INVERSE PROBLEM FOR DETERMINATION OF MICROWAVE PARAMETERS OF ISOTROPIC MEDIUMS BY USING AN OPEN SPHERICAL RESONATOR

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Abstract: The technique for determination of dielectric parameters of substances is submitted on a base of using a high-Q "whispering gallery" oscillations in a quasi-optical spherical resonator. The function ability of a method is shown on an example of measurement of permittivity for several substances in the 8 mm wave band.

Now the spherical dielectric resonators are widely used at studying physical phenomena arising in different science fields and engineering, at creation of stable microwave standards in precision measuring devices. High Q-factor and narrow field of energy localization of "whispering gallery" oscillations allow widely using them at creation of devices of millimeter and submillimeter wave bands and at research of physical mechanisms of the anomalously-small dissipation of energy in dielectric crystals. The studying of dielectric parameters of the medium, for example liquids, can be carried out by measurement of the spectral and energy characteristics of a spherical resonator placed in this medium. The microwave properties of a resonator material can also be determined by measurement of its spectral characteristics.

EIGEN OSCILLATIONS OF AN ISOTROPIC SPHERICAL RESONATOR

In the macroscopic electrodynamics the electromagnetic field in any medium and at any time is determined by four values: by vectors of strength \( \mathbf{E} \) and induction \( \mathbf{D} \) of an electrical field and vectors of strength \( \mathbf{H} \) and induction \( \mathbf{B} \) of a magnetic field. The vectors of an electromagnetic field depend on spherical coordinates \( r, \theta, \varphi \) and time \( t \). They are coupling among themselves by the set of Maxwell equations. The set of Maxwell equations is added by the system of constitutive equations which have values of permittivity \( \varepsilon \) and permeability \( \mu \). They image of medium influence on took place electromagnetic phenomena.

The spectral characteristics of "whispering gallery" oscillations essentially depend on environmental parameters. Let us consider the sphere manufactured from the dielectric with permittivity \( \varepsilon_1 \) and permeability \( \mu_1 \) and placed in the medium with \( \varepsilon_2 \) and \( \mu_2 \).

The independent TE \(_p\) and TM \(_p\) modes of resonance oscillations exist in the isotropic resonators. Each \( p \) mode of such oscillations is characterized by a triple index \( nms \), where \( m \leq n \). The polar index \( n \) determines number of field variations on polar coordinate \( \theta \) for azimuthally homogeneous oscillations \((m=0)\). In turn the azimuthal index \( m \) characterizes number of field variations for azimuthal oscillations on \( \varphi \), which exist at \( m=n \). The radial index \( s \) determines number of field variations on radial coordinate \( r \).

In spherical coordinate system \( (r, \theta, \varphi) \), in such structure the components of the field of TE\(_p\) \((E_\theta = 0)\) and TM\(_p\) \((H_\phi = 0)\) oscillations are determined by following expressions:

For TE oscillations:

\[
E_{\theta p}^{i} = 0; \quad rE_{\phi p}^{i} = \frac{1}{r^{2}} \sin \theta \frac{\partial V_{p}^{i}}{\partial \phi}; \quad rE_{\phi p}^{i} = -\frac{1}{r^{2}} \sin \theta \frac{\partial V_{p}^{i}}{\partial \theta},
\]

\[
H_{\phi p}^{i} = \frac{n(n+1)}{r^{2}} V_{p}^{i}; \quad rH_{\theta p}^{i} = \frac{\partial^{2} V_{p}^{i}}{\partial \theta^{2}}; \quad rH_{\theta p}^{i} = \frac{1}{r^{2}} \sin \theta \frac{\partial^{2} V_{p}^{i}}{\partial \varphi^{2}},
\]

And for TM oscillations:

\[
H_{\phi p}^{i} = 0; \quad rH_{\theta p}^{i} = -ik^{l} \varepsilon_{1} \frac{1}{\sin \theta} \frac{\partial U_{p}^{i}}{\partial \varphi}; \quad rH_{\theta p}^{i} = \frac{ik^{l} \varepsilon_{1}}{\sin \theta} \frac{\partial U_{p}^{i}}{\partial \theta},
\]

\[
E_{\phi p}^{i} = \frac{n(n+1)}{r^{2}} U_{p}^{i}; \quad rE_{\theta p}^{i} = \frac{\partial^{2} U_{p}^{i}}{\partial \theta^{2}}; \quad rE_{\phi p}^{i} = \frac{1}{r^{2}} \sin \theta \frac{\partial^{2} U_{p}^{i}}{\partial \varphi^{2}},
\]

