ACCURACY ANALYSIS AND OPTIMIZATION OF THE METHOD OF AUXILIARY SOURCES (MAS) FOR OBLIQUE INCIDENCE SCATTERING BY A PERFECTLY CONDUCTING, INFINITE CIRCULAR CYLINDER

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Abstract: This paper is presenting a rigorous accuracy analysis of the Method of Auxiliary Sources (MAS) for the problem of oblique incidence, transverse magnetic (TMz) plane wave scattering by a perfectly conducting, infinite circular cylinder. The MAS matrix is inverted analytically, via eigenvalue analysis, yielding exact expressions for the discretization error. It is demonstrated that the error increases very abruptly for source locations associated with the characteristic eigenvalues of the scattering problem. The optimal location of the auxiliary sources is determined on the grounds of error minimization.

Introduction.
The Method of Auxiliary Sources generally serves as a promising alternative to standard integral equation techniques, such as the Moment Method (MoM). Its primary merit is algorithmic simplicity, which allows relatively low computational cost [1]. Nevertheless, its advantages are obscured by the ambiguity associated with the location of the auxiliary sources (AS’s). It has been observed that inadvertent AS’s positioning often leads to slow convergence rates or unacceptably high boundary condition (BC) errors.

A rigorous assessment of the MAS accuracy for normally incident, plane wave scattering from a perfectly conducting (PEC) infinite circular cylinder was carried out in [2]. It was proven that for this particular situation, the MAS matrix can be inverted analytically and the discretization error can be explicitly evaluated. Comparisons showed that the analytically and numerically evaluated errors were identical for a wide range of the AS’s location, whereas locally high BC errors resulted into characteristic “peaks”.

In order that MAS be implemented in a broad spectrum of electromagnetic scattering problems, it needs to be initially optimized for the most generic scatterers possible. The purpose of this work is the MAS error estimation for the most general case of plane wave incidence on a PEC, infinite circular cylinder. The method invoked requires a different set of AS’s than the one in [2] and full vector wave analysis must be implemented for the calculation of the radiated fields. On the other hand, the resulting MAS matrix has a circulant form and a similar technique to [2,3] can be applied towards its analytical inversion.

Analytical Considerations.
Assume a PEC infinite, circular cylinder of radius $a$, illuminated by a transverse magnetic (TMz) plane wave impinging from an elevation angle $\theta$ (see Fig.1). According to the MAS fundamental concept, a set of AS’s is located inside the scatterer, residing on a fictitious auxiliary surface, conformal to the actual boundary. Thus the auxiliary surface is assumed to be a cylinder of radius $a$, hosting a number of $N$ AS’s (see Fig.2).

The $n$-th AS ($1 \leq n \leq N$) is assumed to possess a current distribution of the form:

$$I_n(\rho', \phi', z') = (A_n \hat{\phi} + B_n \hat{z}) \frac{\rho(\theta'-\phi_n)\rho(\rho'-\rho_n)}{\rho'} e^{-\beta z'}$$  (1)
where $A$, $B$ are the unknown current coefficients, which will be determined by the solution of the MAS linear system, $k_0$ is the free space’s wavenumber, $\phi_0 = k_0 \cos \theta$ and $\phi_n = n \cdot 2\pi/N$ is the azimuth angle of the $n$-th source. The response of this source type is computed using the integral operator of the free space dyadic Green’s function [4]. Satisfying the boundary conditions (continuity of the $\phi$ and $\theta$ components of the total electric field) at $N$ collocation points (CP’s) on the $z$-plane projection of the scatterer’s surface, yields the MAS square linear system. This linear system can be written in compact, block matrix form as:

$$
\begin{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
E_\phi \\
E_z
\end{bmatrix}
$$

(2)

where $[U], [V], [W], [Y]$ are $N \times N$ square matrices with elements given by:

$$
U_{mn} = -\frac{j}{k_0} \sum_{j=-\infty}^{\infty} \left\{ J_1(k_0 h_z)(1)^2(k_1, h_z) + k_0^2 k_x^2 \right\} \exp(j(\phi_m - \phi_h)) \exp(-jk_z) (3)
$$

$$
V_{mn} = -\frac{j}{k_0} \sum_{j=-\infty}^{\infty} \left\{ k_0^2 J_1(k_0 h_z)(1)^2(k_1, h_z) \exp(j(\phi_m - \phi_h)) \right\} \exp(-jk_z) (4)
$$

$$
W_{mn} = -\frac{j}{k_0} \sum_{j=-\infty}^{\infty} \left\{ k_0^2 J_1(k_0 h_z)(1)^2(k_1, h_z) \exp(j(\phi_m - \phi_h)) \right\} \exp(-jk_z) (5)
$$

$$
Y_{mn} = -\frac{j}{k_0} \sum_{j=-\infty}^{\infty} \left\{ k_0^2 J_1(k_0 h_z)(1)^2(k_1, h_z) \exp(j(\phi_m - \phi_h)) \right\} \exp(-jk_z) (6)
$$

where the dot denotes differentiation with respect to the entire argument, $k_0 = k_0 \sin \theta$. $A, B$ are $N \times 1$ column vectors containing the unknown AS weights, $E_\phi, E_z$ are $N \times 1$ column vectors, consisting of the incident field’s $\phi$ and $z$ components respectively, sampled at the $N$ CP’s.

