Numerically Efficient and Robust Iterative Algorithm for Microwave Imaging

Tobias Meyer, Andreas Jöstingmeier and Abbas S. Omar

University of Magdeburg, Chair of Microwave and Communication Eng., P.O. Box 4120, Magdeburg, Germany

Abstract

The issues that prevented a widespread practical application of iterative microwave inverse scattering techniques are insufficient robustness, numerical inefficiency and difficulties in collecting accurate measuring data with the proposed systems. All three aspects are addressed in this paper. The robustness of the proposed algorithm is achieved by a novel regularization scheme using a transformation of the object in the space harmonics domain. Using a hybrid Jacobian approximation limiting the number of direct problem evaluations increases the numerical efficiency and allows for using a FDTD solver. The measuring system is using microwave cavities instead of anechoic chambers, which reduces system complexity and enables highly accurate vector measurements. The proposed iterative method incorporating these novel approaches is capable of quantitatively imaging high dielectric contrast and lossy objects. Numerical examples and comparison with measurements show excellent accuracy.

1. Introduction

Since the first proposals for imaging systems using microwaves [1], microwave imaging has been considered having a high potential for medical applications and nondestructive testing. The reason for this is the strong interaction of many materials with microwaves. However, this high potential could not be exploited so far to build a usable imaging system due to several difficulties. Diffraction tomography [1] is based on the Born approximation and therefore limited to weak scattering and low-loss problems [2]. In experimental studies the limited accuracy of measurements is an additional source of image distortion [3]. Iterative schemes that have been presented often use approximations to quickly solve the direct problem. These computationally savings are on the expense of accepting limitations on the range of objects and accuracy. The inverse scattering technique suggested in this article avoids the above problems and limitations and is based on the following new approaches:

Based on the transformation of the object into the space harmonic domain it is possible to develop a new regularization scheme that does not need stabilizing functions and offers additional benefits over other regularization schemes. The fast progress of computing speed and advanced FDTD algorithms allow for the exact solution of the direct problem in an iterative scheme. Hence limitations due to the use of simplistic models for the direct problem solution can be avoided. Taking measurements using a multiport microwave cavity that is loaded with the object under test allows for quick highly accurate and fully automated vector measurements. Multi-frequency scattering data is fitted using a modified least-squares algorithm to further enhance the imaging accuracy and stability.

2. Regularization Using Object Transformation

The reconstruction of an unknown object by its scattered field is an ill-conditioned inverse problem. In order to solve it, a suitable regularization is applied. The corresponding direct problem is the creation of a set of measured field samples \( g \) contained in the image space \( Y \) by the object \( f \) in the object space \( X \). A nonlinear operator \( A \) does this mapping. It can be assumed that the direct problem

\[
Af = g
\]

