Optimisation methods to detect buried objects in a arbitrary media

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Abstract: In this paper we present several optimisation methods to detect buried objects in a arbitrary media. We give the principles of each method and some validation examples. The advantages of these methods are discussed.

INTRODUCTION

During recent years, many methods have been developed to detect buried objects in real-time. To guarantee the constraints of time, the optimisation process and the field model are generally very simple in these methods. The most famous of these methods are imaging methods like GPR or more recently SAR where the processing is an inverse Fourier transform and where the field model is based upon the propagation of plane waves delayed in time. More classical methods based upon non linear optimisation techniques such as least square and Maxwell’s equations for field model are generally avoided because of an important cost in time. However, recently, optimisation methods based about the minimisation of an object distribution are studied and can be a good alternative to the classical GPR or even SAR methods. In this paper, we show two possible methods in the context of the detection of objects, which are based upon a non linear optimisation function and a Maxwell’s equations model and which are very fast.

In the first section of this paper, we present the physical problem studied with the assumptions made. The principles of the different inverse methods used to detect the objects are given in a second section with some numerical examples of validations.

PHYSICAL PROBLEM

The physical problem under study consist in detecting buried objects by means of electromagnetic waves. It is well known that this problem presents a lot of difficulties. In our contribution, several assumptions to simplify the real problem have been done:
- the ground is considered as an homogeneous dielectric material;
- the surface of the soil is flat;
- the dielectric contrast between the soil and the objects to be detected is important;
- the source used is a plane wave;
- the sensors are localised above the soil on a plane parallel to the surface of the soil and there is no interference between them.

PRINCIPLE OF THE INVERSE METHODS

The GPR methods. The technique consists in evaluating the depth of an object located in the soil below a source/sensor by using an inverse Fourier transform of the frequency measurements. The restrictions on the physical model (plane wave) don’t allow to locate and identify exactly the object (in particular when there are very close objects), but gives generally rough estimate at an hyperbola apex.

The SAR method. With the same assumptions on the physical model, this method consists to focalise the hyperbola on a point by using the measurement values taken at various positions of the sensors (migration processing) [1].

The inverse integral method. Let be a distribution of n dielectric objects $B_i$ in the space, the electric fields at a given point $z$ of the space can be written $E(z) = E_{inc}(z) + \sum_{i=1}^{n} \int_{B_i} G(x,z)(k_0^2 -
\[ k^2 E(x) \, dx \] where \( G(x, z) = -g(x, z)I_d - \frac{1}{k^2} H_z(g(x, z)) \), \( H_z(u) = \nabla \nabla^T u \) with \( \nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{pmatrix} \) and \( g(x, z) \)
defines the scalar green function which take into account the soil.

In this formulation, the function \( G(x, z) \) is known and \((k_0^2 - k^2)E(x)\) are the unknowns. We can note that these values are only not equal to zero inside the dielectric objects. So, let us assume that we want to find a distribution of dielectric objects in a given volume. We split the considered volume into several cells where we assume to have a dielectric. The fields at each sensor \( k \) are given by

\[
E(z_k) = E_{inc}(z_k) + \sum_{i=1}^{N} G_i(z_k)u_i \text{ where } u_i = (k_0^2 - k_i^2) \int_{B_i} E_i(x) \, dx, \text{ and } G_i(z_k) = \int_{B_i} G(x,z_k) \, dx.
\]

If we have also some measurements \( E_{mes} \) on the sensors, the problem can be formulated as:

Find \( u = (u_i)_{i=1}^{N} \) so that:

\[
\min_u \left( \sum_{k=1}^{N_s} (E_{mes}(z_k) - E(z_k))^2 \right) = \min_u \| E_{mes} - E \|^2 \leftrightarrow \min_u \| E_{mes} - E_{inc} - G \| \tag{1}
\]

The solution (distribution of objects) is given by \( u = G^+(E_{mes} - E_{inc}) \).

The figure 1 shows an example studied to validate the method. The **Topological Gradient method**.

![Figure 1: configuration studied and solution obtained](image)

In this approach [3], the problem is written as the minimisation of a function \( J(E) = \| E_{mes} - E \|^2 \)
where \( E_{mes} \) and \( E \) are measured and computed values taken at the sensor locations. The unknown of the problem is a distribution of dielectric objects in the space.