Here \( k^{l} = \omega^{l}_{p} / c \), \( \omega^{l}_{p} = \omega^{l}_{p} - i \omega^{r}_{p} \) is the resonance frequency of the \( p \)-th mode of \( j \)-oscillation; \( l = 1 \) at \( r < r_{0} \) and \( l = 2 \) at \( r > r_{0} \), \( r_{0} \) is the radius of sphere. Auxiliary functions \( U_{p}^{i} \) and \( V_{p}^{i} \) is determined by the follow expressions.
\[
U_p = A_1 P_n^m (\cos \theta) e^{(m \phi - \alpha_p r)} \times \begin{bmatrix}
    j_0(x_p r / r_0), & r \leq r_0 \\
    \frac{\varepsilon_1}{\varepsilon_2} h_0(x_p r / r_0), & r \geq r_0
\end{bmatrix},
\]
\[
V_p = A_2 P_n^m (\cos \theta) e^{(m \phi - \alpha_p r)} \times \begin{bmatrix}
    j_0(x_p r / r_0), & r \leq r_0 \\
    \frac{\mu_1}{\mu_2} h_0(x_p r / r_0), & r \geq r_0
\end{bmatrix},
\]
where \( x_p = \sqrt{\varepsilon \mu_k / r_0} \); \( A_i \) are constants determined from the excitation condition; \( j_0(x) = \sqrt{\pi / 2} J_{n+1/2}(x) \), \( h_0(x) = \sqrt{\pi / 2} H_{n+1/2}^{(1)}(x) \). The function \( J_{n+1/2}(x) \) is cylindrical function, which is used at the description of fields inside the sphere \( (r \leq r_0) \), and \( H_{n+1/2}^{(1)}(x) \) is the Hankel function of the first kind, which is used at the description of fields outside the sphere \( (r \geq r_0) \). For \( p \)-th mode the eigen frequency of isotropic dielectric sphere placed in the dispersive medium is determined by the solution of the expression
\[
\alpha_p \frac{J_0(x_p r)}{J_1(x_p r)} = \frac{h_1(x_p r)}{h_0(x_p r)},
\]
where \( \alpha_p = \sqrt{\varepsilon \varepsilon_2 / \varepsilon_1 \mu_2} \) for the TM mode, and \( \alpha_p = 1 / \alpha_p \) for the TE mode. The prime denotes the derivative with respect to the argument.

The Q-factor is determined by the expression \( Q = \alpha_p / 2 \omega_p \) at the resonance frequency \( \omega_p \).

**DETERMINATION OF MEDIUM PERMITTIVITY AND PERMEABILITY**

This theory can be used for determination of the microwave characteristics of materials. The medium permittivity of both isotropic sphere and its environment is complex value \( \varepsilon = \varepsilon_i + i \varepsilon_i' \). For determination its values need experimentally to be determined appropriate values of the resonance frequency \( \omega_p \) and polar index \( n \) of \( j \)-oscillation. The solution of the dispersion equation (1) uniquely determines the characteristics of substance \( \varepsilon_i \) and loss tangent \( \tan \delta_i = \varepsilon_i' / \varepsilon_i \) for the concrete medium \( l \) at known appropriate values of the contact medium and values of \( \mu_l \) at \( l = 1, 2 \). The values of medium permeabilities \( \mu_l \) can be similarly retrieved.

The computing program for research of an isotropic spherical resonator and determination of microwave properties of materials was designed on the basis of this approach.

The program allows:
- to determine the permittivity (or permeability) of the resonator material on experimental data about the resonance frequency and Q-factor of the identified mode of the spherical resonator placed in the environment with known parameters;
- to determine permittivity (or permeability) of the environment, in which the resonator is placed, on experimental data about the resonance frequency and Q-factor of the identified mode of the spherical resonator with known parameters.

**RESULTS OF RESEARCHES**

The approbation of the designed technique was carried out in the electromagnetic band of eight-millimeter wavelengths on the computer-controlled measuring test bench (Fig. 1). Accordingly of the theory the experimental research of the spectral and energy characteristics was made for the isotropic dielectric resonator made from fused quartz as the sphere with radius of \( r_0 = 13,7 \) mm. The "whispering gallery" modes of resonator were identified and their resonance frequencies and Q-factors were measured. The test bench consisted of microwave module and three-coordinate scanner. The dielectric resonator was placed on the regulated platform between units of excitation (Fig. 1). The trial probe or receive antenna were placed on the moving subject table. The control of moving units of test bench, and the registration and the data interpretation were implemented by the control microprocessor complex of steppers and by the eight-channel system of collecting and processing of continuous data.
Spectra of eigen oscillations of the spherical dielectric resonator as the curve resonant transmission factor was registered by the modification of generator frequency of the eight-millimeter band under the linear law. The field distributions of resonance oscillations on the resonator surface were registered as amplitude change of the resonant transmission factor at circle scanning by the small-perturbed probe near to the resonator surface. The identification of resonance oscillations was implemented by the analysis of the obtained curves. The example of the normalized spectrum for $\text{TE}_{nm1}$ modes of the quartz spherical resonator is shown in Fig. 2. The oscillations having larger Q-factor correspond to the "whispering gallery" modes with the radial index of $s = 1$.

On experimental data the permittivity $\varepsilon_1' = 3.6 \pm 0.03$ and loss tangent $\tan \delta_1 = 1.2 \times 10^{-4}$ were calculated for the researched resonator placed in air. The similar calculation was made for the resonator submerged in the transformer oil. The determined value of the oil permittivity $\varepsilon_2 = (2.2 \pm 0.1) + (0.01 \pm 0.003)i$ not bad corresponds to known tabular data.

The spectral and energy characteristics for studied resonator within of polar indexes $n = 11 \div 20$ are shown in Fig. 3 and Fig. 4. The frequent dependences of slow-down factor $\xi = n/k'_p r_0 - 1$ are obtained using the solution of the equation (1). The difference between calculated and measured spectral characteristics of resonator for the $\text{TE}_{nm1}$ mode was no more than 0.3%. This difference is possibly conditioned by infringement of the resonator symmetry.

CONCLUSIONS
The technique of contactless determination of the microwave parameters of materials under testing is written in this paper. As an example, the permittivity of fused quartz and transformer oil were determined on the basis of the measured spectral and energy characteristics of the spherical isotropic resonator.