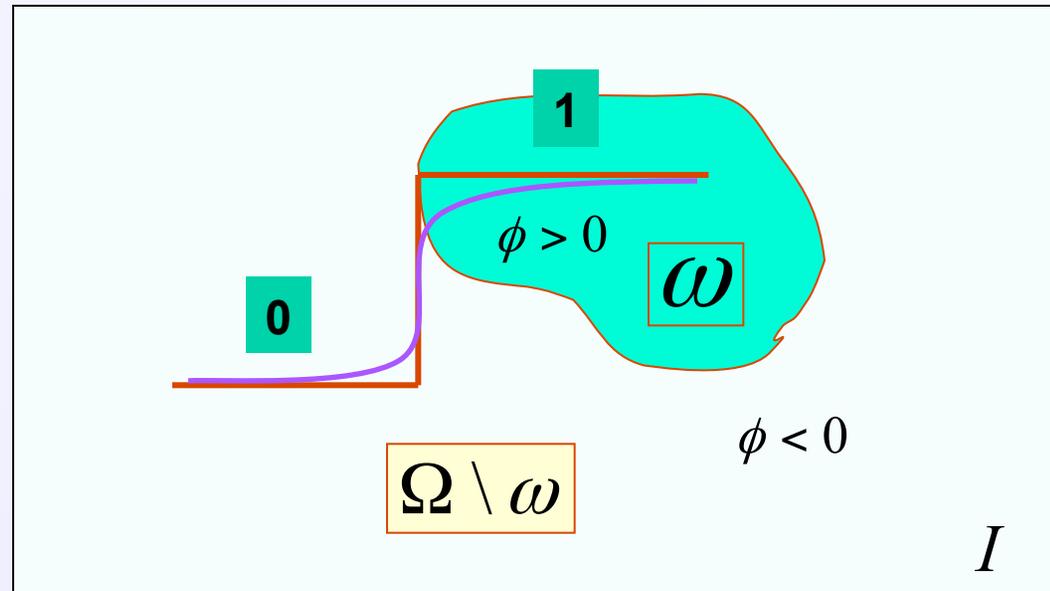
Image domain Ω

Object domain $\omega \in \Omega$

Level-set function $\phi : \Omega \rightarrow \mathfrak{R}$

Embedded contour $C = \{ \mathbf{x} \in \Omega \mid \phi(\mathbf{x}) = 0 \}$

Dynamic Shape Representation



$$H_{\varepsilon}(\phi) = \begin{cases} 1 & \phi \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

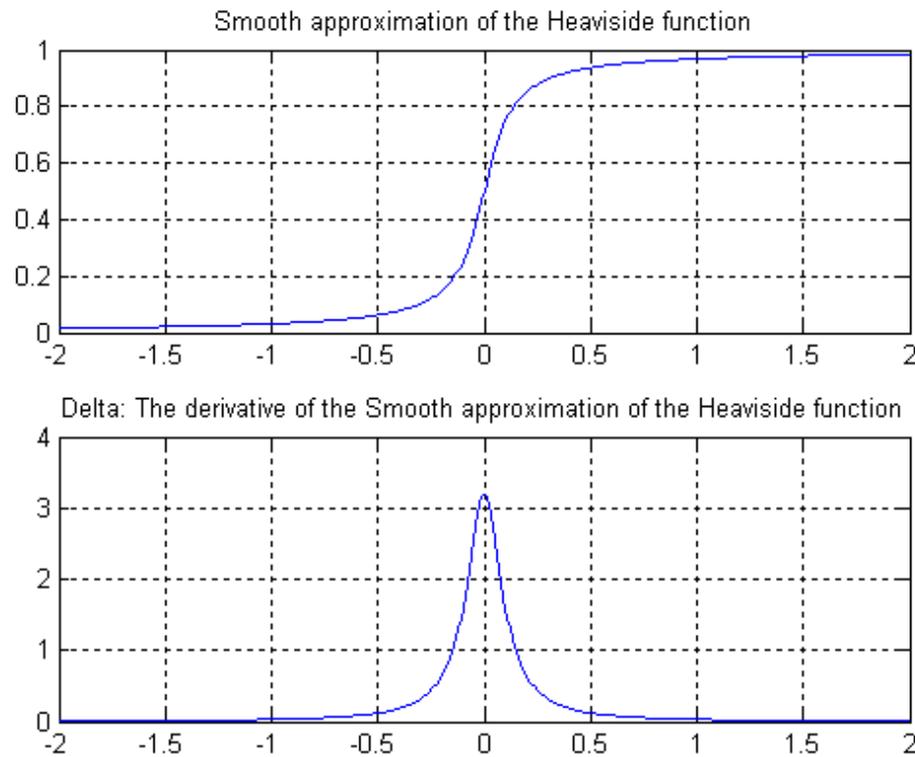
Heaviside function



$$\chi(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in \omega \\ 0 & \mathbf{x} \in \Omega \setminus \omega \end{cases}$$

Object indicator function

Reguralized Heaviside and Delta Functions



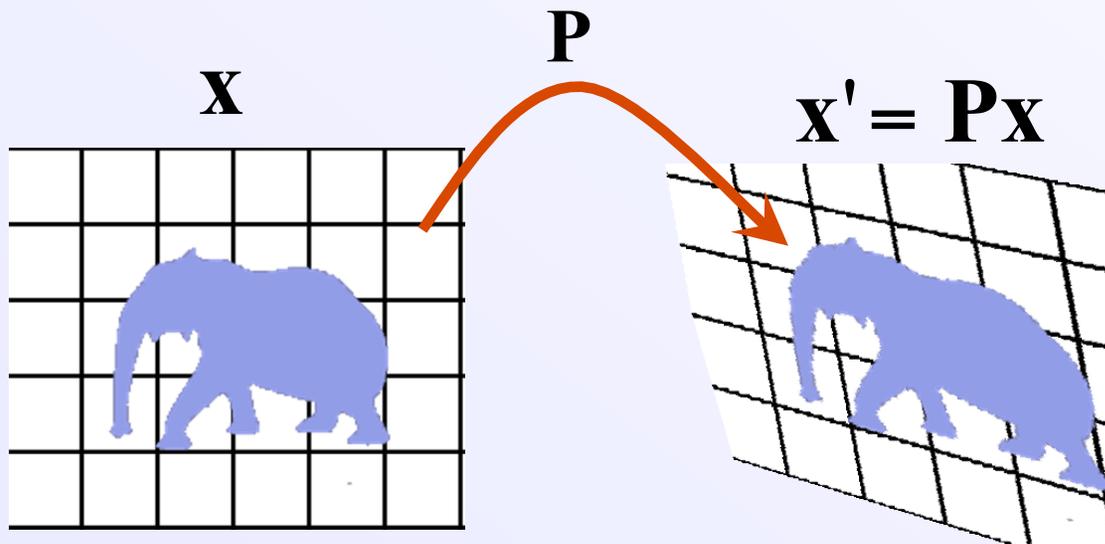
$$H_{\varepsilon}(\phi) = \frac{1}{2} \left(1 + \frac{2}{\pi} \arctan\left(\frac{\phi}{\varepsilon}\right) \right)$$

$$\delta_{\varepsilon}(\phi) = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + \phi^2}$$

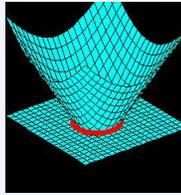
Dissimilarity Measure

$$D(\text{red elephant}, \text{cyan elephant}) = \text{blue elephant} = \int_{\Omega} \square$$

Transformations



Talk Overview



- Level-sets formulation

- Shape representation

- Segmentation

- Prior based segmentation

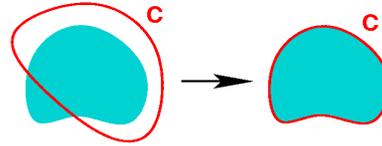
- Mutual segmentation

- Symmetry based segmentation

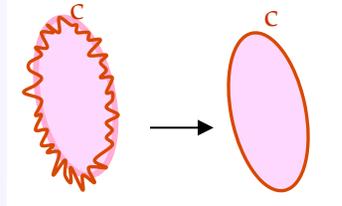
Cost Functional

$$E(\phi, \mathbf{P}) = \int_{\Omega}$$

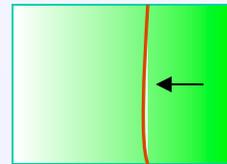
Region based



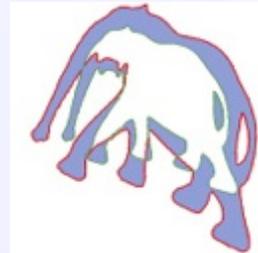
+ Smoothness



+ Edge based



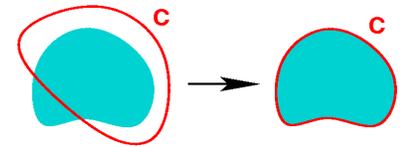
+ Shape



$d\mathbf{x}$

Region based Term

Piecewise constancy assumption



$$E_{RB}(u_{in}, u_{out}, C) = \int_{inside(C)} (I - u_{in})^2 d\mathbf{x} + \int_{outside(C)} (I - u_{out})^2 d\mathbf{x}$$

$$\int_{inside C} d\mathbf{x} \Rightarrow \int_{\Omega} H_{\varepsilon}(\phi) d\mathbf{x}$$

$$\int_{outside C} d\mathbf{x} \Rightarrow \int_{\Omega} [1 - H_{\varepsilon}(\phi)] d\mathbf{x}$$

$$E_{RB}(u_{in}, u_{out}, \phi) = \int_{\Omega} [(I - u_{in})^2 H_{\varepsilon}(\phi) + (I - u_{out})^2 (1 - H_{\varepsilon}(\phi))] d\mathbf{x}$$

Chan and Vese 2001

$$E(\phi, \mathbf{P}) = \int_{\Omega} \boxed{\text{Region based}} + \text{Smoothness} + \text{Edge based} + \text{Shape} d\mathbf{x}$$

Region-based Term

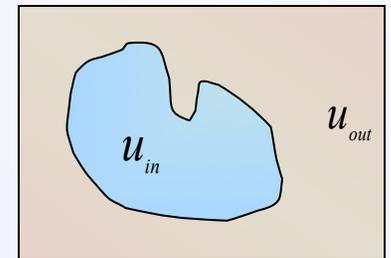
Chan-Vese Two Phase Model

Gradient descent equation (Euler Lagrange)