To derive an expression for the boundary condition error, (2) must be inverted analytically. Such an inversion is feasible because each block of the MAS matrix has a circulant form, and therefore a technique similar to [2] can be invoked for the diagonalization of each block:

$$
[U] = [G] [D_\phi] [G]^{-1}, [V] = [G] [D_\theta] [G]^{-1}, [W] = [G] [D_z] [G]^{-1}, [Y] = [G] [D_\phi] [G]^{-1}
$$

(7)

where $[G]$ is the eigenvector square matrix, $[D_\phi] [D_\theta] [D_z]$ are diagonal matrices containing the eigenvalues of $[U], [V], [W], [Y]$ respectively, given by ($q=1,\ldots,N$):

$$
u_q = \frac{jN}{4} \sum_{j=-\infty}^{\infty} \left\{ J_{q+\sinh(k_0 h_z)}(1)^2(k_1, h_z) + \frac{(q+N)^2 k_0^2 k_x^2}{k_0^2 k_x^2} J_{q+\sinh(k_0 h_z)}(1)^2(k_1, h_z) \right\} \exp(-jk_z) (8)
$$

$$
u_q = \frac{jN}{4} \sum_{j=-\infty}^{\infty} \left\{ \frac{(q+N)^2 k_0^2 k_x^2}{k_0^2 k_x^2} J_{q+\sinh(k_0 h_z)}(1)^2(k_1, h_z) \right\} \exp(-jk_z) (9)
$$

$$
u_q = \frac{jN}{4} \sum_{j=-\infty}^{\infty} \left\{ \frac{(q+N)^2 k_0^2 k_x^2}{k_0^2 k_x^2} J_{q+\sinh(k_0 h_z)}(1)^2(k_1, h_z) \right\} \exp(-jk_z) (10)
$$

$$
u_q = \frac{jN}{4} \sum_{j=-\infty}^{\infty} \left\{ \frac{(q+N)^2 k_0^2 k_x^2}{k_0^2 k_x^2} J_{q+\sinh(k_0 h_z)}(1)^2(k_1, h_z) \right\} \exp(-jk_z) (11)
$$

The final result for the MP’s normalized error can be written in a manner analogous to [3] as:

$$
e(a, b, N) = \sqrt{\frac{\sum_{m=1}^{N} |e_m|^2 + \sum_{m=1}^{N} |e_m|^2}{N(1 + \cot \theta)}}
$$

(12)

where

$$
e_m = \frac{1}{N} \sum_{n=1}^{N} \sum_{p=1}^{N} \exp\{j(m-n)\phi_p\} (l_p^{(1)} E_\phi + l_p^{(2)} E_z) - \tilde{E}_\phi
$$

(13)

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\[
e_{m2} = \frac{1}{N} \sum_{n=1}^{N} \sum_{p=1}^{N} \exp\left(i(m - n)\varphi_p\right) \left(E_p^{(1)} + E_p^{(2)} - \tilde{E}_z^{(2)}\right) \tag{14}
\]

whereas \(\varphi_p = \phi + 2\pi/N\) and

\[
\begin{align*}
I^{(1)}_{q} &= \frac{\tilde{u}_q y_q - \tilde{v}_q w_q}{u_q y_q - v_q w_q}, \\
I^{(2)}_{q} &= \frac{\tilde{u}_q y_q - \tilde{v}_q w_q}{u_q y_q - v_q w_q}, \\
I^{(1)}_{q} &= \frac{\tilde{u}_q u_q - \tilde{v}_q v_q}{u_q y_q - v_q w_q}, \\
I^{(2)}_{q} &= \frac{\tilde{u}_q u_q - \tilde{v}_q v_q}{u_q y_q - v_q w_q}
\end{align*} \tag{15}
\]

the symbol \(\sim\) above each variable denotes the corresponding quantity considered at the MP’s. To achieve the highest possible accuracy for the MAS solution, \(c\) in (12), must be minimized by choosing the most appropriate \(a\) for given \(b\), and \(N\).

**Numerical Results**

To validate the expressions derived above, several cases were examined for various parameters. The analytical error was calculated and plotted by use of (12)-(14). The computational error was calculated by a LU decomposition and numerical inversion of the MAS matrix. Fig.3 and 4 show the comparison between the analytical and the computational error for various incident angles and AS’s numbers. Peaks correspond to resonances, determined by vanishing denominators in (15), and are related to the roots of Bessel functions and their derivatives, as well as to the incidence angle \(\theta\). Frantic fluctuation of the numerical error for very small \(a^2\’s\) is due to a high condition number of the system [2,3].

**REFERENCES**


