RIGOROUS ANALYSIS OF TWO-DIMENSIONAL PHOTONIC CRYSTAL WAVEGUIDES

K. Yasumoto, H. Jia, and S. Kai
Department of Computer Science and Electrical Engineering
Kyushu University, Fukuoka 812-8581, Japan

Abstract: A rigorous approach for modal analysis of two-dimensional photonic crystal waveguides consisting of layered arrays of circular cylinders is presented. The mode propagation constants and the mode field profiles can be accurately obtained by a simpler matrix calculus, using the one-dimensional lattice sums, the T-matrix of an isolated circular cylinder, and the generalized reflection matrices for a multilayered system.

INTRODUCTION
Photonic crystals are periodic dielectric structures in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. A photonic crystal waveguide is formed by introducing a defect layer in a photonic crystal material. The guided fields are strongly confined because any electromagnetic energy can not escape through the surrounding materials. Such waveguides have received growing attention in view of their promising applications to new integrated optical devices [1]. The mode propagation in two-dimensional photonic crystal waveguides consisting of a lattice of circular cylinders has been extensively studied, using the plane wave expansion method or the finite-difference time-domain (FDTD) method. Although these methods can be universally applied to various configurations of photonic crystal waveguides, they yield approximate solutions because the electromagnetic boundary conditions between the circular cylinders and the background medium are not considered. A more rigorous treatment that takes into account the boundary conditions is an important issue to have precise understanding the guided wave phenomena in photonic crystals.

In this paper, we shall present a rigorous approach to modal analysis of a two-dimensional waveguide bounded by photonic crystals consisting of layered periodic arrays of circular cylinders. The method is an extension of the lattice sums technique combined with the T-matrix approach which has been recently developed [2] to analyze the electromagnetic scattering from a periodic array of cylindrical objects. The dispersion equations for TE and TM guided modes are obtained in compact form in terms of the generalized reflection matrices for the layered periodic arrays. Numerical examples of the dispersion characteristics and field distributions are presented for the lowest TE modes in a coupled two-parallel photonic crystal waveguides with a square lattice of dielectric circular cylinders.

FORMULATION
The side view of a two-dimensional waveguide is shown in Fig. 1. The guiding region (−t < x < t) is bounded by two photonic crystals. The upper and lower regions consist of \( N_1 \)-layered and \( N_2 \)-layered arrays of circular cylinders, respectively, which are infinitely long in the \( y \) direction and periodically spaced with a common distance \( h \) in the \( z \) direction. The cylindrical elements should be same along each layer of the arrays but those in difference layers need not be necessarily identical in material properties and dimensions. The background medium is a homogeneous dielectric with permittivity \( \varepsilon_s \) and permeability \( \mu_0 \). The guided waves are assumed to be uniform in the \( y \) direction and vary in the form \( e^{i\beta z} \) in the \( z \) direction where \( \beta \) is a real propagation constant.

The scattering from each layer of the arrays is characterized by the reflection and transmission matrices for the space harmonics with the \( z \)-dependence as \( e^{i\phi_l} \) where \( \delta_l = \beta + 2\pi/l \) and \( l \) is integers. For an isolated array on the \( j \)-th layer in which the local origin of the array is located at \( x = x_j \) and \( z = z_j \), the reflection matrix \( R_j \) and the transmission matrix \( F_j \) are derived as follows [2]:

\[
R_j = W_j^T U T_j P W_j^T, \quad F_j = W_j^T (I + V T_j P) W_j^T
\]  

with

\[
U = [u_{lm}] = [\frac{2(-i)^m}{k_s h \sin \phi_l} e^{im\phi_l}], \quad V = [v_{lm}] = [\frac{2(-i)^m}{k_s h \sin \phi_l} e^{-im\phi_l}]
\]  

\[
T_j = (I - T_j L)^{-1} T_j, \quad W_j = [e^{\pm i k_s z_j} \delta_{lm}]
\]  

\[
P = [p_{mn}] = [i^m e^{im\phi_m}] \quad (l, m, n = 0, \pm 1, \pm 2, \cdots)
\]  

\[
\cos \phi_l = \frac{\beta_l}{k_s}, \quad \sin \phi_l = \frac{k_t}{k_s}, \quad \kappa_l = \sqrt{k_s^2 - \beta_l^2}
\]
where \( k_s = \sqrt{\varepsilon_s \mu_0} \) is the wavenumber of the background medium. In Eq. (3), \( T_j \) is the T-matrix of the circular cylinder in isolation which is obtained in closed form for TE and TM waves, \( I \) is the unit matrix, and \( L \) is a square matrix whose elements are given by \( L_{mn} = S_{m-n}(k_s h, \cos \phi_0) \) where \( S_{m-n}(k_s h, \cos \phi_0) \) is the \((m-n)\)-th order lattice sum [3]. The generalized reflection matrix for the layered arrays are obtained by concatenating \( R_j \) and \( F_j \) in the \( x \) direction. This calculation is efficiently performed using a recursion formula for two-layered arrays or using a concept of Floquet modes propagating in the \( x \) direction.