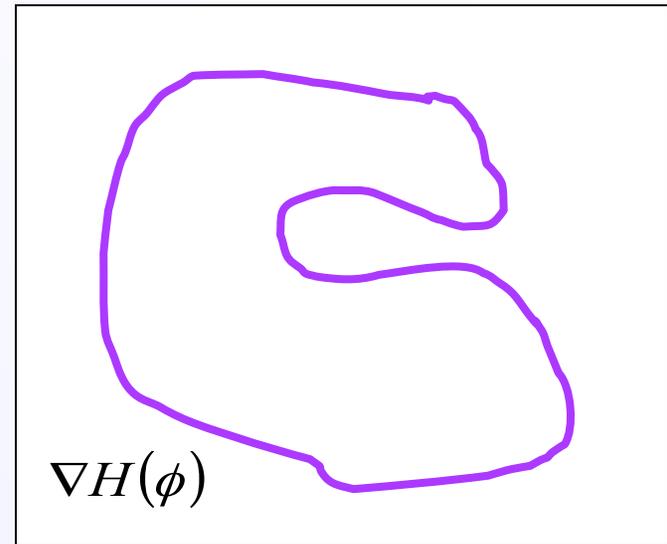
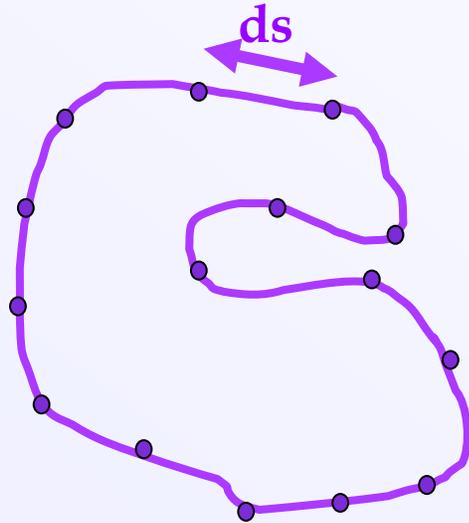
$$\frac{\partial \phi^{\text{RB}}}{\partial t} = \delta_{\varepsilon}(\phi) \left[(I(\mathbf{x}) - u_{out})^2 - (I(\mathbf{x}) - u_{in})^2 \right]$$

$$u_{in} = \frac{\int I(\mathbf{x}) H_{\varepsilon}(\phi) d\mathbf{x}}{\int H_{\varepsilon}(\phi) d\mathbf{x}}$$

$$u_{out} = \frac{\int I(\mathbf{x}) (1 - H_{\varepsilon}(\phi)) d\mathbf{x}}{\int (1 - H_{\varepsilon}(\phi)) d\mathbf{x}}$$



Smoothness Term



$$|c| = \int_0^L ds \quad \Longrightarrow \quad E_{LEN} = \int_{\Omega} \underbrace{|\nabla H(\phi(\mathbf{x}))|}_{ds} d\mathbf{x}$$

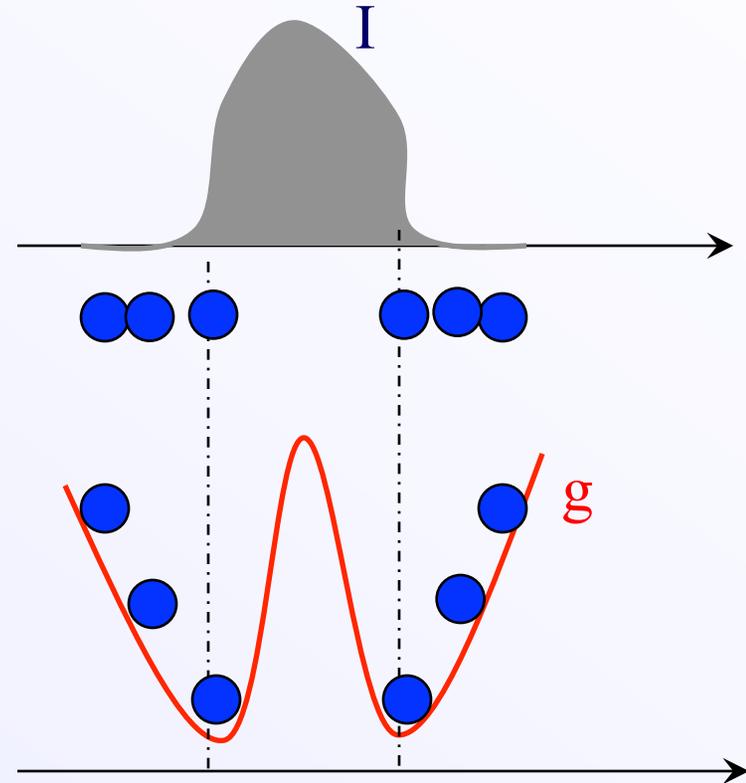
$$\phi_t^{LEN} = \delta_{\varepsilon}(\phi) \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right)$$

$$E(\phi, \mathbf{P}) = \int_{\Omega} \text{Region based} + \boxed{\text{Smoothness}} + \text{Edge based} + \text{Shape} d\mathbf{x}$$

Geodesic Active Contour

$$E_{GAC} = \int_C g(|\nabla I|) ds$$

$$g(\mathbf{x}) = \frac{1}{1 + |\nabla I(\mathbf{x})|^2}$$

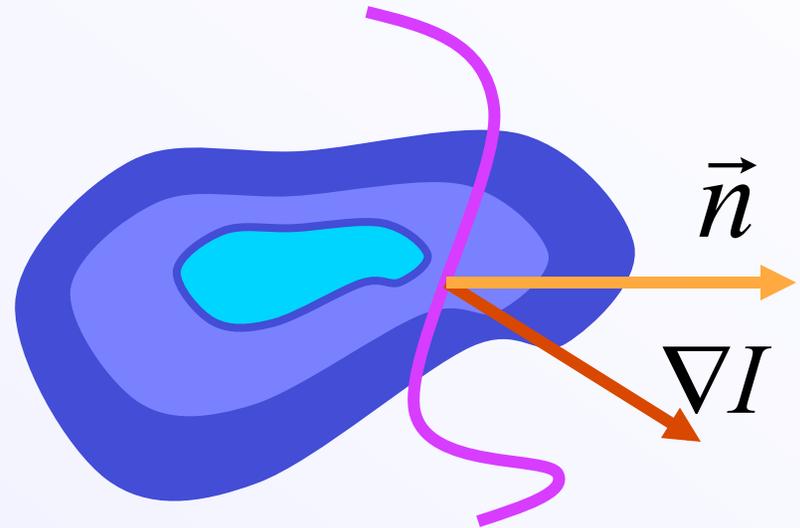


$$E_{GAC} = \int_{\Omega} g(|\nabla I|) \underbrace{|\nabla H_{\varepsilon}(\phi(\mathbf{x}))|}_{ds} d\mathbf{x}$$

$$E(\phi, \mathbf{P}) = \int_{\Omega} \text{Region based} + \text{Smoothness} + \boxed{\text{Edge based}} + \text{Shape} d\mathbf{x}$$

Alignment Term

$$\vec{n} = \frac{\nabla \phi}{|\nabla \phi|}$$



$$E_{RA} = - \int_{\Omega} \left\langle \nabla I, \frac{\nabla \phi}{|\nabla \phi|} \right\rangle |\nabla H_{\varepsilon}(\phi)| d\mathbf{x}$$

Talk Overview

- Level-sets formulation

- Shape representation

- Segmentation



- Prior based segmentation

- Mutual segmentation

- Symmetry based segmentation

Prior Shape Term

$$E_{\text{shape}}(\phi) = \int_{\Omega} \left(H_{\varepsilon}(\phi) - H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}) \right)^2 d\mathbf{x}$$

$$D(\text{red elephant}, \text{cyan elephant}) = \text{blue elephant} = \int_{\Omega} \square$$



$I(\mathbf{x})$



$\tilde{\chi}(\mathbf{x}) = H_{\varepsilon}(\tilde{\phi}(\mathbf{x}))$

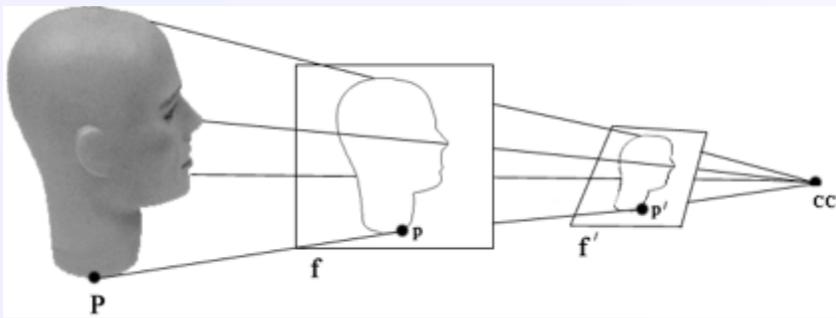


$\tilde{\chi}(\mathbf{P}\mathbf{x}) = H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}})$

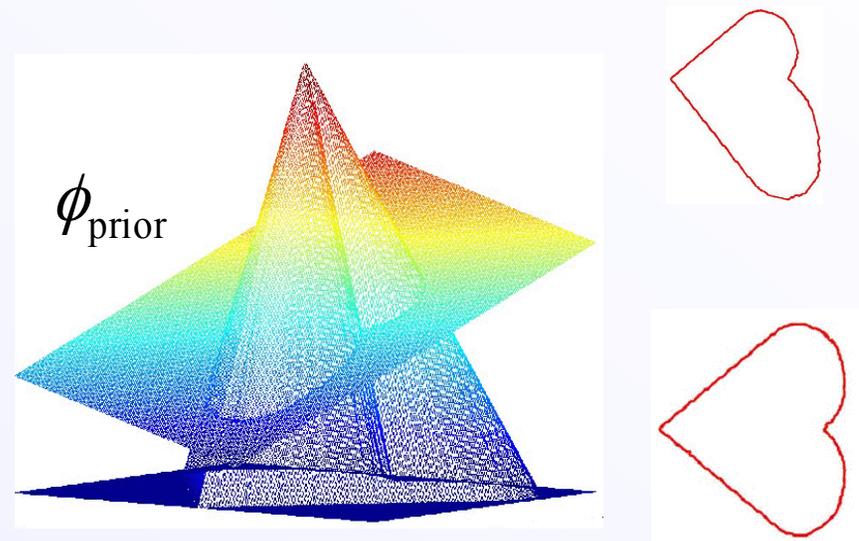
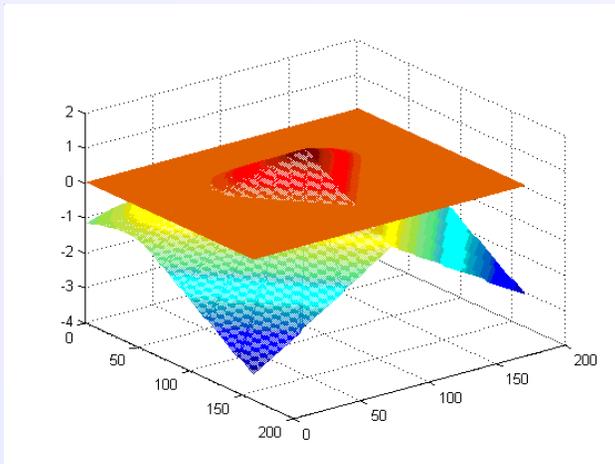
$$E(\phi, \mathbf{P}) = \int_{\Omega} \text{Region based} + \text{Smoothness} + \text{Edge based} + \boxed{\text{Shape}} d\mathbf{x}$$

