APPLICATION OF VOLUMETRIC AND SURFACE DEFECT MODELS FOR THE ANALYSIS OF EDDY CURRENT NONDESTRUCTIVE TESTING PROBLEMS

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Abstract: Defects in eddy current testing measurements can be modeled basically in two different ways. These are the volumetric and surface defect models. In the paper these models are compared theoretically and numerically. By calculating the errors of the surface model compared with the volumetric one for various defects having different thicknesses, shapes and conductivities, one is attempting to show how to select the most appropriate model for a given type of defects.

INTRODUCTION

The modeling of the interaction of eddy current testing (ECT) probes and defects in tested specimens is a key issue in the design and application of ECT systems. Also, this modeling plays an important role when solving the related inverse problem, i.e., characterizing defects from signals collected by an ECT probe. These applications require a fast and robust analysis enabling accurate prediction of the defect-probe interaction.

A defect, i.e., a given volume region of the inspected specimen, can be detected by ECT if the electric conductivity or magnetic permeability of this region differs from the material constants of the otherwise homogeneous specimen. In most real life situations not too much information is available on the effective boundaries and material parameters of a defect. Thus, in many cases the numerical model to put together is not a straightforward matter. In this paper we compare the results of defect models in order to reach some insight into the question of how to select a numerical model for a given defect type.

Evidently, the best defect model should be the one for which one could freely select its conductivity and permeability. This model is the so-called volumetric defect model. Since most defects are concentrated within a small volume region, proper numerical evaluation of the volumetric model requires very dense discretization of this region. This is especially true if not just the volume of the defect is small but the difference between the material parameters of the specimen and those of the defect is very large at the same time. A typical example would be a very thin, crack-like void, for which the application of the so-called surface defect model might be more suitable instead. The goal of this paper is to investigate the extent of the validity of the surface defect model compared to the volumetric model.

In the following, the focus being on non-ferromagnetic specimens and defects, the volumetric and surface defect models are shortly reviewed and numerical aspects and computational costs of the models are highlighted. As an illustration, results of the two models are compared with measurements in a case of interest. Using the calculation codes applied for this example, in the presentation, the errors of the surface model, depending on the defect thickness, shape and conductivity, will be thoroughly investigated.

Fig. 1 Eddy current probe above a plate specimen containing a defect
VOLUMETRIC DEFECT MODEL

A typical eddy current testing arrangement is shown in figure 1. One assumes that the electric field varies in time, \( t \), as the real part of \( \exp(j \omega t) \) where \( \omega \) is the angular frequency of the excitation. The conductivity of the specimen, \( \sigma(r) \), is equal to the conductivity of the defect free specimen, \( \sigma_0 \), outside the volume of the defect, \( V_d \), the relative permeability of both the specimen and the defect being both assumed to be equal to one. The field perturbation caused by the defect can be represented via a secondary source, the current dipole density function, \( P(r) \), which is supported by the volume of the defect, \( V_d \) [1]. This unknown dipole density function can be obtained by solving the vector integral equation

\[
E(r) = E'(r) - j \omega \mu_0 \int_{V_d} G(r \mid r') \cdot P(r') dr'.
\]

where \( E \) is the total electric field and \( E' \) is the incident electric field (i.e., the electric field generated by the exciting coil in the defect-free specimen) [1]. \( G(r \mid r') \) is the dyadic Green's function which transforms the current excitation into the electric field. \( \mu_0 \) is the permeability of vacuum. The dipole density function is defined as

\[
P(r) = [\sigma(r) - \sigma_0] E(r).
\]

From the solution of (1), the impedance change, \( \Delta Z \), of the ECT probe due to the presence of the defect can be calculated as

\[
I^2 \Delta Z = - \int_{V_d} E'(r) \cdot P(r) dr.
\]

where \( I \) is the driving current of the exciting coil [1].

The integral equation (1) is solved numerically by the discretization of the unknown function, \( P \), in the defect volume. The calculations are performed by a computer code described in [2]. This method is henceforth referred to as the volume integral method (VIM).

SURFACE DEFECT MODEL

Let us now assume that the defect is a void (zero conductivity) and that its volume shrinks onto the planar surface \( x = 0 \), with area \( S_d \) on this surface. When the thickness of the defect becomes very small it may be treated as a mathematical surface. The magnetic and electric field must satisfy the following boundary conditions on it [3]: the magnetic field should be continuous whereas the normal component of the electric field, \( E_n \), is cancelled on \( S_d \). These boundary conditions can be satisfied if an \( x \)-directed current dipole layer, \( p \), is placed as secondary source on \( S_d \). This unknown dipole density can be calculated from the following scalar integral equation [3],

\[
0 = E'_x(r_0) - \lim_{r \to r_0} j \omega \mu_0 \int_{S_d} G^{xx}(r \mid r') p(r') dr', \quad r_0 \in S_d,
\]

where \( G^{xx}(r \mid r') \) is the component of the Green's dyad that is transforming the \( x \)-directed current excitation into the \( x \)-directed electric field. Once obtained the solution of this integral equation, the impedance change of the excitation is obtained as before as

\[
I^2 \Delta Z = - \int_{S_d} E'_x(r) p(r) dr.
\]

The integral equation (3) is solved by discretizing the function, \( p \), with first-order triangular elements on a regular grid that is dividing \( S_d \). The computer code used for the solution of (3) is detailed in [4]. This method is henceforth referred to as the boundary integral method (BIM).

COMPARISON OF THE CRACK MODELS

The first difference between the two models is their ability to deal with various kinds of defects. In principle, the VIM is able to handle an arbitrary conductivity pattern when properly discretized. However, one should remember that the VIM is a vector integral equation while the BIM is a scalar integral equation, and as a matter
of fact 3D and 2D objects are to be discretized, respectively. So, the computational burden of BIM is expected to be considerably smaller than the one of VIM. At the same time, when the thickness of the defect becomes smaller and smaller, numerical difficulties may result in the VIM as leading to a worse representation of the defect than the BIM.

In short, the two methods are not competing methods but complementary ones. So, it is of interest to know when to apply one or the other. By studying the results yielded by the VIM and the BIM methods on various examples, a set of guidelines can be proposed. In particular, to that aim, one should compute the error of the BIM compared to the VIM when the conductivity of a reference thin crack is varied from zero to the conductivity of the host material. Also, the error of the BIM depending upon the thickness of the defect should be an useful indicator. Finally, large volumetric defects could be modeled with groups of surface cracks. These several items will be considered in detail in the presentation, the example below being given as an illustration of the applicability of the two methods.

**NUMERICAL EXAMPLE**

The schematic drawing of the configuration under study is shown in figure 1. The inner and outer radiiuses of the exciting coil are 1 mm and 1.62 mm, respectively. The height, lift-off and number of turns of the coil are 2 mm, 0.3 mm and 328, respectively. The coil is fed by a sinusoidal current at 500 kHz frequency. The conductivity, relative permeability and thickness of the investigated plate are 1 MS/m, 1 and 1.55 mm, respectively. The length, depth and thickness of the crack positioned centrally in the x=0 plane are 4 mm, 0.61 mm and 0.12 mm. The crack is ID, i.e., it opens to the side of the plate where the exciting coil is located.

In figure 2 the real and the imaginary parts of the impedance changes of the coil are depicted when it scans along the x and y axes. The results of the VIM and BIM are compared with experiments. One can see that both methods have almost the same, acceptable error compared to measurements.

![Impedance variation](image)

*Fig. 2 Impedance variation of the probe versus the location of the middle of the probe. The probe scans along the a) x and b) y axes.*

**REFERENCES**