Let \( D_0 \) be an initial distribution of objects. \( D_e \) defines the distribution where we add a dielectric ball \( B_e \) to \( D_0 \). We obtain two electric fields \( E_0 \) and \( E_e \) respectively for each distribution, and we can write:

\[
J(E_e) - J(E) = -\left( \frac{\partial J(E_0)}{\partial E} , (E_e - E_0) \right) \tag{2}
\]

In this expression, we see that the variation of the cost function \( J \) is given by \( \langle \frac{\partial J(E_0)}{\partial E} , (E_e - E_0) \rangle \). Let \( D_0 \)
be the empty distribution (no dielectric object in space), the field \( E_0 \) satisfies \( \nabla \times (\nabla \times E_0) - k_0^2 E_0 = 0 \leftrightarrow A_0 E_0 = 0 \). For the distribution \( D_e \), we obtain \( A_0 E_e = (k_e^2 - k_0^2)E_e \), where we have \( k_e = k_0 \) everywhere except in \( B_e \). Now, the minimisation problem \( \min \langle J(E) \rangle \) under the constraint \( AE = b \) can be rewritten as the minimisation of the Lagrangian \( L(E, \lambda) = J(E) + \langle \lambda, AE - b \rangle \). The adjoint value verifies \( -A^*\lambda = \frac{\partial J(E)}{\partial E} \). Then we obtain

\[
J(E_e) - J(E) = -\langle \lambda, (k_e^2 - k_0^2)E_e \rangle \tag{3}
\]

At a given point \( z \) of the space, we have also \( E_e(z) - E_0(z) = \int_{B_e} G(x,z)(k_e^2 - k_0^2)E_e(x) \). Then at each position \( x \) of the ball \( B_e \) we obtain:

\[
E_e(x) = \frac{E_0(x)}{1 - \frac{k_0^2}{k_e^2} g_e} \text{ with } g_e = \int_{B_e} G(x,e) \, dx
\]

For the adjoint value, we have:

\[
\lambda(x) = \sum_{i=1}^{N} G(x_i, x) \frac{\partial J(E_0(x))}{\partial E}
\]

with \( N \) equal to the number of sensors.
So, finally, we obtain at each position in the domain where we search a dielectric object, a value depending of $E_0(x_a)$ and $\frac{\partial J(E)}{\partial x}$ taken at the sensor locations, which gives the variation of the cost function when we add a dielectric ball at this position. Generally, in a first step we obtain with this process a good and fast localisation of the objects. The next steps of the process consist to add dielectric balls at the distribution $D_k$. The problem is more difficult and the value which describes the variations of $J(E)$ is not obvious to obtain. The figure 2 shows a 2D example of localisation obtained with this method.

![Image](image.png)

Figure 2: Real configuration and optimised configuration obtained with the topological gradient

**CONCLUSION**

In this paper, we have shown different methods to detect buried objects in a given medium. Each method presents the same advantage to be fast. The most famous are radar imaging methods (GPR, SAR) where the fields are approximated like plane waves. This approach involves a process which is reduced to an inverse Fourier transform. In this case, the interactions between the objects are generally neglected. Now, it is also possible to take into account exactly the Maxwell-equations in the optimisation process and then to improve the model of the field. In this paper we presented two methods where this is the case and where the non linear cost function to be minimized doesn’t increase dramatically the CPU time required to solve the problem. The first method reduces the non linear cost function to a linear least square problem equivalent to a Fourier transform. However, we need to assume an homogeneous soil and a flat air/soil interface to obtain easily the Green’s function. For the second method, the unknown is the distribution of the objects and a variation of the cost function at a step is given by the product of the fields and the adjoint of the fields. We don’t need derivatives. Generally, we obtain in this method a good estimate of the solution at the first step where the fields can be obtained analytically. This fact improves the efficiency of the method. The examples presented in this paper show on simple cases the validity of the methods. Future work will focus on the comparison with the GPR and SAR methods in order to quantify the gain brought by a better accurate physical model.

**References**