Representation of the Prior shape

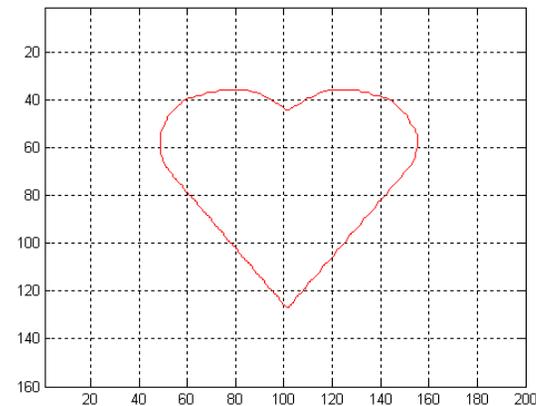
Single View Geometry



Cone of Rays

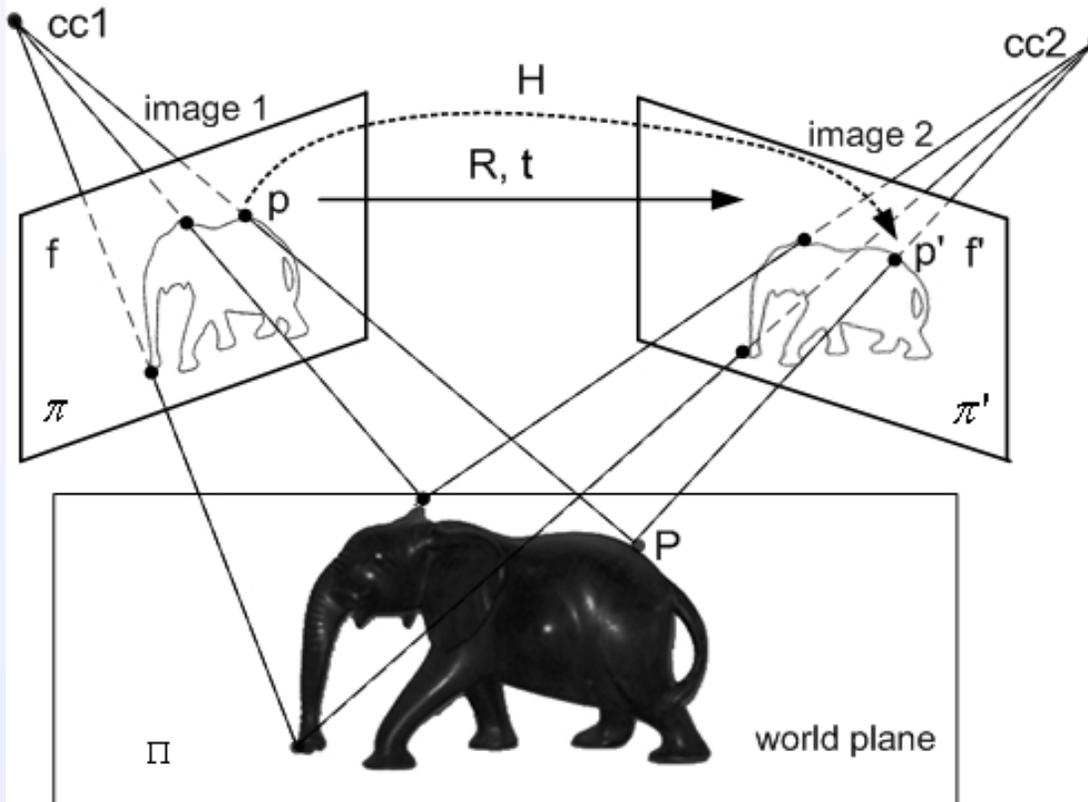


Generalized cone



Representation of the Prior shape

Two View Geometry



The homography

$$\mathbf{x}' = \left(R + \frac{1}{d} \mathbf{t} \mathbf{n}^T \right) \mathbf{x} = \mathbf{P} \mathbf{x}$$

Gradient Descent Equations

$$E_{\text{shape}}(\phi) = \int_{\Omega} \left(\underbrace{H_{\varepsilon}(\phi)}_{\chi} - \underbrace{H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}})}_{\tilde{\chi}_{\mathbf{P}}} \right)^2 d\mathbf{x}$$

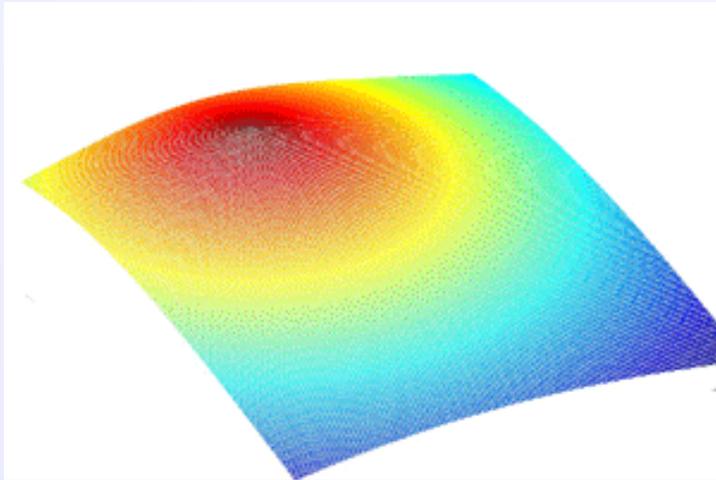
Gradient descent equations:

$$\frac{\partial \phi}{\partial t} = -2\delta_{\varepsilon}(\phi) \left[H_{\varepsilon}(\phi) - H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}) \right]$$

$$\frac{\partial p_{ij}}{\partial t} = \int_{\Omega} \delta_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}) \left[H_{\varepsilon}(\phi) - H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}) \right] \frac{\partial \tilde{\phi}_{\mathbf{P}}}{\partial p_{ij}} d\mathbf{x}$$

Unified Cost Functional

$$E(\phi, \mathbf{P}) = \int_{\Omega} \text{Region based} + \text{Smoothness} + \text{Edge based} + \text{Shape} \, d\mathbf{x}$$