For the waveguide shown in Fig. 1, the upper and lower boundaries viewed from the guiding region are characterized by the generalized reflection matrices \( R_{N_1} \) at \( x = t - 0 \) and \( R_{N_2} \) at \( x = -t + 0 \). Using the method mentioned above, these matrices are calculated as functions of \( \beta \). Denoting the amplitude vectors of upgoing and downgoing space-harmonics by \( a^\pm(x) = [a_j^\pm(x)] \), respectively, the following relations are obtained:

\[
\begin{bmatrix}
I & -X(\beta)R_{N_1}^\dagger(\beta) \\
X(\beta)R_{N_2}^\dagger(\beta) & -I
\end{bmatrix}
\begin{bmatrix}
a^+(t-0) \\
a^-(-t+0)
\end{bmatrix} = 0
\]

(6)

where

\[ X(\beta) = [e^{i2 \kappa(\beta) t} \delta_{l\nu}] \]

(7)

Equation (6) has nontrivial solutions only for discrete values \( \beta_\nu \) of \( \beta \) which satisfies the equation

\[
\det[I - X(\beta_\nu)R_{N_1}^\dagger(\beta_\nu)X(\beta_\nu)R_{N_2}^\dagger(\beta_\nu)] = 0.
\]

(8)

The value of \( \beta_\nu \) gives the propagation constant of the \( \nu \)-th guided mode propagating along the \( z \) direction. The result is substituted into Eq. (6) to determine \( a_j^+(t-0) \) and \( a_j^-(t+0) \) for the \( \nu \)-th mode. The mode-amplitude vectors \( a_j^\pm(x_j \pm 0) \) just above and below the \( j \)-th array plane at \( x = x_j \) are successively calculated from \( a_j^+(t-0) \) and \( a_j^-(t+0) \) using the recursion formula for two layered arrays.

The mode-field distributions inside the homogeneous strip regions parallel to the \( z \) axis in the background medium are obtained by superposing the upgoing space-harmonics with \( a_j^+(x_j + 0) \) and the downgoing space-harmonics with \( a_j^-(x_j + 1 - 0) \). This space-harmonic expansion does not converge inside the grating regions, that is, in the inhomogeneous strip regions including the arrayed circular cylinders. The mode-fields in the grating regions are easily calculated by using the original expressions of scattered fields [2] in terms of cylindrical waves under the incidence of upgoing and downgoing space-harmonics with the amplitude vectors \( a_j^\pm(x_j \mp 0) \). Thus the mode-field distributions are obtained through the entire cross section along the \( x \) axis. The mode-field pattern varies within a unit cell along the \( z \) axis but the same pattern is repeated with the period \( h \).

**NUMERICAL EXAMPLES**

Although a substantial number of numerical examples could be generated, we shall discuss here only the results for the lowest TE modes in a coupled two-waveguides system. When two identical photonic crystal waveguides are brought in close proximity, they form a directional coupler which can be used for a variety of applications in integrated optics such as power division, switching, and wavelength or polarization selection. The characteristics of coupling are determined by the propagation constants and field distributions of two eigenmodes, an even mode and an odd mode, supported by the two-waveguides system. Their precise analysis is significantly important to design the coupler.
The schematic diagram of the coupled photonic crystal waveguides is shown in Fig. 2. Two parallel identical waveguides, obtained by removing one row from a square lattice of dielectric circular cylinders in free-space background, are coupled through two rows of the lattice. The dielectric constant and radius of the cylinders are chosen to be \( \varepsilon_r = 11.56 \) and \( a = 0.2h \), respectively, where \( h \) is the lattice constant. Testing the convergence of solutions, the numerical results were obtained by considering the lowest seven space harmonics and truncating the cylindrical wave expansion at \( m = \pm 10 \) to calculate the T-matrix of isolated circular cylinder.

Figure 3 shows the dispersion curves and field distributions of the even and odd modes in the coupled photonic crystal waveguides. The field distribution is plotted as functions of \( x \) on the plane \( z/h = 0.5 \) for \( h/\lambda_0 = 0.34 \). The same configuration has been analyzed using the FDTD method, and the numerical data of the propagation constants (\( \beta_{\text{even}} \) and \( \beta_{\text{odd}} \)) and the coupling length \( L_c \) have been reported as follows \([4]\): \( \beta_{\text{even}} = 2.34387 \times 10^6\text{m}^{-1}, \beta_{\text{odd}} = 1.9672 \times 10^6\text{m}^{-1}, \) and \( L_c = 2\pi/(\beta_{\text{even}} - \beta_{\text{odd}}) = 16.5\mu\text{m} = 10.63\lambda_0 \) for \( \lambda_0 = 1550\text{nm} \), \( h = 527\text{nm} \), and \( a = 106\text{nm} \).

We shall compare these data with the present results. Following the present analysis, we have \( \beta_{\text{even}} = 2.343387 \times 10^6\text{m}^{-1}, \beta_{\text{odd}} = 2.056928 \times 10^6\text{m}^{-1}, \) and \( L_c = 21.93\mu\text{m} = 14.15\lambda_0 \). It is noted that the coupling length \( L_c \) estimated by the FDTD analysis is about 25% shorter than that obtained by the present analysis. The result of this comparison is very instructive for understanding the degree of accuracy of the FDTD analysis.

**Fig. 3.** Dispersion curves and field distributions of even and odd TE modes in the two-parallel photonic crystal waveguides shown in Fig. 2.

**CONCLUSIONS**

We have presented an analytical approach to analyze the eigenmode fields supported by two-dimensional optical waveguides surrounding by photonic crystals consisting of a lattice of circular cylinders. The method is rigorous since the electromagnetic boundary conditions on the circular cylinders are fully satisfied and can be applied to photonic crystal waveguides with a large difference of refractive indices between the cylinders and the background medium. If the lattice element contains two or more circular cylinders in unit cell, the T-matrix in Eq. (3) is replaced by the aggregate T-matrix for the composite cylindrical system. This extension is straightforward.

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**REFERENCES**


