INVERSE SCATTERING OF AN IMPERFECT CONDUCTING CYLINDER BURIED IN THREE-LAYER STRUCTURE

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Abstract

The imaging of an imperfectly conducting cylinder buried in a three-layer structure by the genetic algorithm is investigated. An imperfectly conducting cylinder of unknown shape and conductivity buried in the second layer scatters the incident wave from the first layer or the third layer. We measure the scattered field in the first and third layers. Based on the boundary condition and the recorded scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. The genetic algorithm is then employed to find out the global extreme solution of the cost function. Numerical results demonstrated that, even when the initial guess is far away from the exact one, good reconstruction can be obtained. In such a case, the gradient-based methods often get trapped in a local extreme. In addition, the effect of uniform noise on the reconstruction is investigated.

I. Introduction

The inverse scattering techniques for imaging the shape of imperfectly conducting objects have attracted considerable attention in recent years. They can apply in noninvasive measurement, medical imaging, and biological application. In the past 20 years, many rigorous methods have been developed to solve the exact equation. However, inverse problems of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient searching schemes such as the Newton-Kantorovitch method [1]. These methods are highly dependent on the initial guess and tend to get trapped in a local extreme. In contrast, the second approach is based on the evolutionary searching schemes [2]. They tend to converge to the global extreme of the problem. Owing to the difficulties in computing the Green’s function by numerical method, the problem of inverse scattering in a three-layer structure has seldom been attacked. In our knowledge, there are still no numerical results by the genetic algorithm for imperfectly conducting scatterers buried in a three-layer structure.

II. Theoretical Formulation

Let us consider a two-dimensional three-layer structure as shown in Fig. 1, where \((\varepsilon_i, \sigma_i)\) \(i = 1, 2, 3\), denote the permittivities and conductivities in each layer and an imperfectly
conducting cylinder is buried in second layer. The metallic cylinder with cross section described by the equation \( \rho = F(\theta) \) is illuminated by an incident plane wave whose electric field vector is parallel to the \( Z \) axis (i.e., TM polarization).

At an arbitrary point \((x, y)\) or \((r, \theta)\) in polar coordinates in regions 1 and 3 the scattered field, can be expressed as

\[
E_s(\bar{r}) = -\int_0^{2\pi} G(\bar{r}, F(\theta'), \theta') J(\theta') d\theta'
\tag{1}
\]

where \( G \) that denotes the Green’s function which can be obtained by tedious mathematic manipulation for the line source in region 2. \( J(\theta) \) is proportional to the induced surface current density which is proportional to the normal derivative of the electric field on the conductor surface. The boundary condition at the surface of the scatterer yields an integral equation for \( J(\theta) \)

\[
E_s(\bar{r}) = \int_0^{2\pi} G(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' + j \sqrt{\omega \mu \sigma} \int_0^{2\pi} \frac{J(\theta)}{\sqrt{F^2(\theta) + F^{'2}(\theta)}} d\theta'
\tag{2}
\]

For the direct scattering problem, the scattered field \( E_s \) is calculated by assuming that the shape and conductivity are known. This can be achieved by first solving \( J \) in (2) and then calculating \( E_s \) using (1). For the inverse problem, assume the approximate center of scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function \( F(\theta) \) can be expanded as:

\[
F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta)
\tag{3}
\]

where \( B_n \) and \( C_n \) are real coefficients to be determined, and \( N+1 \) is the number of unknowns for the shape function. In the inversion procedure, the steady state genetic algorithm is used to minimize the following cost function:

\[
CF = \left( \frac{1}{M_t} \sum_{m=1}^{M_t} \left| E^\text{exp}(\bar{r}_m) - E^\text{cal}(\bar{r}_m) \right|^2 + \alpha \left| F'(\theta) \right|^2 \right)^{1/2}
\tag{4}
\]

where \( M_t \) is the total number of measurement points. \( E^\text{exp}(\bar{r}) \) and \( E^\text{cal}(\bar{r}) \) are the measured and calculated scattered fields, respectively. The factor \( \alpha \left| F'(\theta) \right|^2 \) can be interpreted as the smoothness requirement for the boundary \( F(\theta) \).

III. Numerical Results

In the example, the shape function is chosen to be \( F(\theta) = (0.03) \) m. The chosen conductivity is 100 S/m. The reconstructed shape function for the best population member is plotted in Fig. 2 with the shape and the conductivity error shown in Fig. 3 Relative error of shape and conductivity as a function of noise in Fig. 4.
IV. Conclusions

We have reported a study of applying the genetic algorithm to reconstruct the shapes and the conductivity of an embedded conducting cylinder. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization one. The genetic algorithm is then employed to de-embed the microwave image of metallic cylinder. Numerical results illustrate that the conductivity is more sensitive to noise than the shape function is. Numerical results also show that good shape reconstruction can be achieved as long as the normalized noise level is \( < 10^{-3} \). But the good conductivity reconstruction can be achieved only the normalized noise level is \( 10^{-5} \).

References