Evolution of ϕ



Registration

Results



Prior

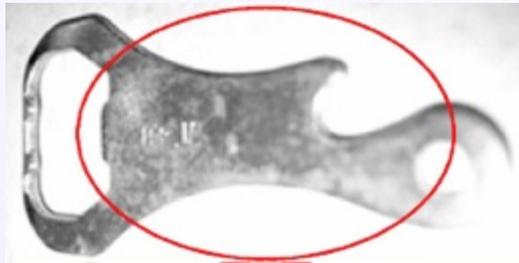


Segmentation

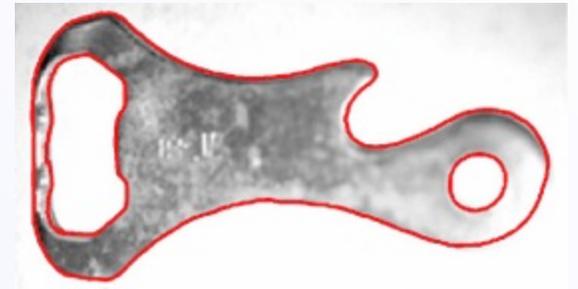
Results



Prior Image



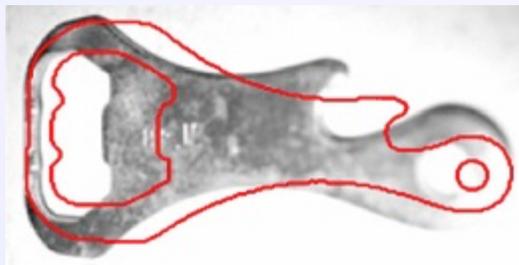
Initial contour



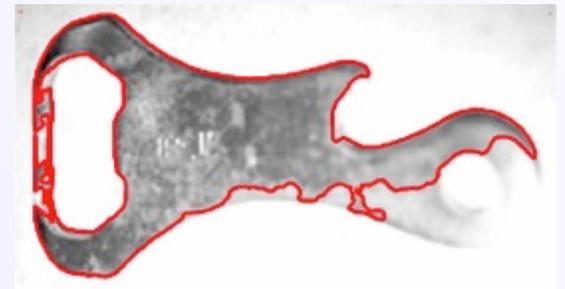
Final (desired) segmentation



Verification:
Final contour on
transformed prior



Prior contour on image



Segmentation without
prior

Results



Prior Image



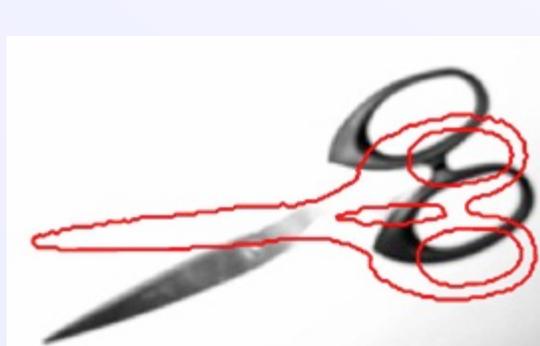
Initial contour



Final (desired) segmentation



Verification:
Final contour on transformed
prior



Prior contour on image



Segmentation without prior

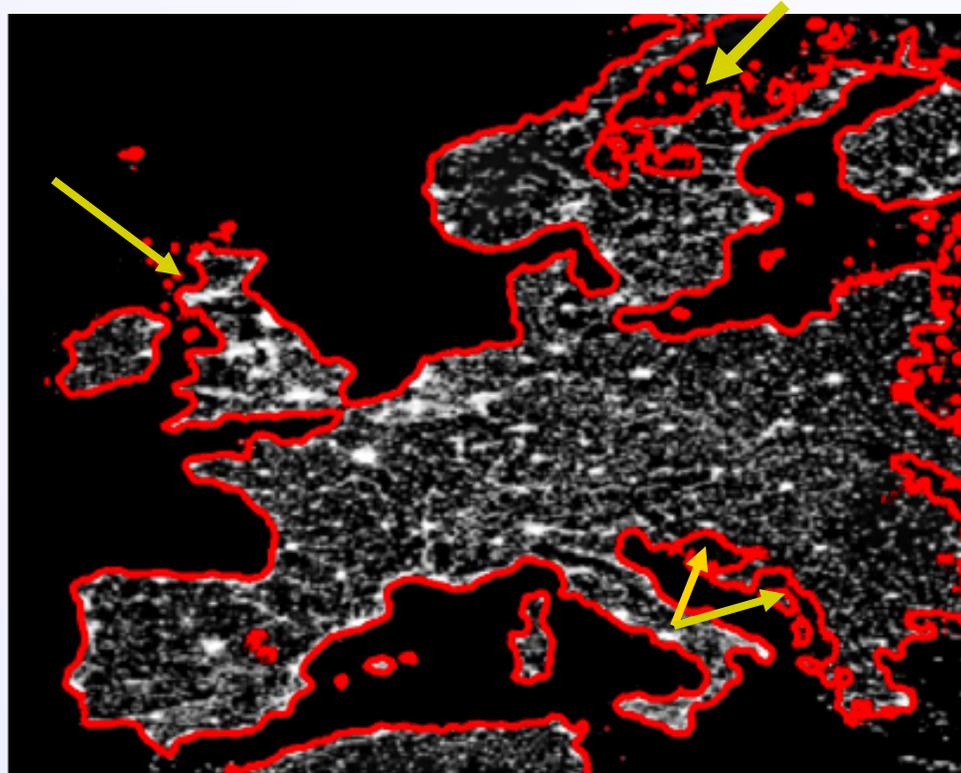
Results



Image to segment



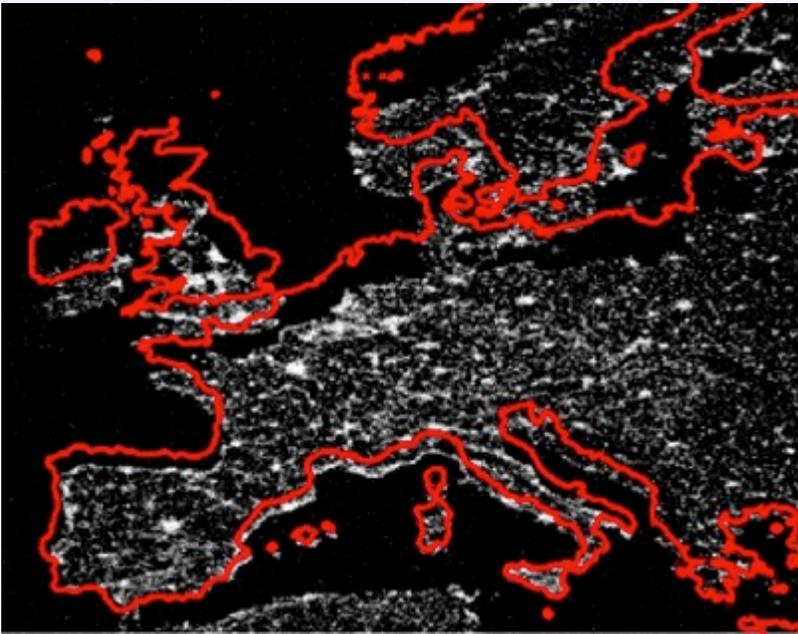
Results



Region based segmentation



Results



Misalignment



Prior image



Results



ICCV05 Riklin-Raviv, Kiryati, Sochen

Talk Overview

- Level-sets formulation

- Shape representation
- Segmentation

- Prior based segmentation



- Mutual segmentation

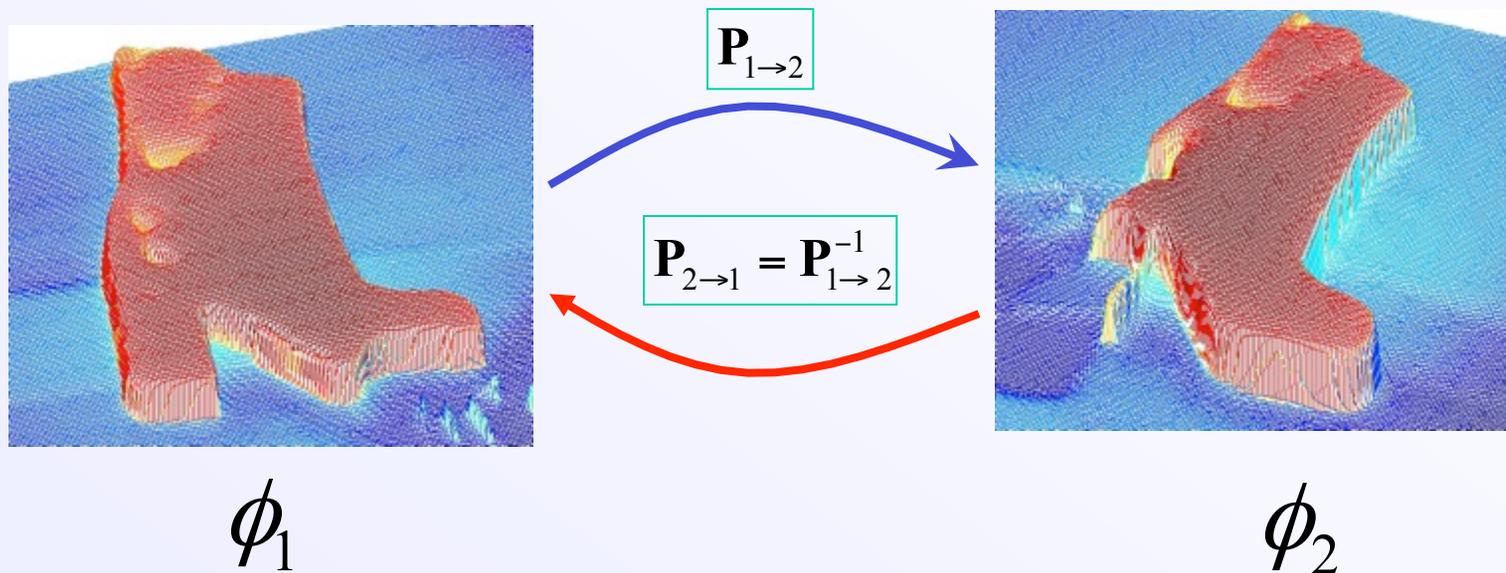
- Symmetry based segmentation

Problem setting



- Two views of the same object are given
- Their contours are related by planar projective transformation
- Each object cannot be segmented based on image data alone
- Together both images contain sufficient information for the extraction of the objects

Shape constraint



Alternately evolve the level set functions of the two object instances using both images data.

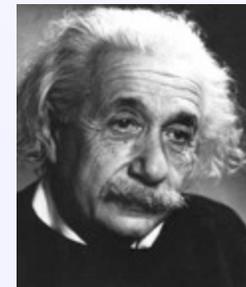
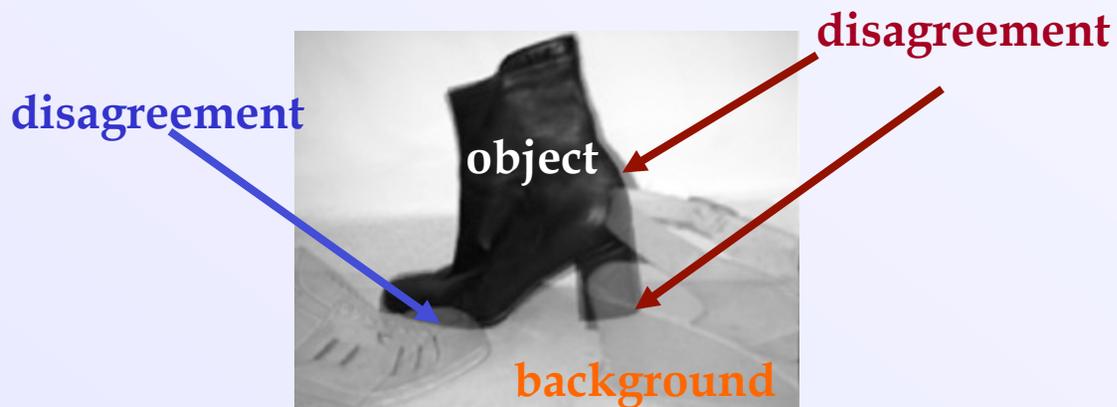
Evolve ϕ_1 based on the data of image I_1 and on ϕ_2 .

Evolve ϕ_2 based on the data of image I_2 and on ϕ_1 .

Oops ...



There is an inherent ambiguity in regions where the two object views disagree.