can always be solved using numerical methods. In our case the direct problem is the computation of the scattering parameters for a given material distribution inside the cavity. The corresponding
inverse problem is to determine the object \( f \) from the image \( g \). Since \( A \) cannot be explicitly inverted we formulate the inverse problem in terms of
\[
\min_{f \in X} \| A f - g \|^2.
\] (2)
Reconstruction methods based on (2) do not reliably converge to the desired solution unless additional constraints are added to limit the object space \( X \) to a subdomain \( \Omega \subset X \) containing physical acceptable solutions. This constraint minimization problem cannot be treated efficiently. It can be converted to the form of an unconstrained one by the method of Lagrange multipliers.
\[
\min_{f \in X} \left\{ 4f - g + \mu h \right\}
\] (3)
where \( \mu \) is the regularization parameter and \( h \) is the stabilizing function ensuring convergence to a solution in \( \Omega \) by making outside solutions expensive. A more efficient regularization can be applied if the object can be transformed to the space harmonics domain first.
\[
\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} D_{nml} \cos(k_n x) \cos(k_m y) \cos(k_l z). \] (4)
The spatial frequencies are \( k_n = n \pi / a \), \( k_m = m \pi / b \) and \( k_l = l \pi / c \) where \( a, b \) and \( c \) are the dimensions of the space to image. The object is now represented by its set of expansion coefficients \( D_{nml} \). Many known methods use the constraint \( h = \| f \|^2 \) to require smoothness and solve equation (3). The new approach allows to formulate a smoothness constraint by limiting the object space in its dimension. This is because the number of space harmonics directly determines the smoothness of the object. We thus simply have
\[
\min_{s \in \Omega^N} \| A s - g \|^2_M,
\] (5)
where \( s \) is the spectral representation of the object, \( \Omega^N \) is the space of all objects which can be represented by a certain number \( N \) of harmonics and \( M \) is the number of data points to match. As all these objects are smooth this simple unconstraint optimization problem converges to the desired solution when the total number of space harmonics is set to a suitable value. This value is determined by the measuring bandwidth. Lower order space harmonics influence the scattering data at lower frequencies while higher order space harmonics have their frequency range of maximum effect at higher frequencies. Low frequency information is therefore necessary to calculate a good initial guess and achieve stable convergence to a good object estimate, while the extension of the measuring frequency band determines the achievable resolution.
In the iterative scheme it is beneficial to successively relax this smoothness constraint during the imaging process. This leads to the successively relaxed smoothness constraint regularization scheme
\[
\min_{s \in \Omega^N} \| A s - g \|^2_M, \quad N = 1, 2 \ldots N_{\text{max}}. \] (6)
Here \( N \) is the total number of space harmonics determining the dimension of the object domain \( \Omega \), and \( M \) is the number of data points corresponding to the current number of parameters.

3. Efficient Iterative Algorithm Using Hybrid Jacobian Approximations
The imaging process is at each iteration step a nonlinear least-squares problem. Because of its very fast convergence a Gauss-Newton algorithm with quadratic backtracking [4] has been selected for the solution of this problem. Fast convergence is essential, as the operator \( A \) must be evaluated numerically. The Jacobian must be calculated by the finite difference method and is computationally very expensive. On the other hand a secant approximation [4] of the Jacobian is beneficial, as it does not require any additional evaluations of the direct problem. However, it can only be used for a set of simultaneous equations, where the size of the Jacobian is constant for the whole iterative process. For the successively relaxed regularization the number of parameters is according to (6) increasing. We therefore suggest a hybrid Jacobian approximation. For the
nonlinear least squares problem (5) we have M equations and N parameters. Using a standard secant approximation we can estimate the Jacobian for the next iteration step \( \tilde{J} \), which is an M by N matrix. As the mapping of the current object is known, the evaluation of derivates for the new parameter \( D_{N+1} \) using the finite difference approximation requires only one additional evaluation of the direct problem. The secant approximation and the finite difference approximation vector can now be combined to form a hybrid Jacobian approximation

\[
\hat{J} = [\tilde{J} \ J],
\]

when the number of data points remains unchanged. The number of data points is therefore kept constant for a number of subsequent iterations while the number of parameters is constantly increasing. When a new set of data points is included into the optimization, the full Jacobian is calculated using finite differences. As this must be done after a number of secant updates to avoid inaccurate approximations, no additional computing time is required using this approach.

4. Experimental Results

To evaluate the stability and accuracy of the imaging system several numerical examples and measurements using an automated tomography system [5] have been carried out. Objects having a dielectric contrast of up to 400% and conductivities up to 0.4 S/m could be successfully imaged with RMS errors smaller than 10% in most cases. Figures 2 and 3 show the image of a staircase profile.

![Image of a staircase profile](image)

Fig. 1. Actual real part of the permittivity.

Fig. 2. Image of the object in Fig. 1 using 2.5GHz to 9.5GHz measuring range.

5. Conclusion

The iterative microwave imaging algorithm proposed in this article is capable of quantitatively imaging high contrast lossy objects. This is achieved by the application of an efficient regularization scheme and a fast and stable iterative algorithm. The approach of using cavities allows convenient collection of highly accurate multifrequency scattering data enabling easy adoption to practical applications. It is stable under all tested conditions so far and achieves accuracies by far superior to other systems. It is computationally more expensive than other methods due to the exact solution of the direct problem using a FDTD solver.

REFERENCES