Biased dissimilarity measure

$$D(\text{blue silhouette}, \text{red silhouette}) = \text{green silhouette} = \int_{\Omega} \text{green square}$$

$$D(\chi, \tilde{\chi}_P) = \int_{\Omega} (\chi(\mathbf{x}) - \tilde{\chi}_P(\mathbf{x}))^2 d\mathbf{x} \cong \int_{\Omega} \chi(\mathbf{x}) \oplus \tilde{\chi}_P(\mathbf{x}) d\mathbf{x}$$

$$D_{biased}(\chi, \tilde{\chi}_P) = \int_{\Omega} [(1 - \chi) \cdot \tilde{\chi}_P + \eta \chi \cdot (1 - \tilde{\chi}_P)] d\mathbf{x}$$

$$0 < \eta < 1$$



Currently evolved



Superposition



Aligned reference shape

Biased dissimilarity measure

$$D(\text{blue silhouette}, \text{red silhouette}) = \text{green silhouette} = \int_{\Omega} \text{green square}$$

$$D(\chi, \tilde{\chi}_P) = \int_{\Omega} (\chi(\mathbf{x}) - \tilde{\chi}_P(\mathbf{x}))^2 d\mathbf{x} \cong \int_{\Omega} \chi(\mathbf{x}) \oplus \tilde{\chi}_P(\mathbf{x}) d\mathbf{x}$$

$$D_{biased}(\chi, \tilde{\chi}_P) = \int_{\Omega} [(1 - \chi) \cdot \tilde{\chi}_P + \eta \chi \cdot (1 - \tilde{\chi}_P)] d\mathbf{x}$$

$$1 < \eta$$



Currently evolved



Superposition



Aligned reference shape

Unified Cost Functional

$$E(\phi, \mathbf{P}) = \int_{\Omega} \text{Region based} + \text{Edge based} + \text{Alignment} + \text{Shape} \, d\mathbf{x}$$

$$E_{Shape}(\phi | \tilde{\phi}, \mathbf{P}) = \int_{\Omega} [(1 - H_{\varepsilon}(\phi)) H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}) + \eta H_{\varepsilon}(\phi) (1 - H_{\varepsilon}(\tilde{\phi}_{\mathbf{P}}))] d\mathbf{x}$$

$$\tilde{\phi}_{\mathbf{P}} = \tilde{\phi}(\mathbf{P}\mathbf{x})$$

The contribution of the shape term to ϕ

$$\phi_t^{Shape} = \delta_{\varepsilon}(\phi) [H(\tilde{\phi}_{\mathbf{P}}) - \eta (1 - H(\tilde{\phi}_{\mathbf{P}}))]$$

Recovery of the transformation parameters:

$$\frac{\partial p_{i,j}}{\partial t} = \int_{\Omega} \delta(\tilde{\phi}_{\mathbf{P}}) [(1 - H(\phi)) - \eta H(\phi)] \frac{\partial \tilde{\phi}_{\mathbf{P}}}{\partial p_{i,j}} d\mathbf{x}$$

Mutual Segmentation Results



Initial contour

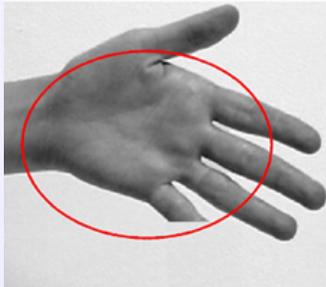


Superposition

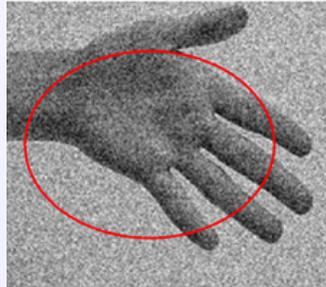


Mutual segmentation

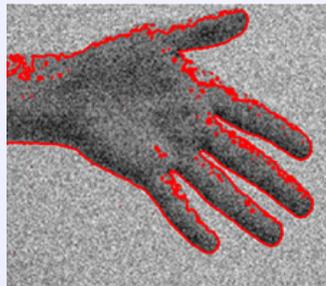
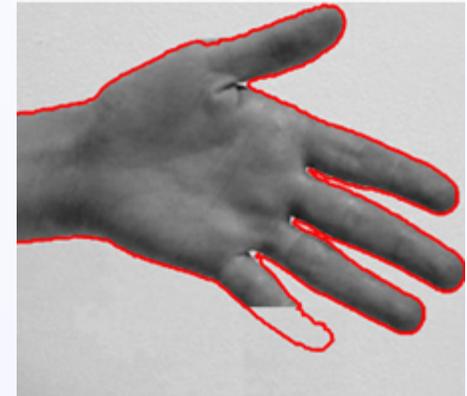
Mutual Segmentation Results



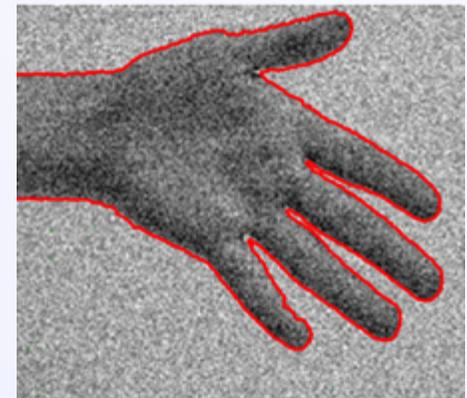
Initial contour



Superposition



Self segmentation



Mutual segmentation

Mutual Segmentation Results



Initial contour



Initial contour



Superposition



Mutual segmentation



Mutual segmentation

Mutual Segmentation Results



Mutual segmentation



Self segmentation

Mutual Segmentation: Results

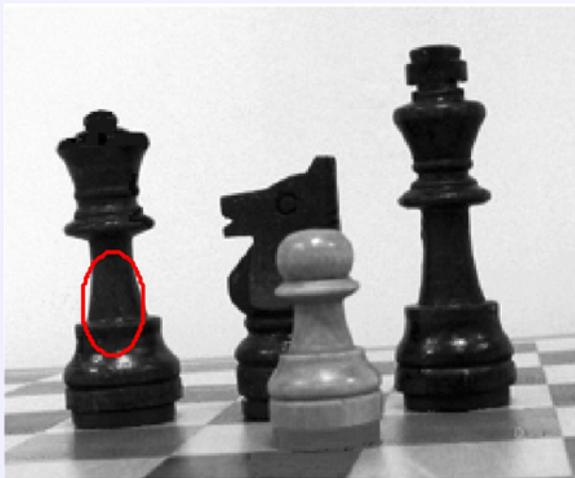


Initial contour

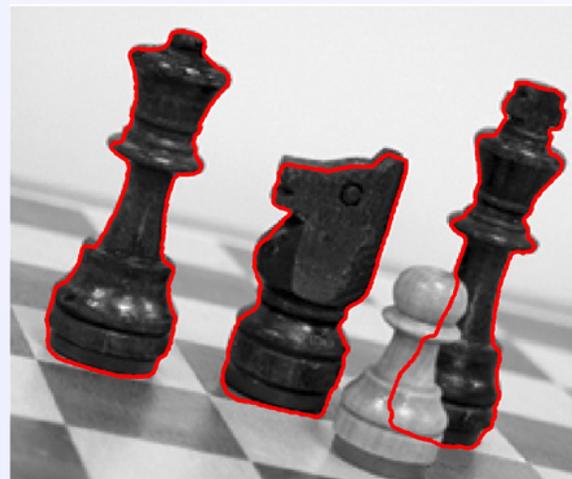
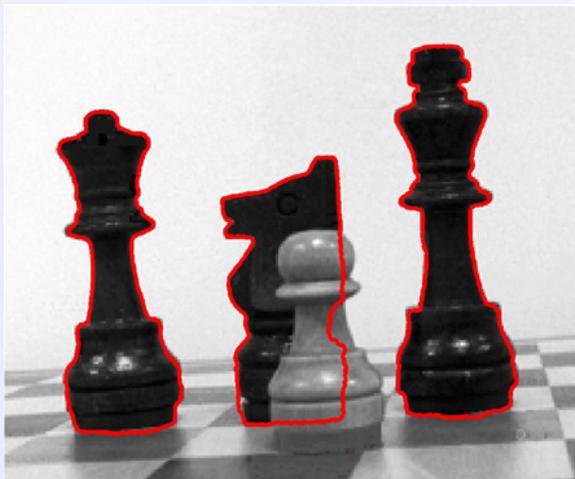


Final contour

Mutual Segmentation: Results



Initial contour



Final contour

Talk Overview

- Level-sets formulation

- Shape representation

- Segmentation

- Prior based segmentation

- Mutual segmentation



- Symmetry based segmentation

Shape Symmetry

The Prior is Inside



Symmetry as Shape Constraint

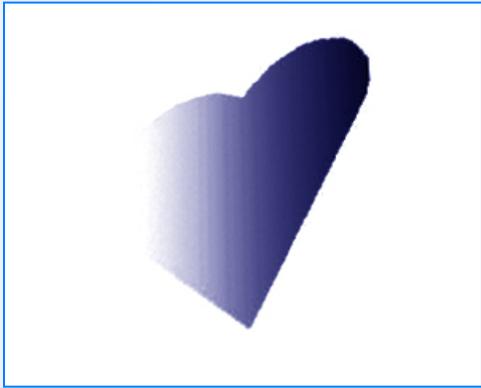
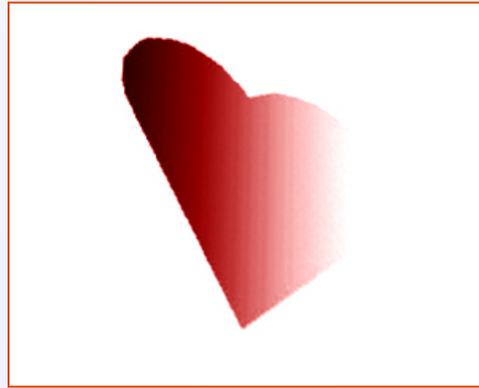
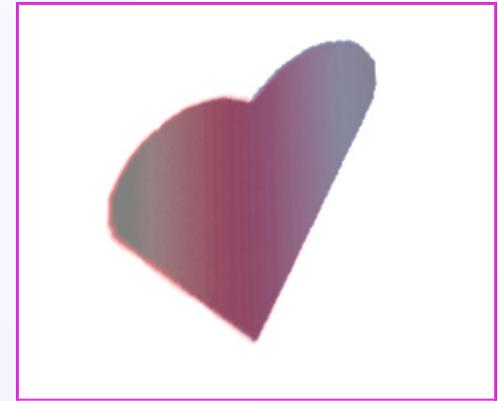


Image to segment



Symmetrical counterpart



Registration

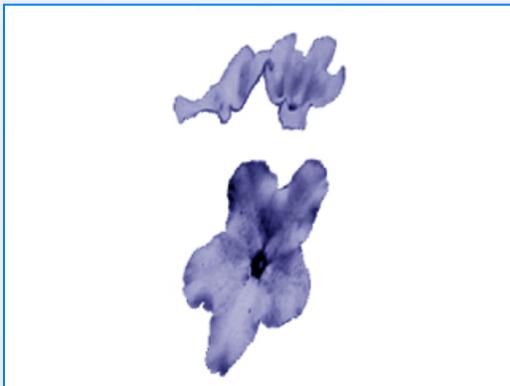
Symmetry as Shape Constraint



Original image



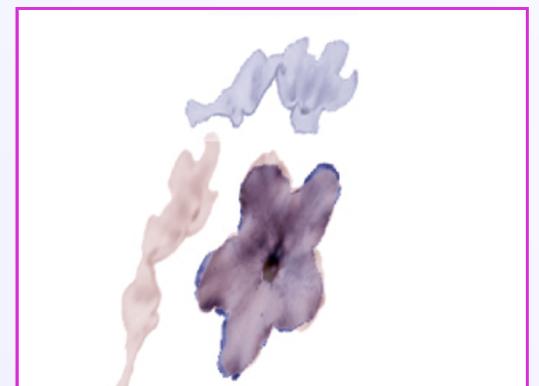
Color-based segmentation



Distorted segmentation

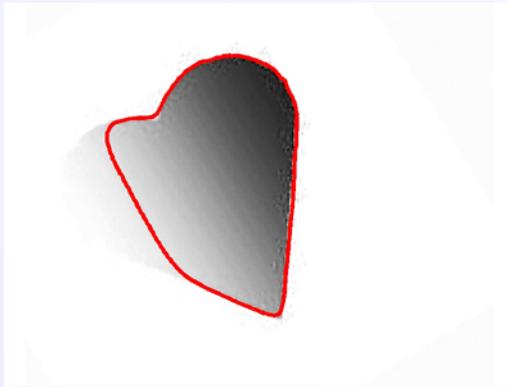


Symmetrical counterpart

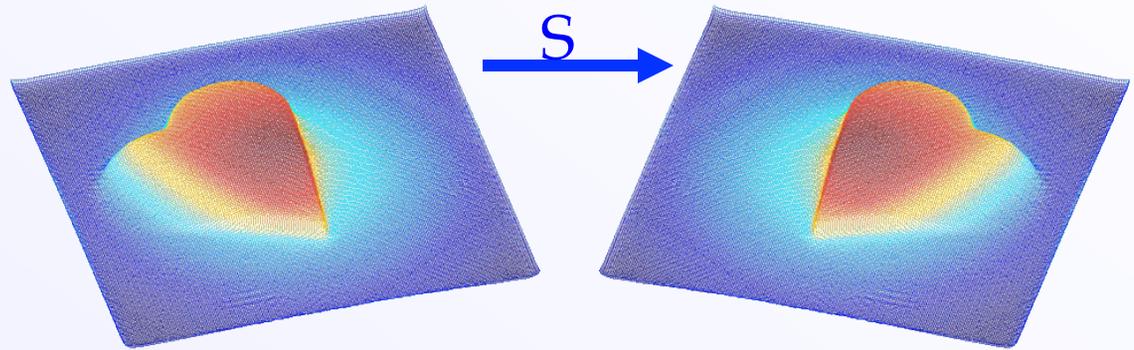


Registration

Symmetrical Counterparts



Segmentation



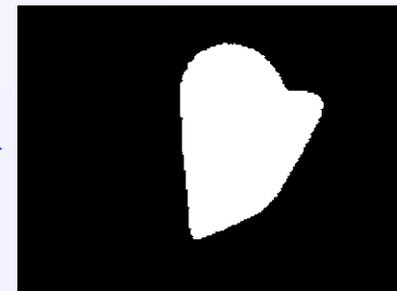
Evolving level-set
function $\phi(t)$

Symmetrical
counterpart $\hat{\phi}(t)$

χ is symmetrical
when $\chi = \hat{\chi}$



$$\chi(\mathbf{x}) = H_\varepsilon(\phi(\mathbf{x}))$$



$$\hat{\chi}(\mathbf{x}) = H_\varepsilon(\phi(S\mathbf{x}))$$

Symmetry Matrices

$$S_{LR} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{UD} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection

$$S_{ROT} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

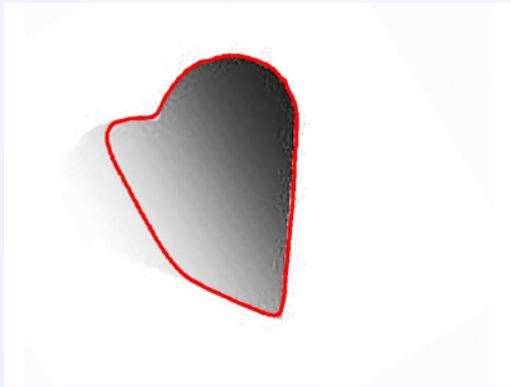
Rotation

$$S_{Tx} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{Ty} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Translation

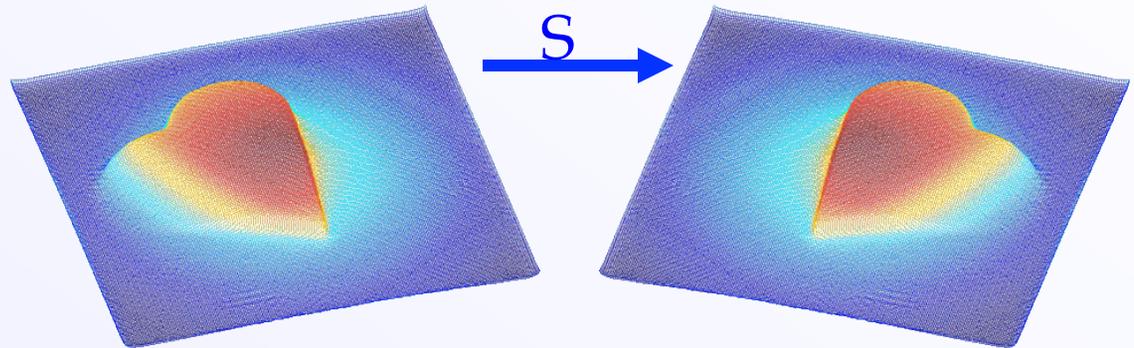
Symmetrical Counterparts



Segmentation

M is a planar projective Homography.

S is a 3x3 matrix.



Evolving level-set function $\phi(t)$

Symmetrical counterpart $\hat{\phi}(t)$



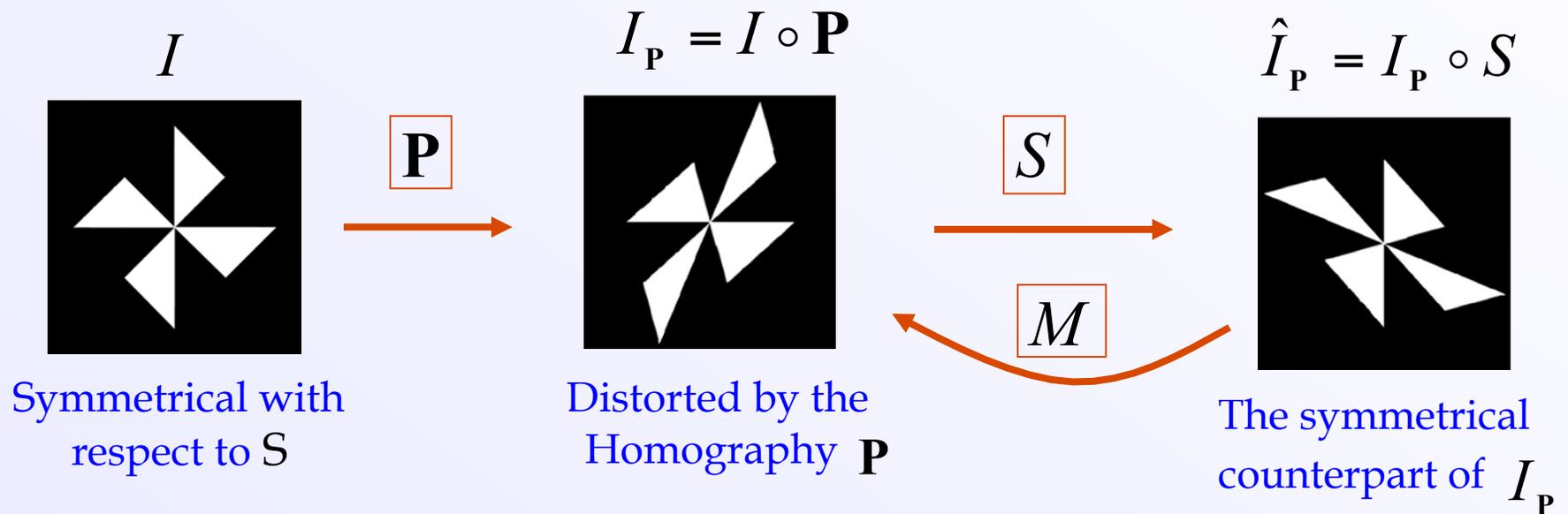
$$\chi(\mathbf{x}) = H_\varepsilon(\phi(\mathbf{x}))$$



$$\hat{\chi}(\mathbf{x}) = H_\varepsilon(\phi(S\mathbf{x}))$$

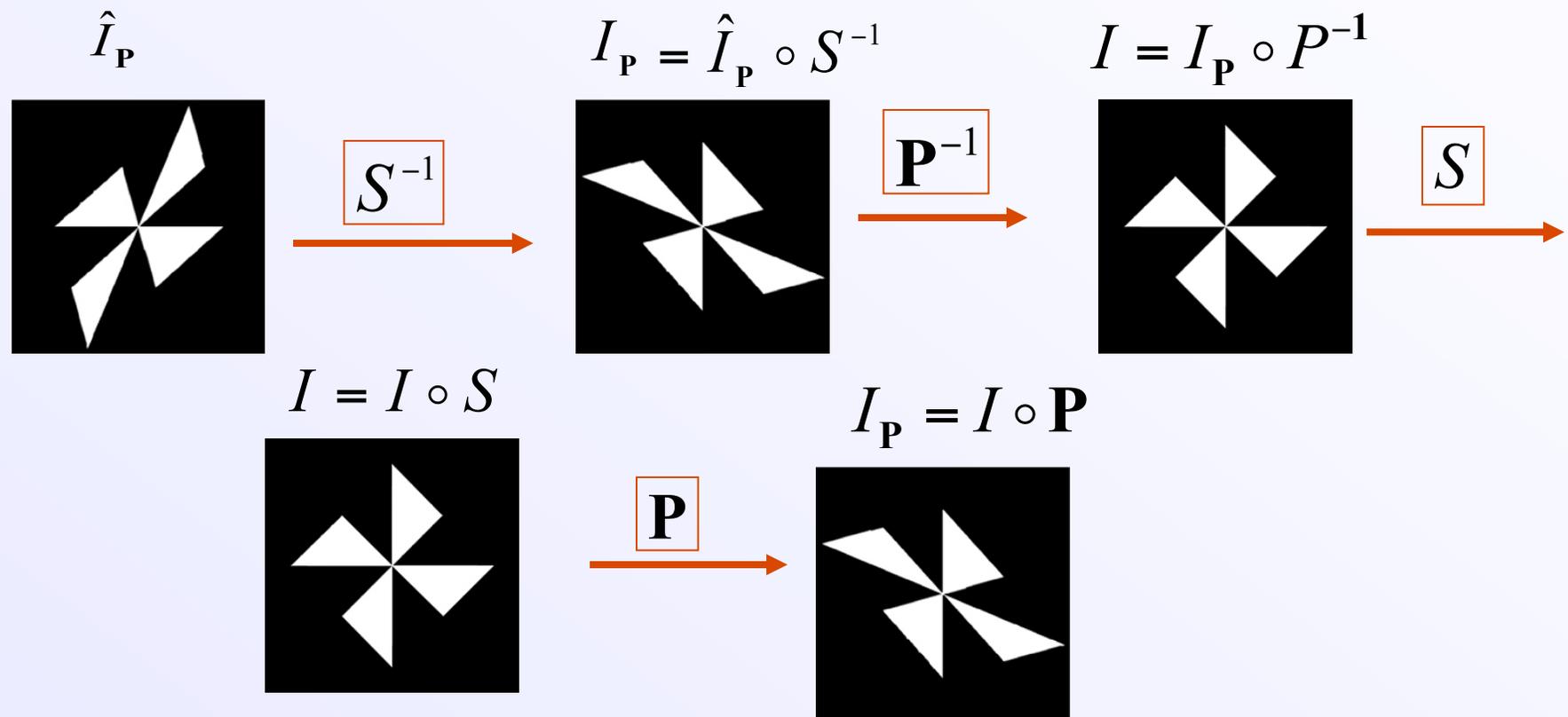
Theoretical Results:

The Recovery of \mathbf{P} from \mathbf{M}



$$\mathbf{M} = \mathbf{S}^{-1} \mathbf{P}^{-1} \mathbf{S} \mathbf{P}$$

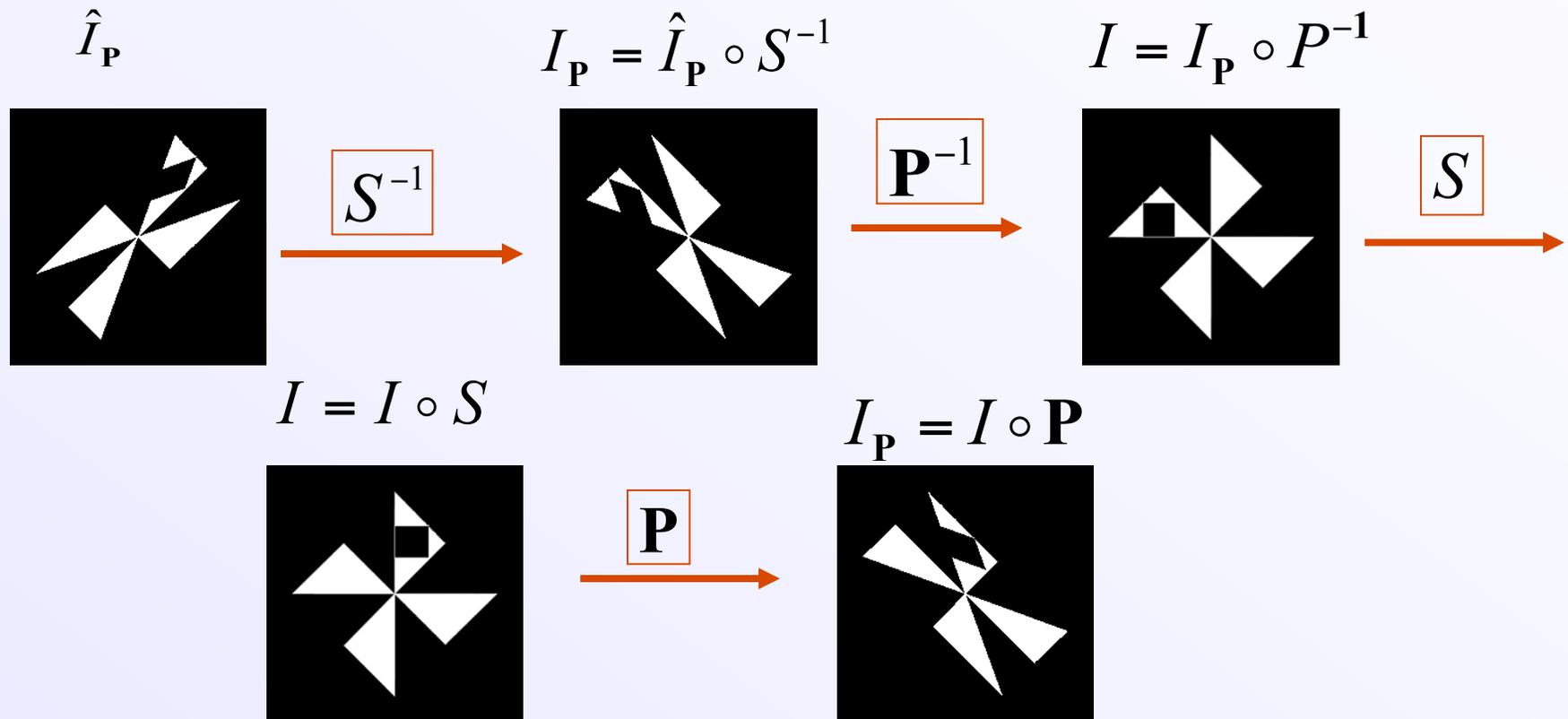
The Transformation between Symmetrical Counterparts



$$S_P = P^{-1} S P = P^{-1} S^{-1} P$$

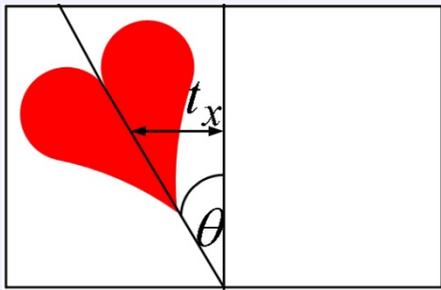
$$M = S^{-1} S_P = S^{-1} P^{-1} S P$$

The Transformation between Symmetrical Counterparts

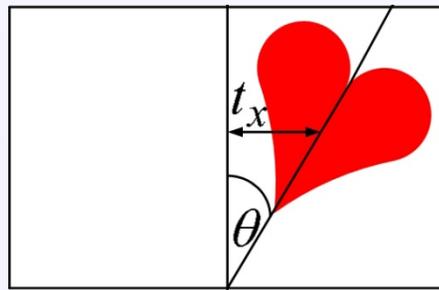


$$M = S^{-1} P^{-1} S P$$

Examples



$I_{\mathbf{P}_E}$



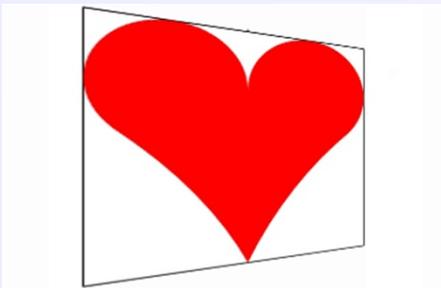
$\hat{I}_{\mathbf{P}_E}$

$$S = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

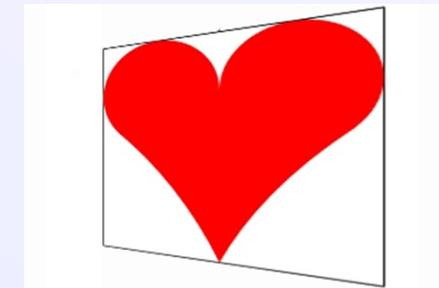
$$\mathbf{P}_E = \begin{bmatrix} R(\theta) & \mathbf{t} \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} R(2\theta) & \tilde{\mathbf{t}} \\ 0 & 1 \end{bmatrix}$$

$$\tilde{\mathbf{t}} = 2R(\theta) \begin{bmatrix} t_x \\ 0 \end{bmatrix}$$



$I_{\mathbf{P}}$

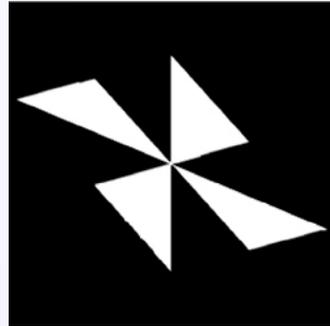
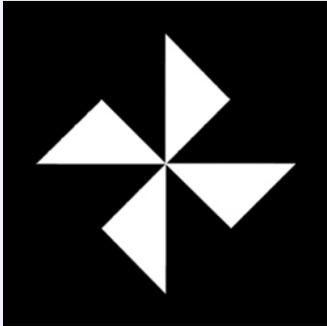


$\hat{I}_{\mathbf{P}}$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ v_1 & v_2 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2v_1 & 0 & 1 \end{bmatrix}$$

Examples



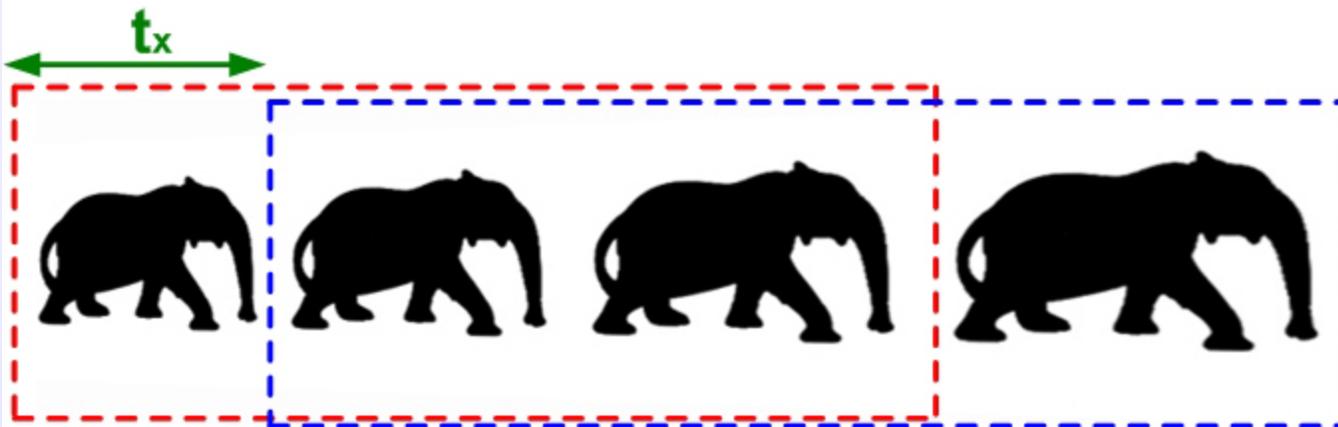
$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation by $\pi/2$

$$H = \begin{bmatrix} R(\theta) & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} K & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

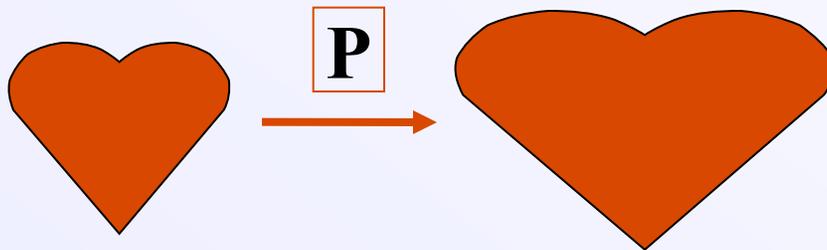
$$K = \begin{bmatrix} k_1 & k_2 \\ 0 & 1/k_1 \end{bmatrix}$$

$$M = \begin{bmatrix} k_1^2 & k_1 k_2 & 0 \\ k_1 k_2 & 1/k_1^2 + k_2^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Limits on the Recovery of \mathbf{P} from \mathbf{M}

\mathbf{P} is symmetry preserving if $I(\mathbf{P}\mathbf{S}\mathbf{X}) = I(\mathbf{S}\mathbf{P}\mathbf{X})$



\mathbf{P} cannot be recovered from \mathbf{M} if $\mathbf{P} = \mathbf{P}^* \tilde{\mathbf{P}}$
and \mathbf{P}^* is a symmetry preserving transformation.

$$\begin{aligned}
 \mathbf{M} &= \mathbf{S}^{-1} \mathbf{P}^{-1} \mathbf{S} \mathbf{P} = \mathbf{S}^{-1} (\mathbf{P}^* \tilde{\mathbf{P}})^{-1} \mathbf{S} (\mathbf{P}^* \tilde{\mathbf{P}}) = \mathbf{S}^{-1} \tilde{\mathbf{P}}^{-1} \mathbf{P}^{*-1} \mathbf{S} \mathbf{P}^* \tilde{\mathbf{P}} = \\
 &\mathbf{S}^{-1} \tilde{\mathbf{P}}^{-1} \mathbf{P}^{*-1} \mathbf{P}^* \mathbf{S} \tilde{\mathbf{P}} = \mathbf{S}^{-1} \tilde{\mathbf{P}}^{-1} \mathbf{S} \tilde{\mathbf{P}}
 \end{aligned}$$

↑
↑

Unified Cost Functional

$$E(\phi, \mathbf{M}) = \int_{\Omega} \text{Region based} + \text{Edge based} + \text{Smoothness} + \text{Symmetry} d\mathbf{x}$$

$$E_{\text{Symmetry}}(\phi, M) = H_{\varepsilon}(\phi)(1 - H_{\varepsilon}(\hat{\phi}_M)) + \eta H_{\varepsilon}(\hat{\phi}_M)(1 - H_{\varepsilon}(\phi))$$

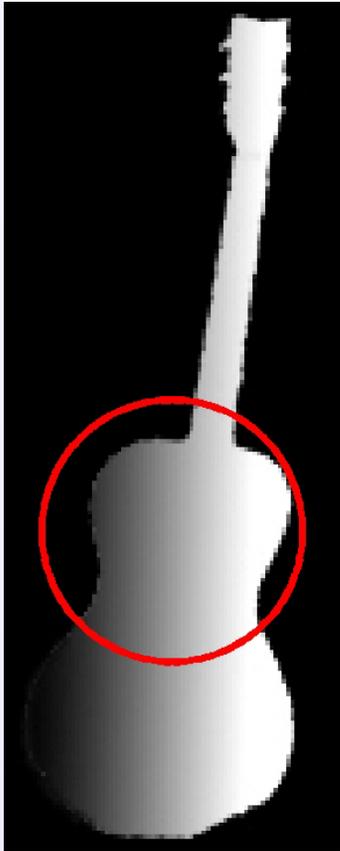
The contribution of the symmetry term to ϕ

$$\phi_t = \delta_{\varepsilon}(\phi) = [H_{\varepsilon}(\hat{\phi}_M) - \eta(1 - H_{\varepsilon}(\hat{\phi}_M))]$$

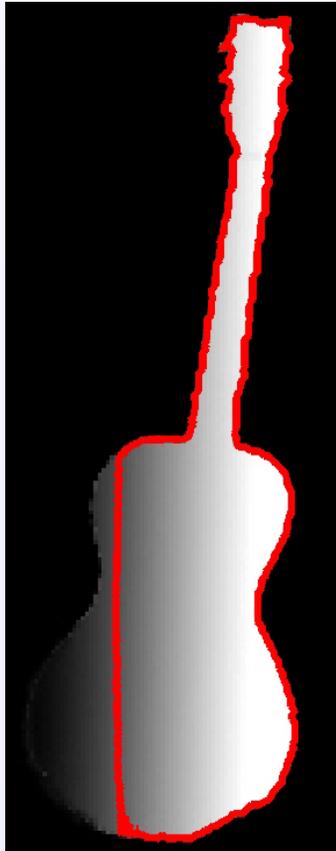
Gradient descent equations for M

$$\frac{\partial m_{i,j}}{\partial t} = \int_{\Omega} \delta_{\varepsilon}(\hat{\phi}_M) [(1 - H_{\varepsilon}(\phi)) - \eta H_{\varepsilon}(\phi)] \frac{\partial \hat{\phi}_M}{\partial m_{i,j}} d\mathbf{x}$$

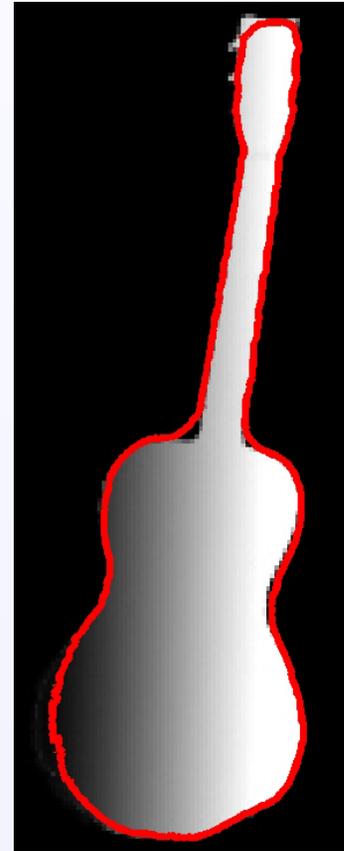
Symmetry Results



Initial contour



Region-Edge based
segmentation



Symmetry-aided
segmentation

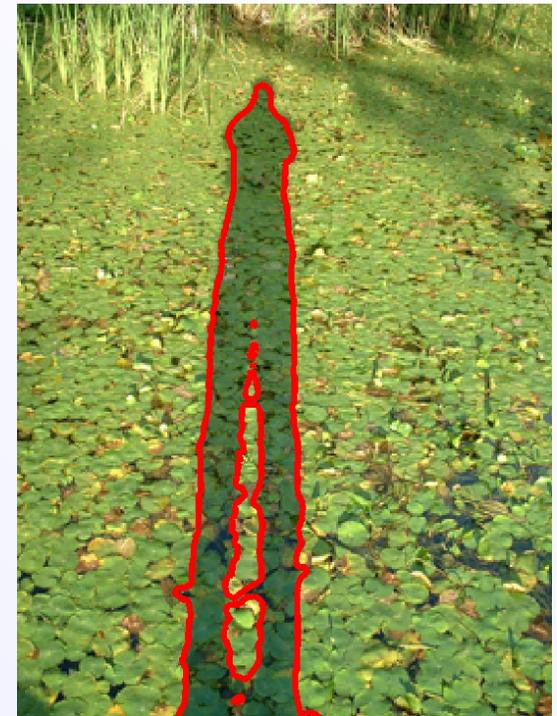
Symmetry Results



Initial contour



Region-Edge based
segmentation



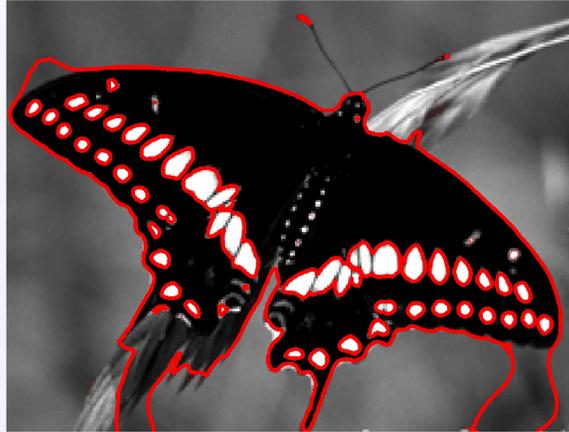
Symmetry-aided
segmentation

Original image courtesy of Amit Jayant Deshpande

Symmetry Results



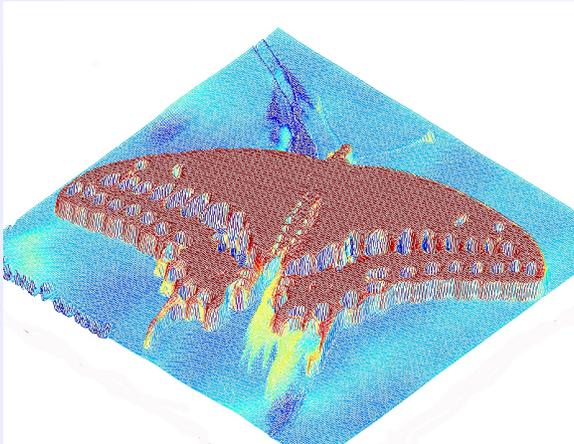
Initial contour



Region-Edge based
segmentation



Symmetry-aided
segmentation



Final level-set function

Original image courtesy of George Payne

Symmetry Results



Initial contour



Region-Edge based
segmentation



Symmetry-aided
segmentation

Original image courtesy of Richard Lindley

Symmetry Results



Symmetry-aided segmentation

Original image courtesy of Kenneth R. Robertson

Symmetry Results



Initial contour



Region-Edge based segmentation



Symmetry-aided

Segmentation: 1 symmetrical counterpart



Symmetry-aided

Segmentation: 2 symmetrical counterparts

Symmetry Results



Region-Edge based segmentation



Symmetry-aided

Original image courtesy of Allen Matheson

Summary



Segmentation using a prior shape in the presence of perspective distortion



Mutual segmentation of two object views



Supporting segmentation by perspective-distorted symmetry

Thank you 😊